Lensing in an elastically strained space-time

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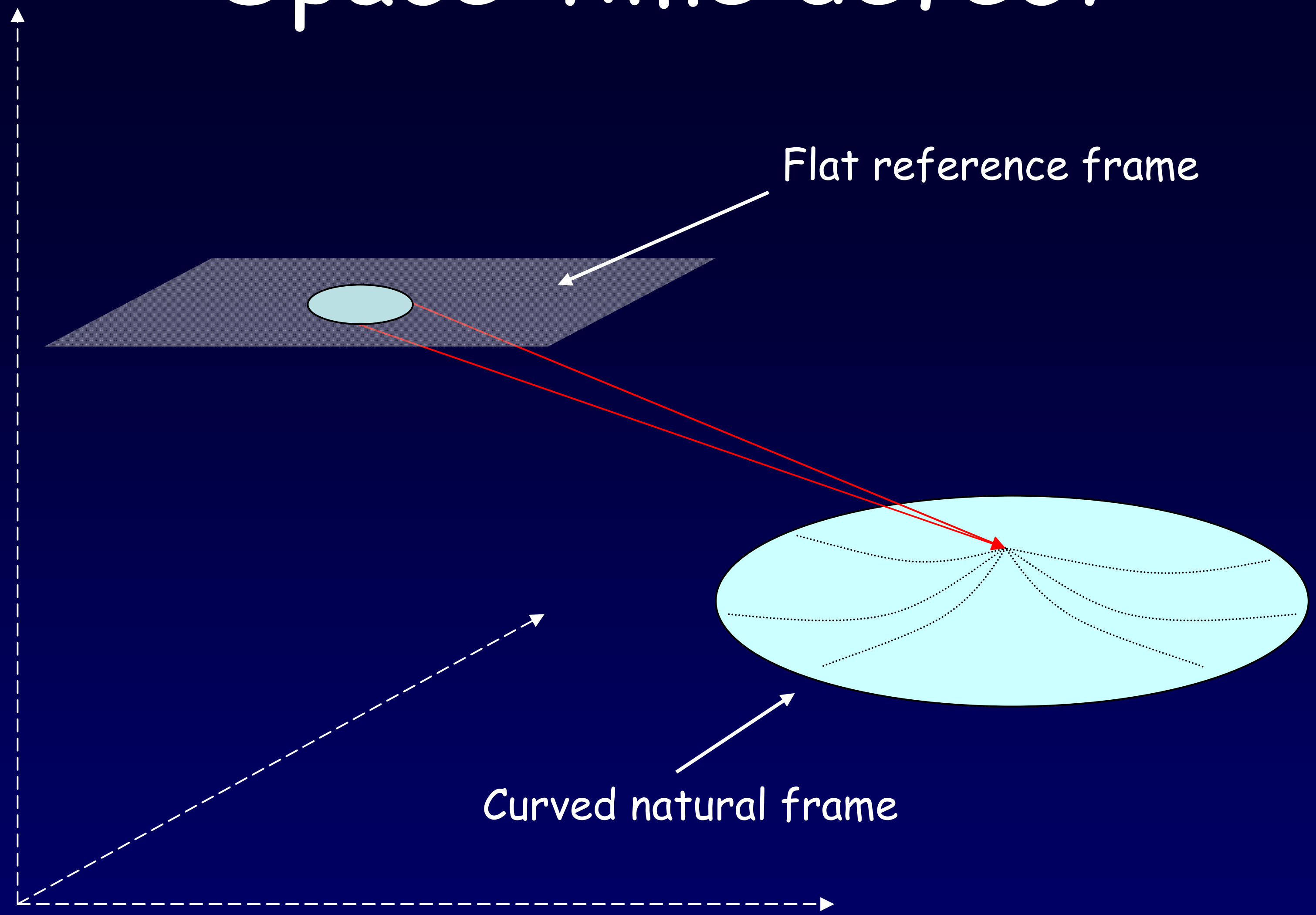
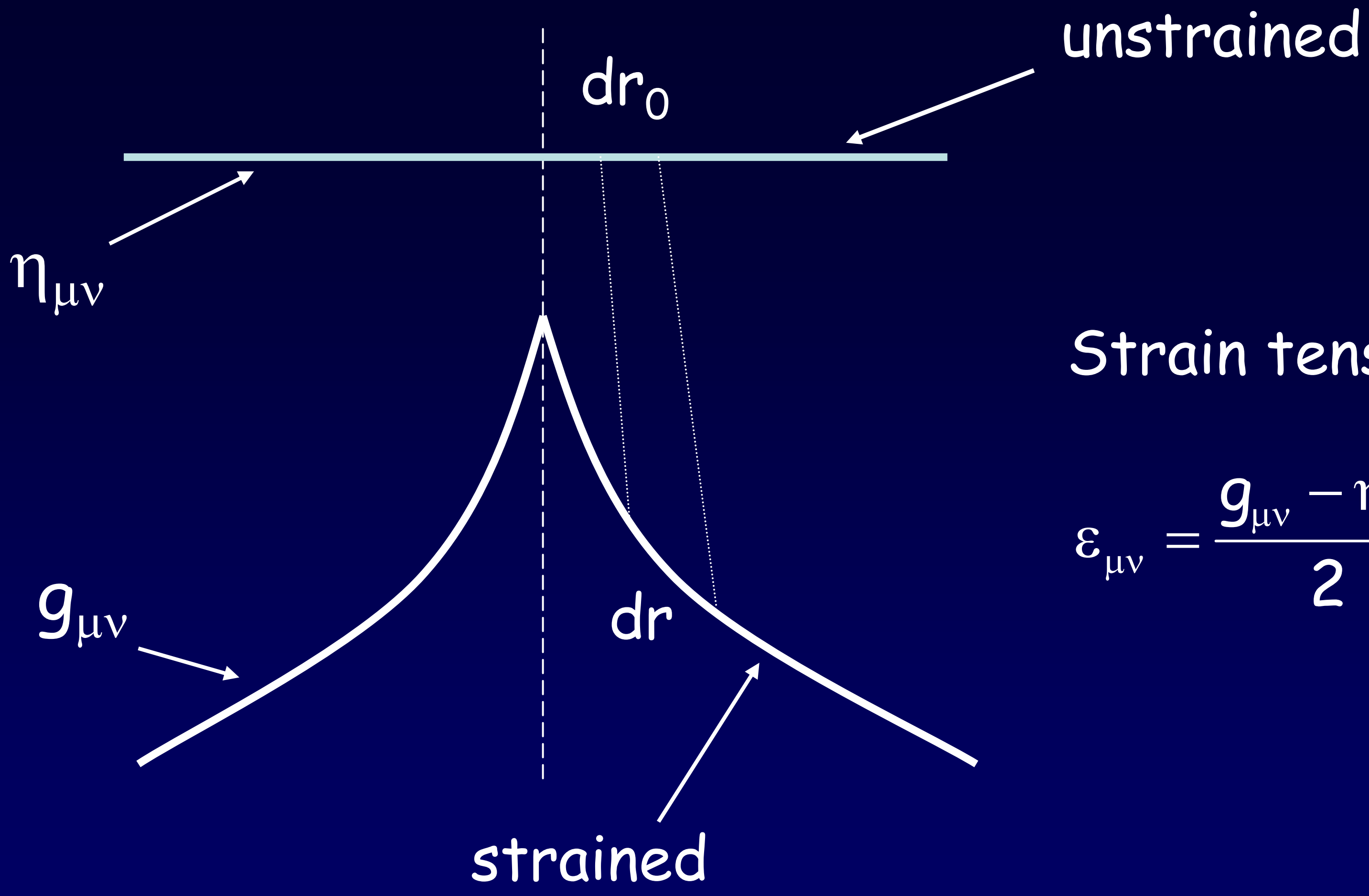
Space-time defect



Strained space-time



Strain tensor

$$\varepsilon_{\mu\nu} = \frac{g_{\mu\nu} - \eta_{\mu\nu}}{2}$$

The Lagrangian density

$$S = \int \left[R + \frac{1}{2}\left(\lambda \varepsilon^2 + 2\mu \varepsilon_{\mu\nu}\varepsilon^{\mu\nu}\right) + \kappa \mathcal{L}_{\text{matter}} \right] \sqrt{-g}\, d^4x$$

"Kinetic" term

Potential term

Geometry

A Robertson-Walker symmetry

$$H=\frac{\dot{a}}{a}=c\sqrt{\frac{B}{16}}\left\{3\left(1-\frac{(1+z)^2}{a_0{}^2}\right)^2+\frac{8\kappa}{3B}(1+z)^3\left[\rho_{m0}+\rho_{r0}(1+z)\right]\right\}^{1/2}$$

$$\kappa=\frac{16\pi}{c^2}G \qquad B=\frac{\mu}{4}\frac{2\lambda+\mu}{2\mu+\lambda}$$

A. Tartaglia and N. Radicella, *CQG*, **27**, 035001 (2010)

Good cosmic performance

- Reproduces well the accelerated expansion (dimming of type Ia supernovae)
- Consistent with the primordial nucleosynthesis (correct proportion between He, D and hydrogen)
- Consistent with structure formation after the recombination era.

Optimal value of the parameters

$$B = (2.28 \pm 0.08) \times 10^{-52}\ m^{-2}$$

$$\rho_{m0} = (2.45 \pm 0.15) \times 10^{-27}\ kg/m^{3}$$

$$B_{a_0}{}^{-1} = (0.012 \pm 0.06) \times 10^{52}\ m^{2}$$

$$B_{a_0} = \frac{8}{9}\kappa \rho_{r0} a_0{}^{4}$$

Schwarzschild symmetry

Natural frame

$$ds^2 = f^2 d\tau^2 - h^2 dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

Reference frame (Minkowski)

$$ds^2 = d\tau^2 - \left(\frac{dw}{dr}\right)^2 dr^2 - w^2 d\theta^2 - w^2 \sin^2\theta d\phi^2$$

Gauge function

The strain tensor

$$\begin{cases} \varepsilon_{00} = \dfrac{f^2 - 1}{2} \\ \varepsilon_{rr} = \dfrac{w'^2 - h^2}{2} \\ \varepsilon_{\theta\theta} = \dfrac{w^2 - r^2}{2} \\ \varepsilon_{\phi\phi} = \dfrac{w^2 - r^2}{2} \sin^2 \theta \end{cases}$$

$$w' = \frac{dw}{dr}$$

The field equations

1)

$$-2\left(\frac{1}{h}-h-2\frac{h'}{h^2}r\right)-(2\lambda+\mu)\left(hr^2-hw^2+\frac{h}{f^2}\frac{r^2}{2}+\frac{1}{2}\frac{h}{r^2}w^4-\frac{r^2}{2h}w'^2\right)$$

$$+\frac{r^2}{4}\left(\frac{\lambda}{2}+\mu\right)\left(3\frac{h}{f^4}-\frac{w'^4}{h^3}\right)-\frac{\lambda}{2}\left(\frac{w^2}{h}w'^2-\frac{h}{f^2}w^2-\frac{r^2}{f^2h}\frac{w'^2}{2}\right)=0$$

field equations

2)

$$2\left(f-\frac{f}{h^2}-2\frac{f'}{h^2}r\right)+(2\lambda+\mu)\left(\frac{r^2}{2f}-fr^2+fw^2-\frac{1}{2}\frac{f}{r^2}w^4-\frac{fr^2}{2h^2}w'^2\right)$$
$$+\frac{r^2}{4}\left(\frac{\lambda}{2}+\mu\right)\left(3\frac{f}{h^4}w'^4-\frac{1}{f^3}\right)-\frac{\lambda}{2}\left(\frac{w^2}{f}-\frac{f}{h^2}w^2w'^2-\frac{r^2}{fh^2}\frac{w'^2}{2}\right)=0$$

field equations

3)

$$\lambda\left(\frac{r}{fh}+\frac{f'}{h}w^2-\frac{f'}{2f^2h}r^2-\frac{fh'}{h^2}w^2-\frac{h'}{2fh^2}r^2\right)w'-\lambda\left(\frac{h}{f}+\frac{f}{h}w'^2-2\frac{f}{h}ww'^2\right)$$

$$+\lambda\left(\frac{f}{h}w^2+\frac{r^2}{2fh}\right)w''+\left(\frac{\lambda}{2}+\mu\right)\left(2\frac{f}{h^3}r+\frac{f'}{h^3}r^2-3\frac{fh'}{h^4}r^2\right)w'^3-3\left(\frac{\lambda}{2}+\mu\right)\frac{f}{h^3}r^2w'^2w''$$

$$-2(2\lambda+\mu)\left(\frac{w^2}{r^2}-1\right)fhw-(2\lambda+\mu)\left(2\frac{f}{h}r+\frac{f'}{h}r^2-\frac{fh'}{h^2}r^2\right)w'-(2\lambda+\mu)\frac{f}{h}r^2w''=0$$

Weak field

$$\begin{cases} f = f_0 + f_1 \\ h = h_0 + h_1 \\ w = r(1+\chi) \end{cases}$$

$$f_0 = \frac{1}{h_0} = \sqrt{1 - 2\frac{m}{r}}$$

$$\frac{m}{r}; f_1, h_1, \chi, \lambda r^2, \mu r^2 << 1$$

Approximate solutions

$$\begin{cases} g_{00} = f^2 \cong 1 - 2\dfrac{M}{r} - \dfrac{1}{12}\lambda r^2 \\ g_{rr} = -h^2 \cong -\left(\dfrac{1}{1 - 2\dfrac{M}{r}} + \dfrac{1}{12}\lambda r^2 \right) \end{cases}$$

Approximation ranges

Balance between M/r and λr^2

- Stars: $r \sim 10^{18}$ m
- Galaxy: $r \sim 10^{22}$ m
- Black hole

at the galactic center: $r \sim 10^{20}$ m

Bending of light rays

From the general null spherical line element:

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{r^4}{b^2 h^2 f^2} - \frac{r^2}{h^2}$$

b is the apparent geometric size of the source

The strained space-time case

$$\phi \cong \int_{r_0}^{r} \frac{dr}{r\sqrt{\frac{r^2}{b^2}+2\frac{M}{r}-1}} - \frac{\lambda}{24}\int_{r_0}^{r} \frac{r^4}{\left(\frac{r^4}{b^2}+2rM-r^2\right)^{\frac{3}{2}}}dr$$

Bending due to strain

$r, r_0 \gg b$

$$\Delta\phi_\lambda \cong \frac{\lambda}{24}\frac{b r_0^2}{\sqrt{r^2 - r_0^2}}$$

$$\Delta\phi_M \cong 2M\frac{r_0 - r}{r_0^2}$$

$$\Delta\phi_0 \cong b\frac{r_0 - r}{r_0^2}$$

Pure strain solutions

M = 0

$$ds^2 = \left(1 - \frac{1}{12}\lambda r^2\right)d\tau^2 - \left(1 + \frac{1}{12}\lambda r^2\right)dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

$$\left.\frac{d^2 r}{d\tau^2}\right|_{dr/d\tau=0} \cong \frac{1}{12}\lambda r$$

Repulsive acceleration

Conclusion

- The strained space-time theory introduces a strain energy of vacuum depending on curvature
- The theory is consistent with BBN, structure formation and SnIa's
- The strain bends light on scales of the order of 1 Mpc
- Vacuum spherically symmetric solutions imply repulsion (voids?)