Nonlocal Models of Cosmology

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“Fundamental theorists need to provide some guidance …”

3 Simple Ideas:

- Λ isn’t small ($G\Lambda \sim 10^{-6}$)
- No scalars, no fine tuning
- Quantum IR from $\mathcal{L} = (16\pi G)^{-1} (R-2\Lambda)\sqrt{-g}$

3 Simple Consequences:

- Λ starts inflation
- IR gravitons eventually stop it
- Long Inflation because gravity is weak

Establishment View

No problem fine tuning

- $\varphi(t_I,x)$ to make inflation start
- $V(\varphi)$ for long, & $\delta\rho/\rho$, & end with $\Lambda\sim 0$
- $\Delta\mathcal{L} = g\varphi\Psi^2$ for reheating
  - then re-tune $V_{eff}(\varphi)$ . . .

But QG inflation is nonsense

Small ≠ Zero Can Matter

Establishment view of redshifting IR gravitons

- $k > H(t)\, a(t)$ ➔ physical
- $k < H(t)\, a(t)$ ➔ pure gauge, can do nothing
- $k > H(t)\, a(t)$ ➔ physical, let's find the signal!

Big volume can beat small ρ

- Const. ρ over radius R ➔ $M \sim \rho R^3$
- $U \sim -GM^2/R \sim -G\rho^2 R^5$
- $\rho_U \sim U/R^3 \sim -G\rho^2 R^2$

Perturbative Results

- Perturb around
  - $ds^2 = -dt^2 + a^2(t)\, dx^2$ with $a(t) = e^{Ht}$
- $3[H_{eff}(t)]^2 = \Lambda + 8\pi G\rho(t)$
  - $\rho_1 \sim +\Lambda^2$
  - $\rho_2 \sim -G\Lambda^3 \ln[a(t)]$
  - $\rho_L \sim -\Lambda^2 [G\Lambda \ln(a)]^{L-1}$
- $d\rho/dt = -3H_{eff}(\rho+p)$ ➔ $p(t) \sim -\rho(t)$
- Hence $p \sim -\rho \sim \Lambda^2 f[G\Lambda \ln(a)]$

Need Phenomenological Model

- Advantages of QG Inflation
  - Natural initial conditions
  - No fine tuning
  - Unique predictions
- But tough to USE!
- Try guessing most cosmologically significant part of effective field eqns

$G_{\mu\nu} = -\Lambda g_{\mu\nu} + 8\pi G T_{\mu\nu}[g]$

- $T_{\mu\nu}[g] = p g_{\mu\nu} + (\rho + p) u_\mu u_\nu$
  - Posit p[g]
  - Infer ρ and u_μ from conservation
- Getting p[de Sitter] = Λ^2 f[GΛ ln(a)]
  - [...] must be nonlocal because

$$R_{\mu\nu\rho\sigma} = \Lambda/3\ [g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}]$$

  - Simplest is X = 1/□ R

R & $\Box \equiv (-g)^{-1/2}\partial_\mu[(-g)^{1/2}g^{\mu\nu}\partial_\nu]$

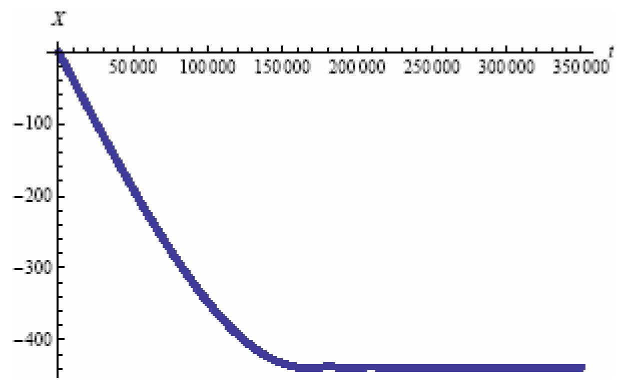
- $R = 6\ dH/dt + 12\ H^2$ for flat FRW
- $\Box f(t) = -a^{-3}\ d/dt\ [a^3\ df/dt]$
  - Hence $1/\Box\ f = -\int^t du\ a^{-3} \int^u dv\ a^3\ f(v)$
- For de Sitter $a(t) = e^{Ht}$ and $dH/dt = 0$
  - $1/\Box\ R = -\ 4\ Ht + 4/3\ [1 - e^{-3Ht}] \sim -4\ \ln(a)$

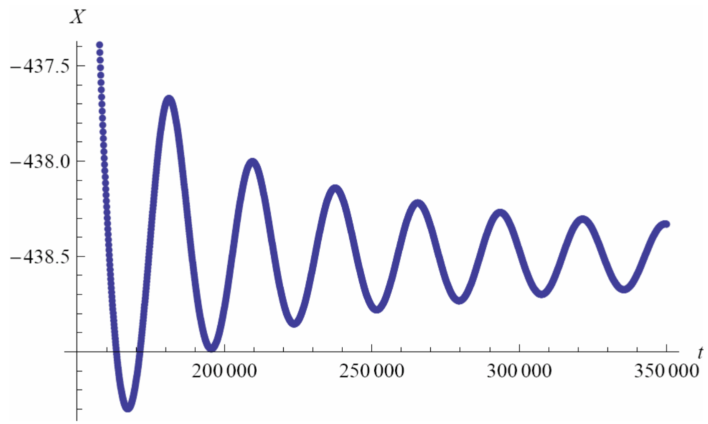
Spatially Homogeneous Case

- $G_{\mu\nu} = (p-\Lambda)g_{\mu\nu} + (\rho+p)\, u_\mu u_\nu$
  - $X = 1/\Box\, R = -\int^t du\, a^{-3} \int^u dv\, a^3\, [12H^2 + 6dH/dv]$
  - $p = \Lambda^2 f(-G\Lambda\, X)$
  - $\rho+p = a^{-3}\int^t du\, a^3\, dp/du$ and $u^\mu = \delta^\mu{}_0$
- Two Eqns
  - $3H^2 = \Lambda + 8\pi G\, \rho$
  - $-2dH/dt - 3H^2 = -\Lambda + 8\pi G\, p$ (easier)
- Parameters
  - 1 Number: $G\Lambda$ (nominally $\sim 10^{-6}$)
  - 1 Function: f(x) (needs to grow w/o bound)

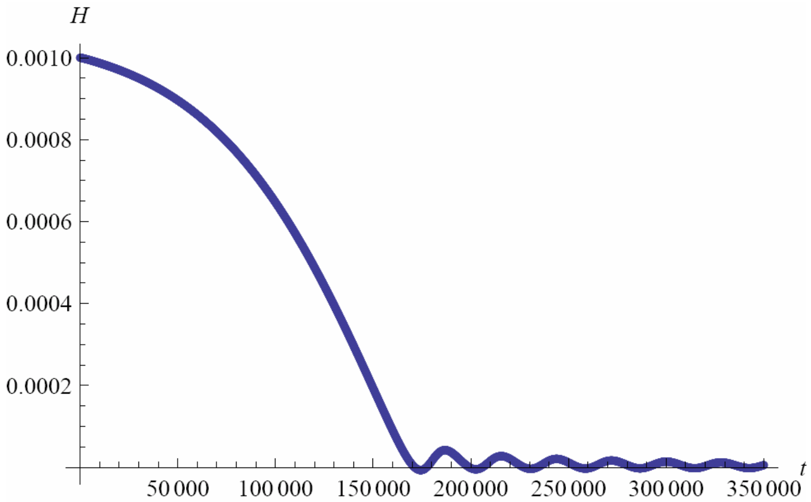
Numerical Results for $G\Lambda=1/300$ and $f(x) = e^x-1$

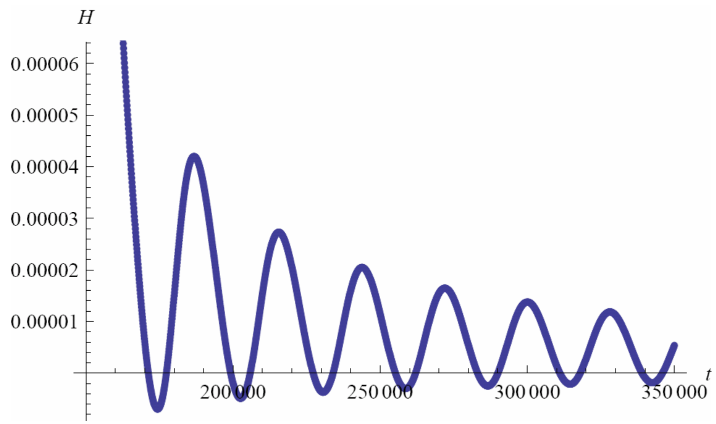
- $X= -\int^t du\, a^{-3} \int^u dv\, a^3 R$
- Criticality
  $p = \Lambda^2 f(-G\Lambda X) = \Lambda/8\pi G$
- Evolution of X(t)
  - Falls steadily to X_c
  - Then oscillates with constant period and decreasing amplitude
  - For all f(x) growing w/o bound

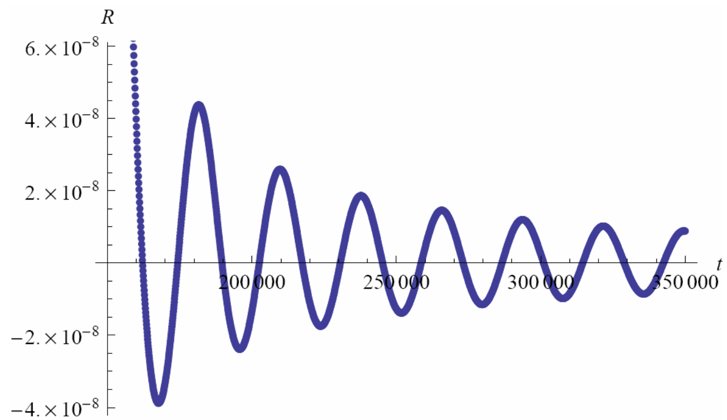




Inflation Ends, H(t) goes < 0, R(t) oscillates about 0







Analytic Treatment ($\epsilon \equiv G\Lambda$)

- $2\, dH/dt + 3 H^2 = \Lambda[1 - 8\pi\epsilon f(-\epsilon X)]$
- $X(t) = X_c + \Delta X(t)$
  - $f \approx f_c - \epsilon \Delta X f'_c$
  - $2dH/dt + 3 H^2 \approx 24\pi\epsilon^2 f'_c \Delta X$
- Use $R = 6\, dH/dt + 12 H^2$
  - L.H.S. $= R/3 - H^2$
  - $\Delta X = 1/\Box R - X_c$
- Act $\Box = -[d/dt + 3H]d/dt$ to localize
  - $[(d/dt)^2 + 2H(d/dt) + \omega^2]R \approx 0$
  - $R(t) \approx \sin(\omega t)/a(t)$
  - $\omega^2 = 24\pi\epsilon^2\Lambda f'_c$ (agrees with plots!)

Generic Expansion Histories with only $\omega^2 = 24\pi f_{cr}'(G\Lambda)^2\Lambda$

During Inflation

- $a(t) = a_{cr}\, e^{-N}$
- $H^2(t) \approx \omega^2/9\ (4N + 4/3)$

During Oscillations, with $\Delta t = t - t_{cr}$

- $a(t) \approx a_{cr}\, [1 + \omega\Delta t - \sqrt{8}\, \sin^2(\tfrac{1}{2}\, \omega\Delta t)]$
- $H(t) \approx \omega\ [1 - \sqrt{2}\, \sin(\omega\Delta t)]/[a(t)/a_{cr}]$

Perturbations Summary

Scalars exotic

- Need action to fix normalization
- Sub-horizon → redshift
- Super-hor. during infl. → approx. const.
- Super-hor. after inflation → oscillate at ω
  - Rapid reheating from n ~ ω^3 $10^{1000000}$ modes

Tensors normal for exotic a(t)

- Oscillations too late for Δ_h^2
- Leave bump at $f_{now} \sim 10^{10}$ Hz

After Inflation

- Model driven by $X = 1/\Box\, R$
  - Oscillations & $H < 0$ ➔ efficient reheating
  - $H = 1/2t$ ➔ $R = 6\, dH/dt + 12\, H^2 = 0$
- QG ends inflation, reheats & then turns off for most of cosmological history
  - $X(t) = -\int^t du\, a^{-3} \int^u dv\, a^3\, R \rightarrow X_c$

Two Problems at Late Times

Eventually matter dominates

- $H(t)$ goes from $1/(2t)$ to $2/(3t)$
- $R = 6dH/dt + 12H^2$ from 0 to $3/(4t^2)$
- $X = 1/\Box\, R$ from X_c to $X_c - 4/3 \ln(t/t_{eq})$

1. The Sign Problem:

   This gives further screening!

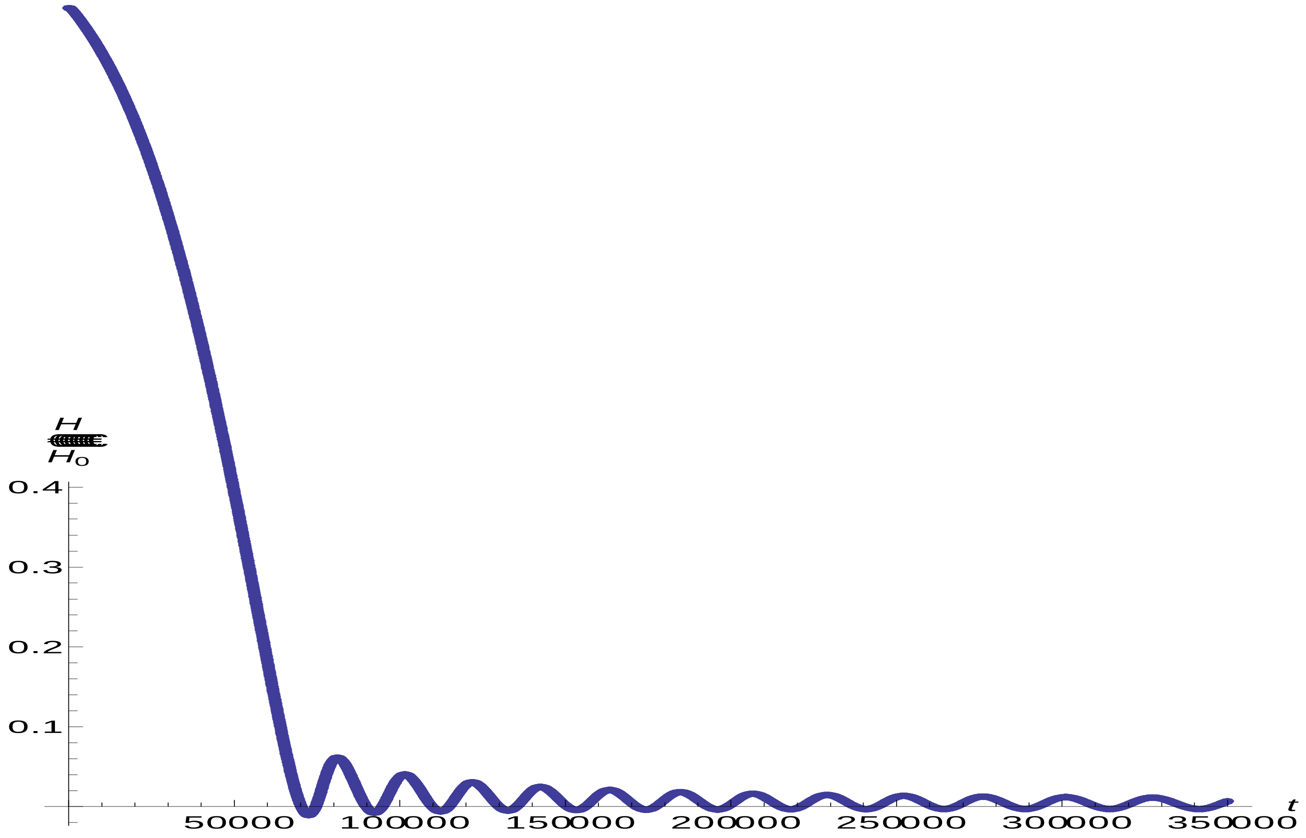
2. The Magnitude Problem:

   $p \approx -\Lambda/G\, (G\Lambda)^2 f_c'\, \Delta X \approx -10^{86}\, p_0 \times f_c'\, \Delta X$

Magnitude Problem: Too many Λ's

- $p = \Lambda^2 f(-G\Lambda\, 1/\Box\, R)$
  - Dangerous changing initial Λ^2
  - But can do $-G\Lambda\, 1/\Box[R] \rightarrow -G/\Box[\text{“}\Lambda\text{”}R]$
- Properties of "Λ"
  - Approximately Λ during inflation
  - Approx. R by onset of matter domination
  - No change to initial value problem
  - Invariant functional of metric
- Many choices but "Λ" = R(t/10) works
  - Can specify invariantly

Same as before with "Λ" = ¼ R(t/10)



Sign Problem: $R(t) > 0$

- $p = \Lambda^2 f(-G/\square[\text{“}\Lambda\text{”} R])$
- Need to add term to “Λ” R inside [ ]
  - Nearly zero during inflation & radiation
  - Comparable to R^2 after matter
  - Opposite sign
- Many choices but $-R_{00} \rightarrow -3qH^2$ works
  - $-R_{00} \approx +\Lambda$ during inflation
  - $-R_{00} \approx -2/(3t^4)$ during matter domination

Why Late Acceleration from $p_{tot} \approx -\omega^2 \Delta X/(24\pi G)$

Why Acceleration with w = -1?

- $\Delta X = 1/\Box\ [R \times -R_{00}/\Lambda]$
- $1/\Box = -\int dt'\ a^{-3} \int dt''\ a^3$
- $R \times -R_{00} \sim -1/t''^4$
- Hence ΔX dominated by const. lower limit

Why Late?

- $\Delta X \sim (H_m)^2/\Lambda$
- $\Lambda_{now} \sim (H_m)^2 \omega^2/\Lambda << (H_m)^2$

Conclusions

- Advantages of QG Inflation
  1. Based on fundamental IR theory ➔ GR
  2. Λ not unreasonably small!
  3. Λ starts inflation naturally
  4. QG back-reaction stops

     Simple idea: Grav. Int. E. grows faster than V
  5. 1 free parameter: Λ
- But tough to use ➔ Phenom. Model

$T_{\mu\nu}[g] = p\, g_{\mu\nu} + (\rho + p)\, u_\mu u_\nu$

- Guess $p[g] = \Lambda^2 f(-G\Lambda X)$
  - $X_1 = 1/\Box R$
  - Infer ρ and u_i from conservation
- Homogeneous evolution: (generic f)
  - X falls to make p cancel $-\Lambda/8\pi G$
  - Then oscillates with const. period & decreasing amp.
- Reheats to radiation dom. (R=0)
  - Matter dom. ➔ $R \neq 0$
  - $X_2 = 1/\Box\, [R \times -R_{00}/\Lambda]$ can give late acceleration
- Perturbations
  - No change to cosmo. tensors, bump at $f \sim 10^{10}$ Hz
  - Scalar norm. not predicted but ok time dependence