Gauss-Bonnet Perturbations in Co-Dimension-2 Brane Worlds

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Outline

- codimension-2 defects
- codimension-2 brane-worlds
- Solutions
- Stability

codimension-2 defects

- Point sources of mass m in 3d

$$dl^2 = \gamma_{ij}(r)dx^i dx^j = d\rho^2 + a^2\rho^2 d\theta^2$$

$$a = 1 - 4Gm$$

Point mass
as δ source

Deser,Jackiw,t'Hooft (1984)

- Cosmic Strings

$$T_{\mu}^{\nu} = \mu\delta\left(x\right)\delta\left(y\right)\left(1,0,0,1\right)$$

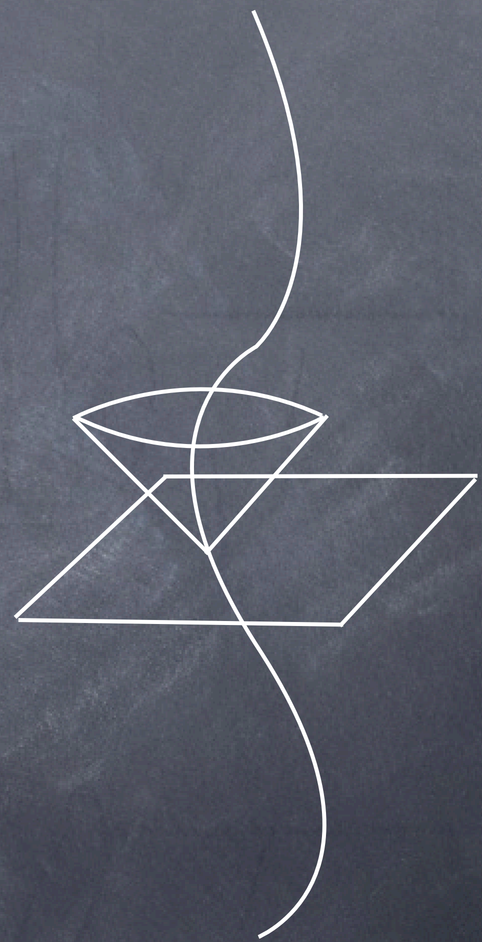
- in cylindrical coordinates the metric is

$$ds^2 = -dt^2 + dz^2 + dr^2 + \left(1 - 4G\mu\right)r^2 d\theta^2$$

$$\theta^{'} = \left(1 - 4G\mu\right)\theta$$

$$ds^2 = -dt^2 + dz^2 + dr^2 + r^2 d\theta^{'2}$$

$$0 \le \theta^{'} \le 2\pi\left(1 - 4Gm\right)$$



Vilenkin (1981)

codimension-2 brane-worlds

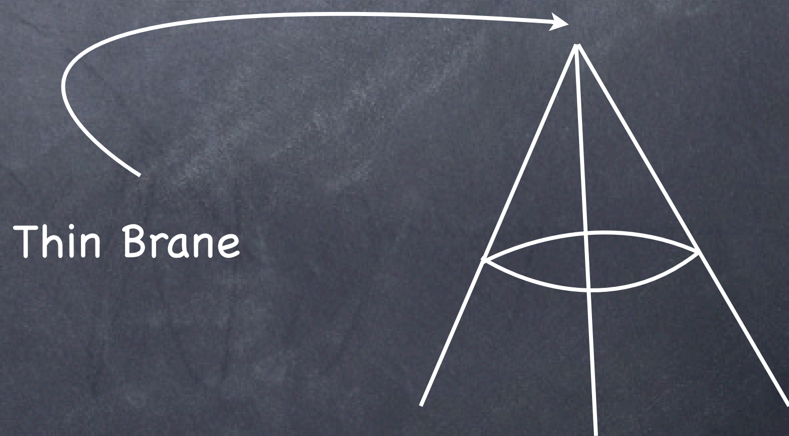
- $$ds^2 = \omega^2(r)\eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + \rho^2(r)d\theta^2$$
  $$\Delta\theta = 2\pi(\rho^{'}(0) - 1) = \frac{T_4}{M_6^4}$$

Chen,Luty,Ponton (2000)

- Any matter on the brane other than tension causes singularities more severe than conical

Cline et.al. (2003)

- Thick Branes
- Curvature Terms



Introduction of a Gauss-Bonnet Term

- $ds^2 = g_{\mu\nu}(x,t)dx^\mu dx^\nu - dr^2 - L^2(x,r)d\theta^2$ $\quad L(x,r) = \beta(x)r + O(r^2)$
- $M_*^4(G_{MN} + H_{MN}) = T_{MN} + S_{MN}$ $\quad T_{MN} = \begin{pmatrix} \widehat{T}_{\mu\nu}\frac{\delta(r)}{2\pi L} & 0 \\ 0 & 0 \end{pmatrix}$
- Conical-anomaly $\quad K_{\mu\nu} \sim \partial_r g_{\mu\nu}|_0 = 0 \quad \beta = constant$

$$\widehat{G}_{\mu\nu} = \frac{1}{M_{Pl}^2}\widehat{T}_{\mu\nu} - \frac{1}{4a}\widehat{g}_{\mu\nu}$$

Bostock et.al. (2004)

Introduction of an Induced Gravity Term

- $$S = \frac{M_4^6}{2}\left[\int d^6x\sqrt{G}R^6 + r_c^2\int d^4x\sqrt{g}R^{(4)}\frac{\delta(r)}{2\pi L}\right] + \int d^6x\mathcal{L}_{bulk} + \int d^4x\mathcal{L}_{brane}\frac{\delta(r)}{2\pi L}$$

- $$ds^2 = g_{\mu\nu}(x,t)dx^\mu dx^\nu + dr^2 + L^2(x,r)d\theta^2$$

- $$G_\mu^{(4)\nu}|_0 = \frac{1}{r_c^2 M_6^4}T_\mu^\nu + \frac{2\pi}{r_c^2}\left(1-\beta\right)\delta_\mu^\nu$$

Papantonopoulos,Papazoglou (2005)

g-b plus induced gravity in 5d

- $$S_{\rm grav} = \frac{M_5^3}{2}\left\{\int d^5x\sqrt{-g^{(5)}}\left[R^{(5)} + \alpha\left(R^{(5)2} - 4R^{(5)}_{MN}R^{(5)MN} + R^{(5)}_{MNKL}R^{(5)MNKL}\right)\right]\right.$$
$$\left. + \; r_c^2\int d^3x\sqrt{-g^{(3)}}\,R^{(3)}\right\} + \int d^5x\mathcal{L}_{bulk} + \int d^3x\mathcal{L}_{brane}$$

- $ds_5^2 = g_{\mu\nu}(x,\rho)dx^\mu dx^\nu + a^2(x,\rho)d\rho^2 + L^2(x,\rho)d\theta^2$

- Bulk equations

$$G^{(5)}_{MN} - \alpha H_{MN} = -\frac{\Lambda_5}{M_5^3}g_{MN}$$

$$H_M^N = \left[\frac{1}{2}g_M^N(R^{(5)\;2} - 4R^{(5)\;2}_{KL} + R^{(5)\;2}_{ABKL}) - 2R^{(5)}R^{(5)N}_M\right.$$
$$\left. +4R^{(5)}_{MP}R^{NP}_{(5)} \quad +4R^{(5)\;\;N}_{KMP}R^{KP}_{(5)} - 2R^{(5)}_{MKLP}R^{NKLP}_{(5)}\right]$$

BTZ-Like String Solutions

- $$ds^2 = cosh^2\left(\frac{\rho}{2\sqrt{\alpha}}\right)\left[-\left(-M+\frac{r^2}{l^2}\right)dt^2+\left(-M+\frac{r^2}{l^2}\right)^{-1}dr^2+r^2d\phi^2\right]$$
$$+d\rho^2+\left(2\,\beta\,\sqrt{\alpha}\,sinh\left(\frac{\rho}{2\sqrt{\alpha}}\right)\right)^2 d\theta^2$$
$$\Lambda_5 = -\frac{3}{4\,\alpha} \qquad l^2 = 4\alpha$$

- $$ds^2 = cosh^2\left(\frac{\rho}{2\sqrt{\alpha}}\right)\left[-\left(-M+\frac{r^2}{l^2}-\frac{\zeta}{r}\right)dt^2+\left(-M+\frac{r^2}{l^2}-\frac{\zeta}{r}\right)^{-1}dr^2+r^2d\phi^2\right]$$
$$+d\rho^2+\left(2\,\beta\,\sqrt{\alpha}\,sinh\left(\frac{\rho}{2\sqrt{\alpha}}\right)\right)^2 d\theta^2$$
$$\Lambda_5 = -\frac{1}{4\,\alpha} \qquad l^2 = 12\alpha$$

B.Cuardos-Melgar,E.Papantonopoulos,M.T.,V.Zamarias (2008)

stability

- In order to check stability issues, we have to see the behavior of small deviations from the background solution

- $$g_{AB} \to g'_{AB} = g_{AB} + h_{AB}$$

- we choose the de Donger gauge, namely the traceless and transverse gauge conditions

$$g^{MN}\, h_{MN} = 0, \qquad \nabla^M\, h_{MN} = 0$$

- $$\delta R_{MN} = 0 \to \ \Box\, h_{MN} + 2\, R_{KMNL}\, h^{KL} = 0$$

- When we consider gauss-bonnet corrections things get comlicated

$$\Box\, h_{MN} + 2R_{KMNL}\, h^{KL} = \alpha\, \mathcal{B}_{MN}$$

$$\mathcal{B}_{MN} = -2\delta H_{MN} + \frac{2}{3}\, g_{MN}\, g^{KL}\delta H_{KL} - \frac{2}{3}\, g_{MN}\, H_{KL}\, h^{KL}$$
$$+ H^{K}{}_{M}\, h_{KN} + H^{K}_{N}\, h_{MK}$$

ansatz

- $$h_{AB} = e^{\Omega t} \begin{pmatrix} h_{tt}(r,\rho) & h_{tr}(r,\rho) & 0 & h_{t\rho}(r,\rho) & 0 \\ h_{tr}(r,\rho) & h_{rr}(r,\rho) & 0 & h_{r\rho}(r,\rho) & 0 \\ 0 & 0 & h_{\phi\phi}(r,\rho) & 0 & 0 \\ h_{t\rho}(r,\rho) & h_{r\rho}(r,\rho) & 0 & h_{\rho\rho}(r,\rho) & 0 \\ 0 & 0 & 0 & 0 & h_{\theta\theta}(r,\rho) \end{pmatrix}$$

- substituting the above tensor to the perturbation equation, we can see that all equations have the same pre-factor

$$\left(l^2 - 4\alpha\right)$$

- as long as we are away from this limit we can investigate the perturbation equations without any strong coupling phenomena

scalar perturbation

- $$h_{\theta\theta} = \left(2\,\beta\,\sqrt{\alpha}\,sinh\left(\frac{\rho}{2\sqrt{\alpha}}\right)\right)^2 y(\rho)\,f(r)$$

$$\frac{4\,\alpha^{3/2}\,coth\left(\frac{\rho}{2\sqrt{\alpha}}\right)}{l^2}\left(3\,cosh\left(\frac{\rho}{\sqrt{\alpha}}\right)\frac{\partial\,y(\rho)}{\partial\,\rho} + \sqrt{\alpha}\,sinh\left(\frac{\rho}{\sqrt{\alpha}}\right)\frac{\partial^2\,y(\rho)}{\partial\,\rho^2}\right) + m\,y(\rho) = 0$$

$$\left(m - \frac{8\,\alpha^2\,\Omega^2}{l^2\,M - r^2}\right) f(r) + \frac{8\,\alpha^2}{l^4\,r}\left(\left(l^2\,M - 3r^2\right)\frac{\partial\,f(r)}{\partial\,r} + r\left(l^2\,M - r^2\right)\frac{\partial^2\,f(r)}{\partial\,r^2}\right) = 0$$

- going to tortoise coordinates we can get a Schrondinger equation with a potential V

$$\frac{d^2\psi(r)}{dr^2} + \left(V(r) - \Omega^2\right)\psi(r) = 0$$

$$V(r) = \frac{M}{2l^2} + \frac{M^2}{4r^2} - \frac{3r^2}{4l^4} + \frac{l^2 mM}{8\alpha^2} - \frac{mr^2}{8\alpha^2}$$

$$\Omega = -\sqrt{M}\left(1 + 2n + \sqrt{1 + \frac{ml^2}{8\alpha^2}}\right)$$

conclusions

- curvature terms can reproduce Einstein gravity on the brane
- Interesting string-like solutions
- Perturbations analysis reveals strong coupling problem at a certain limit
- Ignoring this limit at least scalar perturbations are stable