BLACK HOLE SPACETIMES WITH SELF-GRAVITATING, MASSIVE ACCRETION TORI

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Plan of Talk

- Overview of disk models without self-gravity
- Main Equations and Numerical Methods
- 3-D Nonlinear Simulations of Nonaxisymmetric Instabilities

References/Collaborators

- Stergioulas, N. (2010), in preparation
- Korobkin, O., Schnetter , E., Zink, B., Stergioulas, N., Abdikamalov, E.B., (2010) in preparation

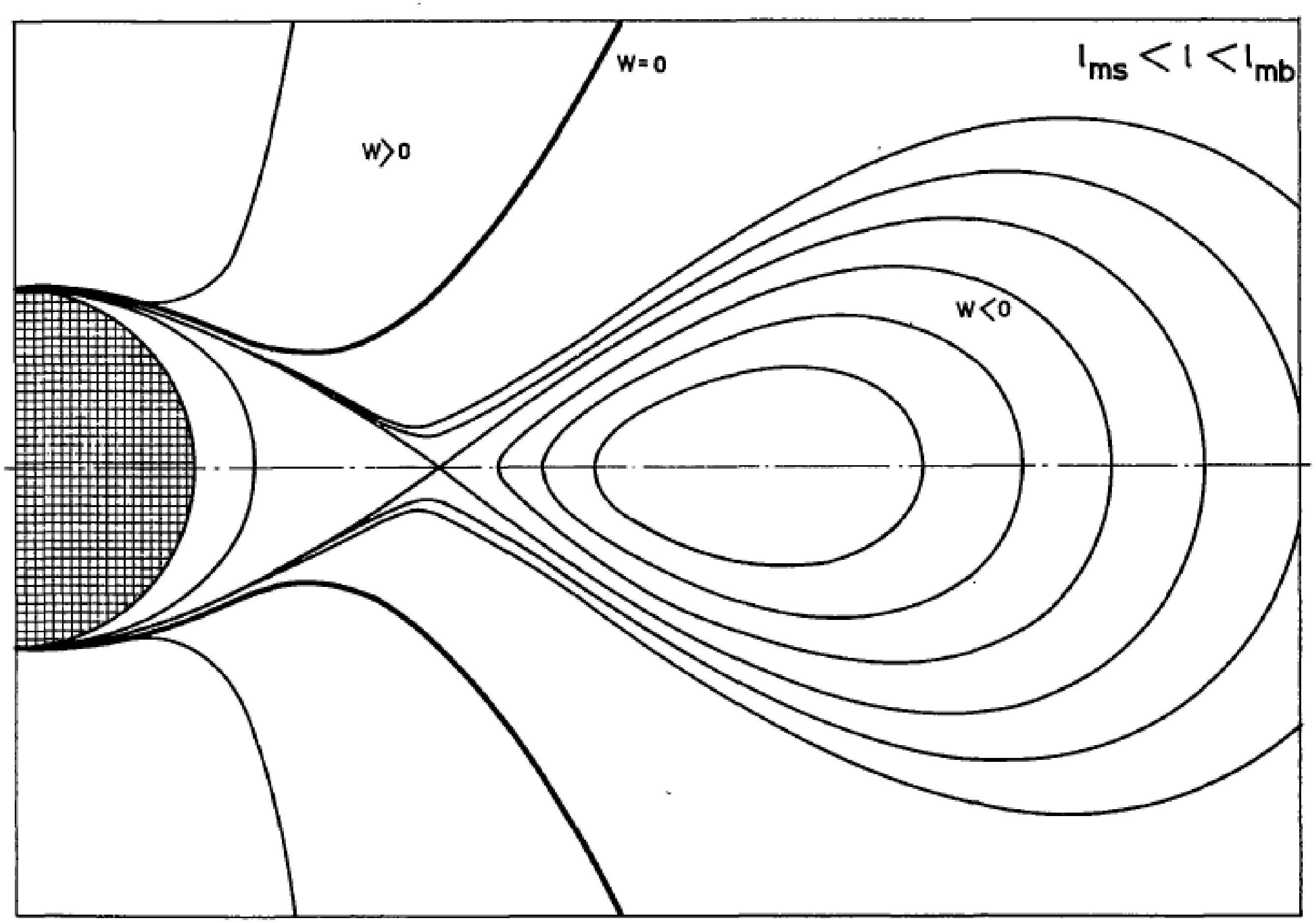
(U. of Tuebingen, Luisiana State University)

BH-Torus Models

Without self-gravity: *Abramowicz, Jaroszynski, Sikora, 1978*

AJS disks: analytic solutions with simple rotational profile, e.g.

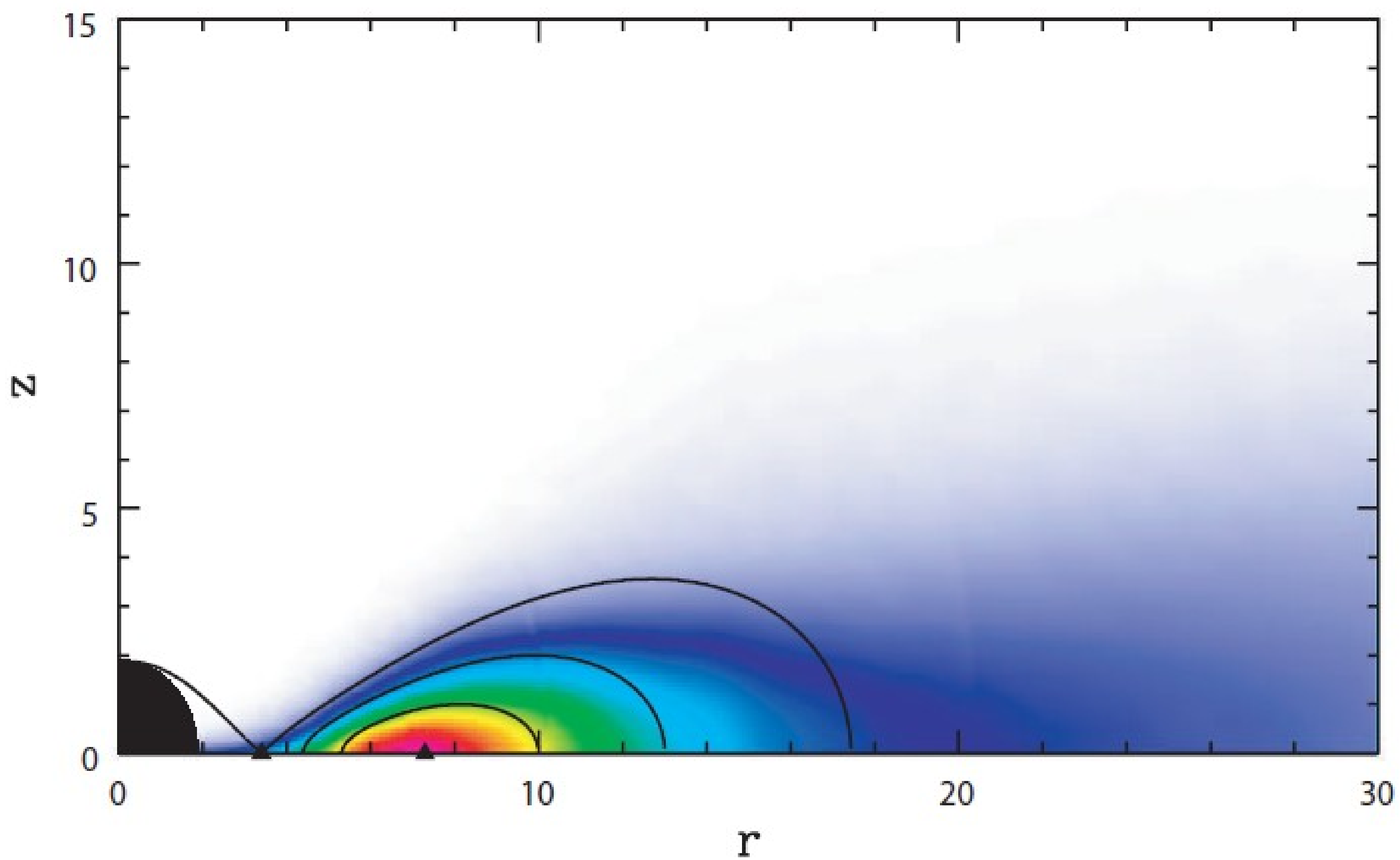
$$l = -u_\varphi / u_t = \text{const.}$$



BH-Torus Models

Without self-gravity: *(Qian et al. 2009)*

A new ansatz for *non-constant*, $l = l(r,\theta)$ distribution.
Agrees well with outcome of MHD simulations.



NONAXISYMMETRIC INSTABILITIES

Without self-gravity (Papaloizou-Pringle 1984) :

P-modes due to *corotation resonance* near the density maximum
becomes weaker as self-gravity becomes more important

With self-gravity: two more modes (Goodman & Narayan 1988)

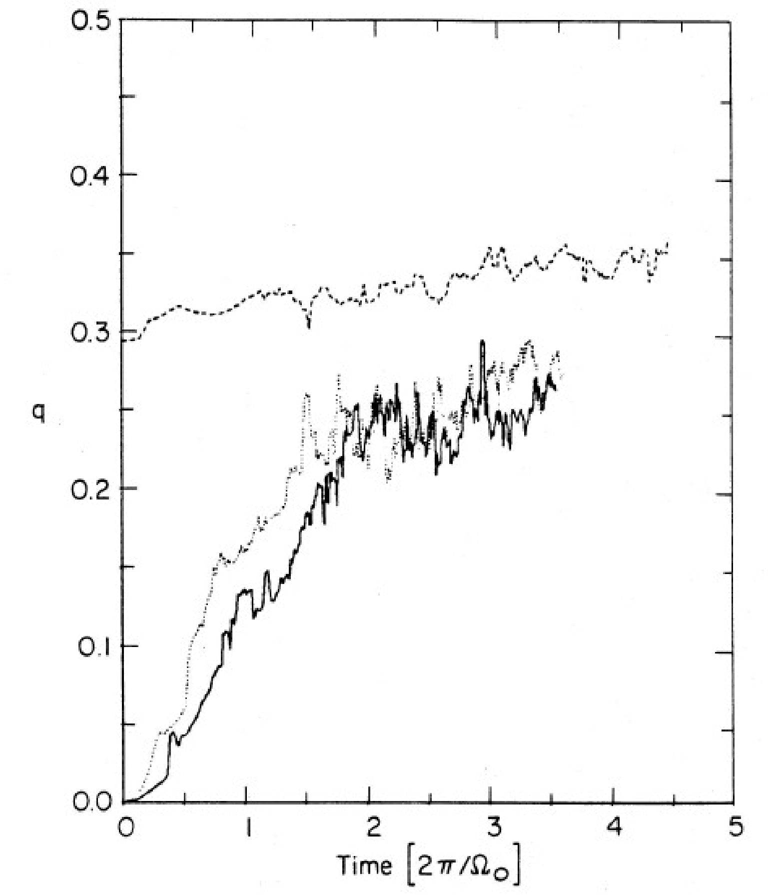
I-modes: elliptic-type deformation
need *moderate degrees* of self-gravity to appear

J-modes: essentially the Jeans instability when self-gravity is *dominant*

EFFECT OF INSTABILITIES

P-mode instability may cause a *dramatic redistribution of angular momentum* within a few dynamical timescales (Zurek & Benz)

An initial l=const. disk approaches a profile that scales as $l \sim r^{0.25}$.



INITIAL DATA

Stationary, axisymmetric models (Nishida and Eriguchi, 1994).

Our method improves by using a compactified radial grid.

Metric in *quasi-isotropic coordinates*:

$$ds^2 = -e^{2\nu}dt^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2) + e^{2(\gamma-\nu)} r^2 \sin^2\theta (d\phi - \omega dt)^2$$

The horizon is at a const. $r = h_0$.

Boundary conditions at horizon:

$$\begin{aligned} B \equiv e^{\gamma} &= 0 \\ e^{\nu} &= 0 \\ \omega = \omega_h &= \text{const.} \end{aligned}$$

Local flatness condition on axis of symmetry:

$$\alpha = \gamma - \nu$$

INITIAL DATA

Field equations:

$$\Delta\lambda = S_\lambda(r, \mu) \ ,$$

$$\left(\Delta + \frac{1}{2}\frac{\partial}{\partial r} - \frac{1}{r^2}\mu\frac{\partial}{\partial\mu}\right)B = S_B(r, \mu) \ ,$$

$$\left(\Delta + \frac{2}{r}\frac{\partial}{\partial r} - \frac{2}{r^2}\mu\frac{\partial}{\partial\mu}\right)\omega = S_w(r, \mu) \ ,$$

where Δ is the flat Laplacian and the *source terms* are:

$$S_\lambda = 4\pi\lambda e^{2\alpha}\left[(\epsilon + p)\frac{1 + v^2}{1 - v^2} + 2p\right] + \frac{1}{2}r^2(1 - \mu^2)B^2\lambda^{-3}\nabla\omega \cdot \nabla\omega - \nabla(\gamma - \nu) \cdot \nabla\lambda \ ,$$

$$S_B = 16\pi p B e^{2\alpha} \ ,$$

$$S_\omega = \nabla(4\nu - 3\gamma) \cdot \nabla\omega - 16\pi e^{2\alpha}(\epsilon + p)\frac{\Omega - \omega}{1 - v^2} \ ,$$

INITIAL DATA

Invert 3 elliptic-type PDES using Green's functions:

$$\lambda = 1 - \frac{h_0}{r} - \sum_{n=0}^{\infty} \int_{h_0}^{\infty} dr' \int_0^1 d\mu' \, r'^2 f_{2n}^2(r, r') P_{2n}(\mu) P_{2n}(\mu') S_\lambda(r', \mu') \, ,$$

$$r \sin\theta B = \left(1 - \frac{h_0^2}{r^2}\right) r \sin\theta - \frac{2}{\pi} \sum_{n=1}^{\infty} \int_{h_0}^{\infty} dr' \int_0^1 d\mu' \, r'^2 f_{2n-1}^1(r, r') \frac{1}{2n-1} \sin(2n-1)\theta \sin(2n-1)\theta' S_B(r', \mu') \, ,$$

$$r \sin\theta\omega = \frac{\omega_h h_0^3}{r^2} \sin\theta - \sum_{n=1}^{\infty} \int_{h_0}^{\infty} dr' \int_0^1 d\mu' \, r'^3 \sin\theta' f_{2n-1}^2(r, r') \frac{1}{2n(2n-1)} P_{2n-1}^1(\mu) P_{2n-1}^1(\mu') S_\omega(r', \mu') \, ,$$

where

$$f_n^1(r, r') = \begin{cases} \left(\frac{r'}{r}\right)^n - \frac{h_0^{2n}}{(rr')^n} \, , & \text{for } \frac{r'}{r} \le 1 \, , \\ \left(\frac{r}{r'}\right)^n - \frac{h_0^{2n}}{(rr')^n} \, , & \text{for } \frac{r'}{r} > 1 \, , \end{cases}$$

$$f_n^2(r, r') = \begin{cases} \left(\frac{1}{r}\right)\left(\frac{r'}{r}\right)^n - \frac{h_0^{2n+1}}{(rr')^{n+1}} \, , & \text{for } \frac{r'}{r} \le 1 \, , \\ \left(\frac{1}{r'}\right)\left(\frac{r}{r'}\right)^n - \frac{h_0^{2n+1}}{(rr')^{n+1}} \, , & \text{for } \frac{r'}{r} > 1 \, . \end{cases}$$

INITIAL DATA

Transform coordinates to compactified radial grid, using

$$r = r_e \frac{s}{1-s}$$

Assume polytropic EOS and $l = l_0 = \text{const.}$

Starting from an analytic AJS solution, iterate between 4 field equations and hydrostationary equilibrium equation:

$$H - H_{\text{in}} \equiv \int_0^p \frac{dp}{\epsilon + p} = \ln\left(\frac{u^t}{u^t_{\text{in}}}\right) + \ln\left(\frac{1 - l_0 \Omega}{1 - l_0 \Omega_{\text{in}}}\right)$$

where the angular velocity is defined as

$$\Omega \equiv \frac{u^\phi}{u^t} = -\frac{g_{t\phi} + l_0 g_{tt}}{g_{\phi\phi} + l_0 g_{t\phi}}$$

During the iteration, the parameters h_0/r_{out}, r_{in}/r_{out}, ω_h and $\varepsilon_{\max}$ are held fixed and define an equilibrium model for given EOS and l_0.

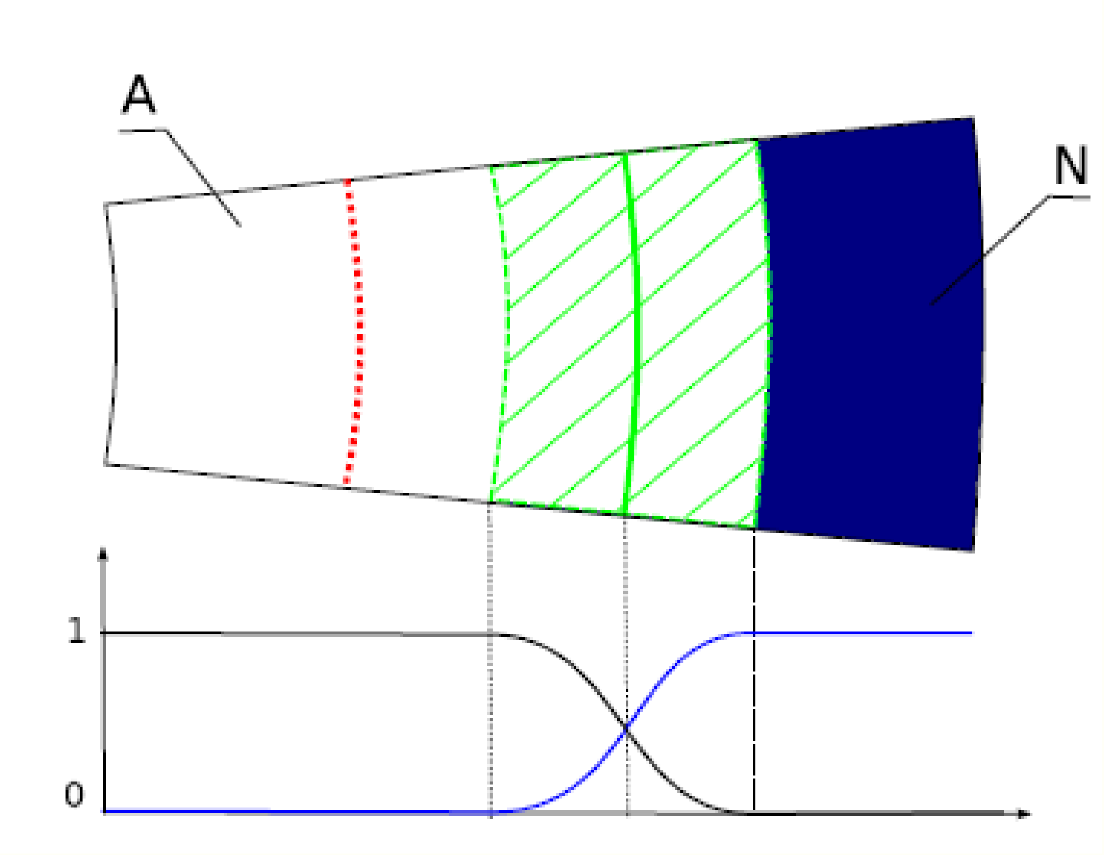
Example

Equilibrium torus with constant specific angular momentum:

	dimensionless units	astrophysical units
Γ	4/3	4/3
K	0.06	$1.1 \times 10^{14}\ \mathrm{g}^{-1/3}\mathrm{cm}^3\mathrm{s}^{-2}$
ρ_c	0.000159	$9.8 \times 10^{13}\ \mathrm{g\,cm}^{-3}$
r_c	11.2	38 km
r_{ISCO}	5.2	17 km
M_{BH}	0.866	$2\ \mathcal{M}_\odot$
$\mu := M_D/M_{BH}$	0.24	0.24
$d := \frac{r_c - r_{in}}{r_c}$	0.32	0.32
ℓ	3.9	$13\ \mathrm{km} \cdot c$
P	≈ 210	2.3 ms

METRIC BLENDING NEAR HORIZON

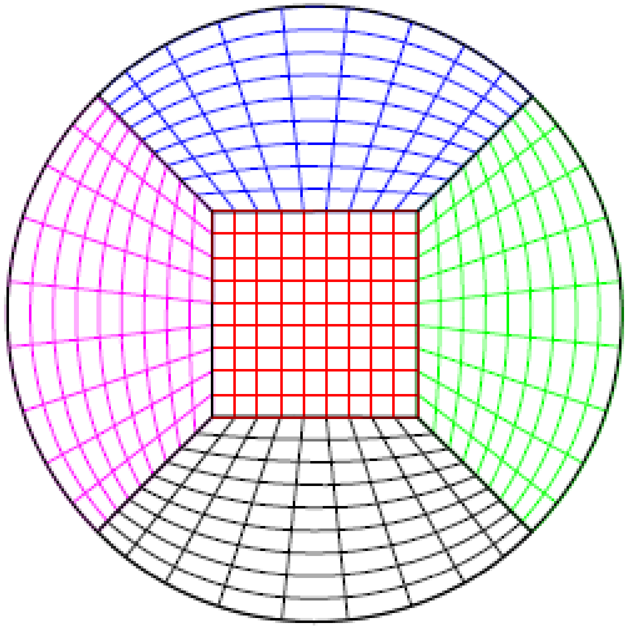
Because some source terms are numerically unstable near the horizon during the iteration process, *the solution is matched to Schwarzschild or Kerr at the innermost 5 grid points*. To smooth out the differences, the two solutions are blended smoothly in a transition zone.

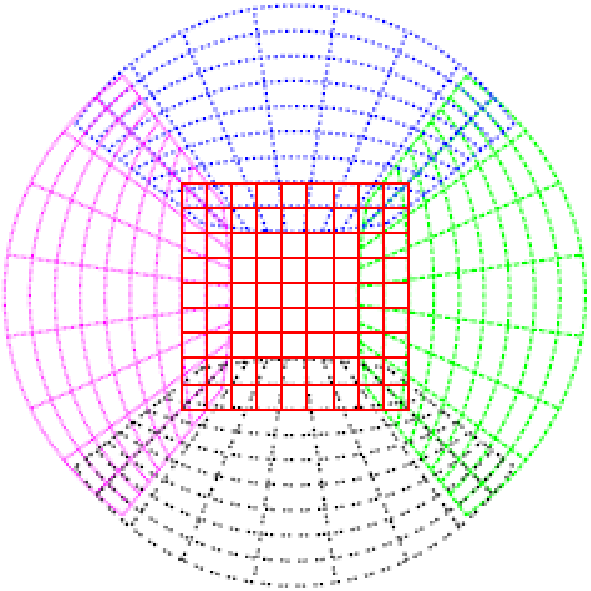


3D NUMERICAL SIMULATIONS

MultiPatch Domain Representation

- The domain is divided into blocks
- Each block comes equipped with a rectilinear grid
- Blocks share common boundary for GR
- Additional overlapping boundary zones for hydro





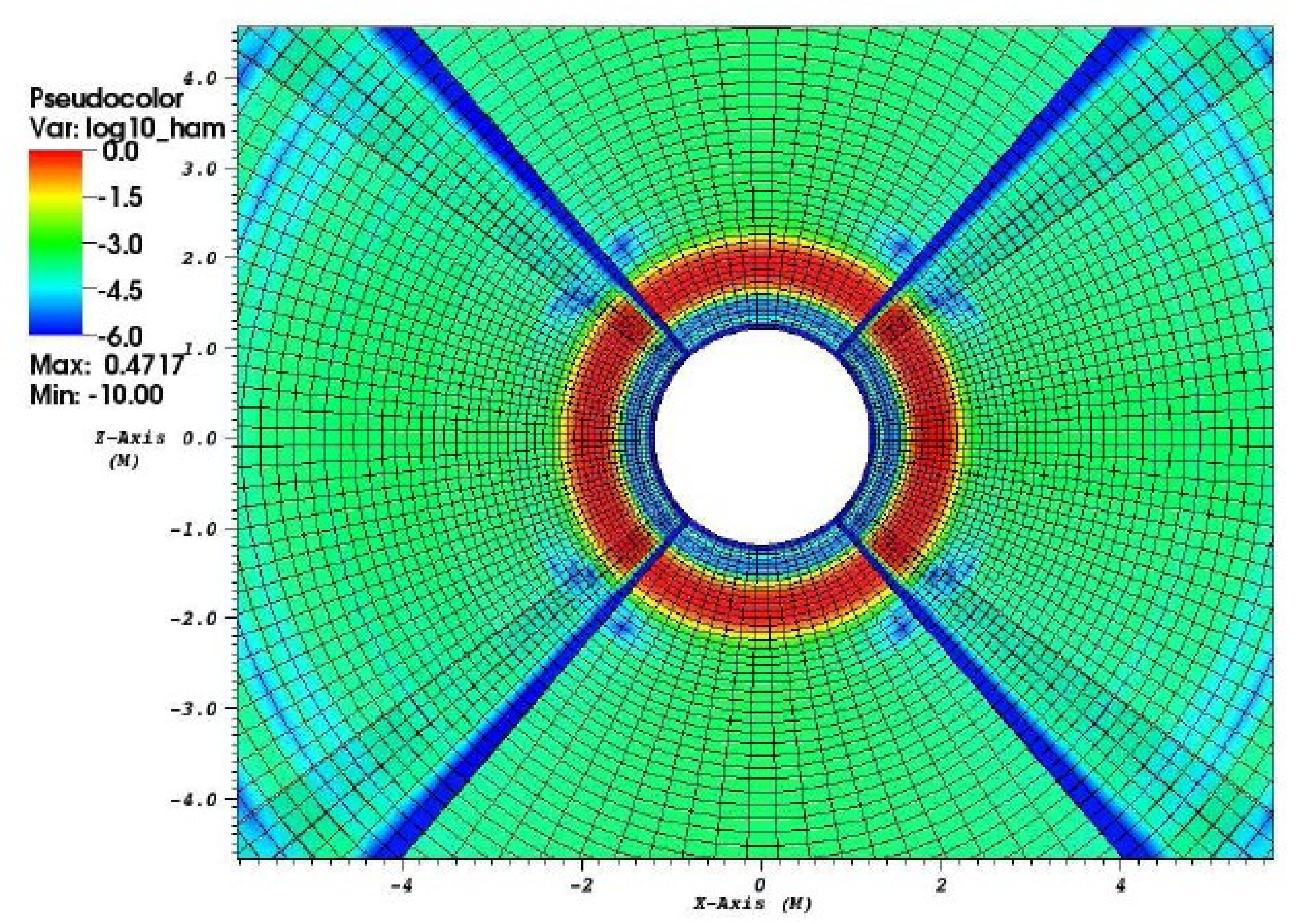
SIMULATION METHOD

The code consists of two main parts: GR ($G_{\mu\nu} = 8\pi T_{\mu\nu}$) and MHD ($\nabla_\nu T^{\mu\nu} = 0$), coupled via energy-momentum tensor $T_{\mu\nu}$.

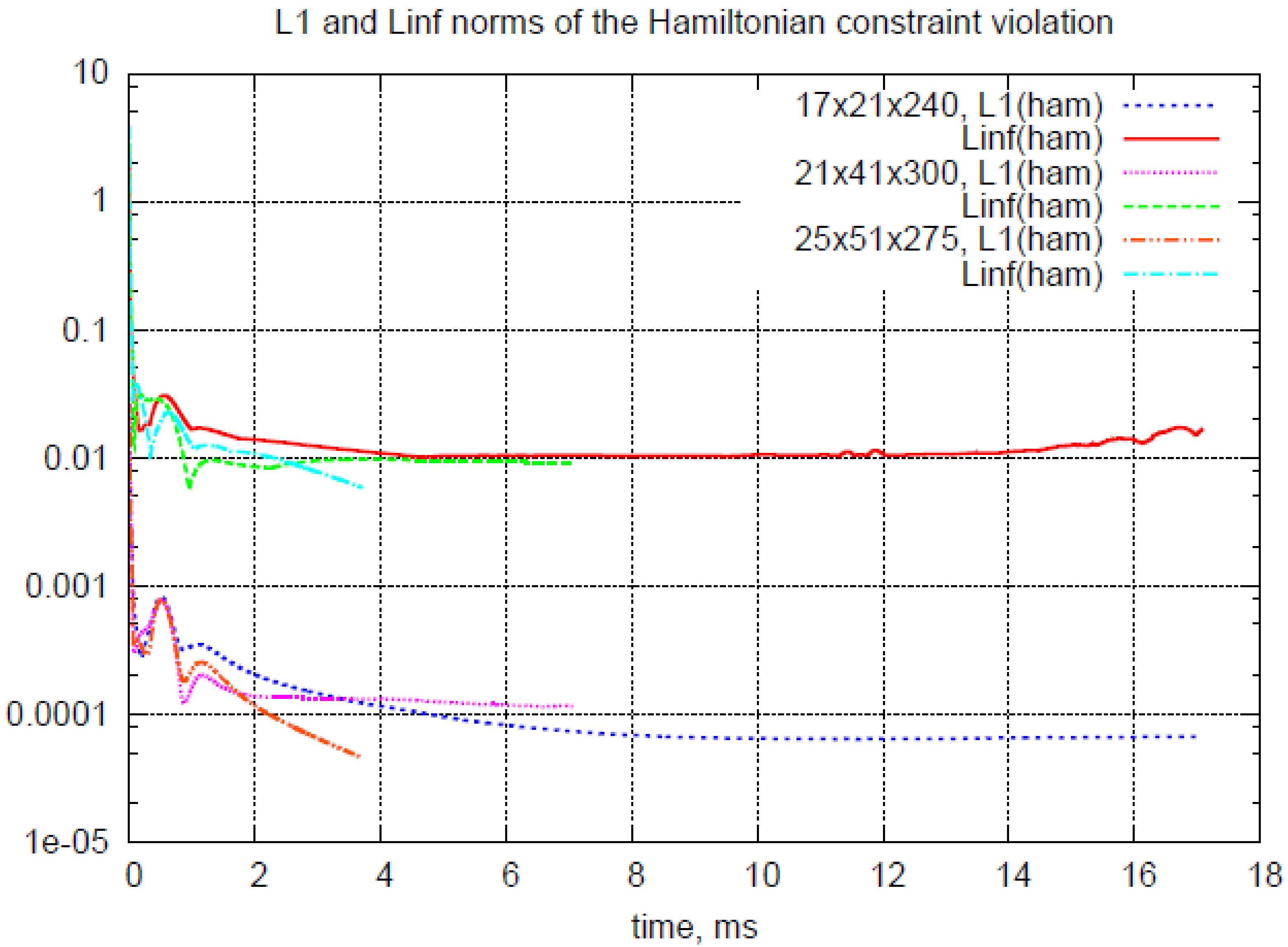
GR part of the code:

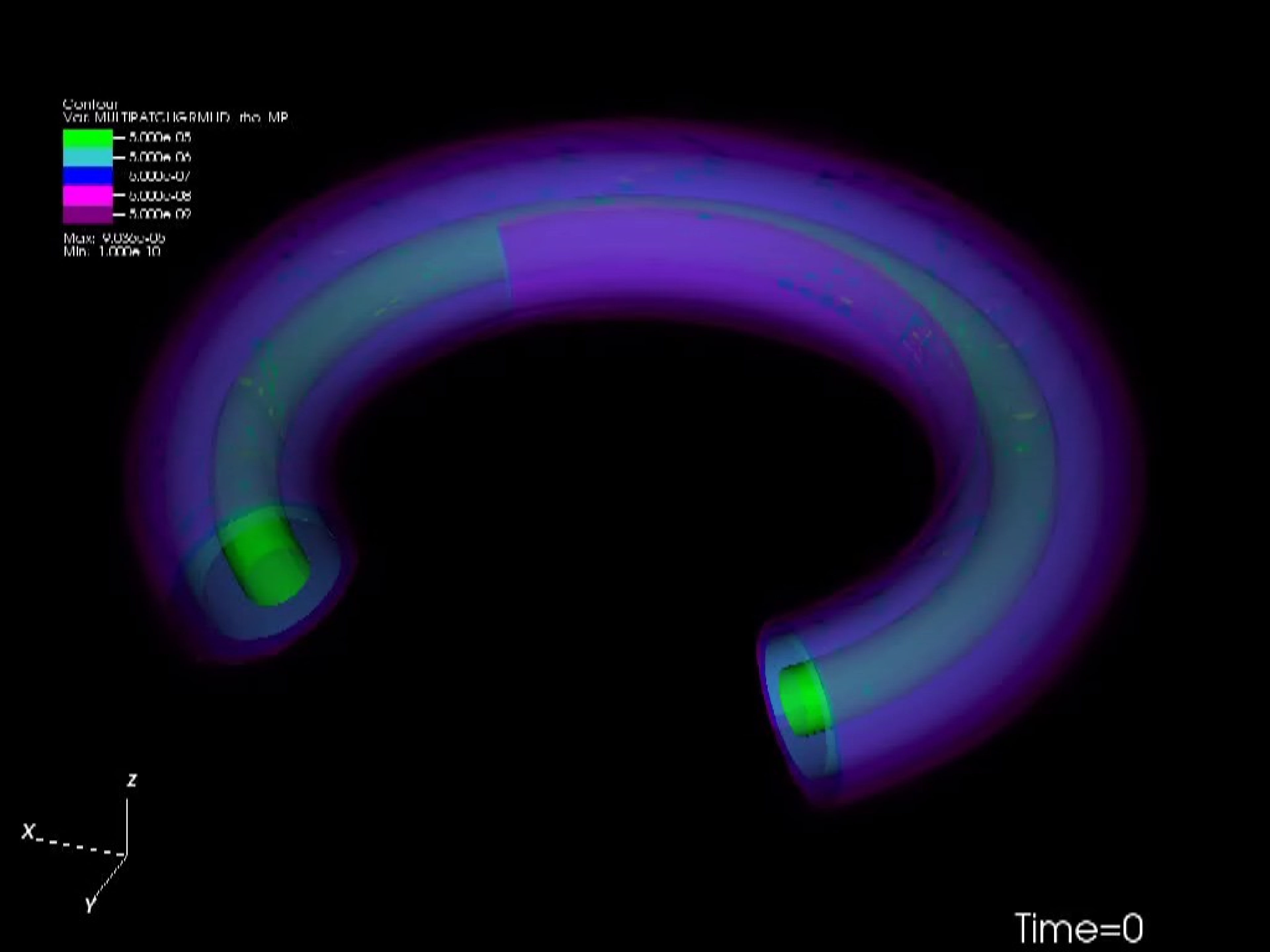
- Generalized Harmonic formulation:
  $R_{ab} = -\frac{1}{2} g^{cd} \partial_c \partial_d g_{ab} + \nabla_{(a} \Gamma_{b)} + \ldots$
- Efficient constraint damping scheme
- Harmonic gauge source functions set to constant values
- Uses finite differences and touching patches
- Penalty boundary conditions
- High-order dissipative SBP differential operators

INITIAL CONSTRAINT VIOLATIONS

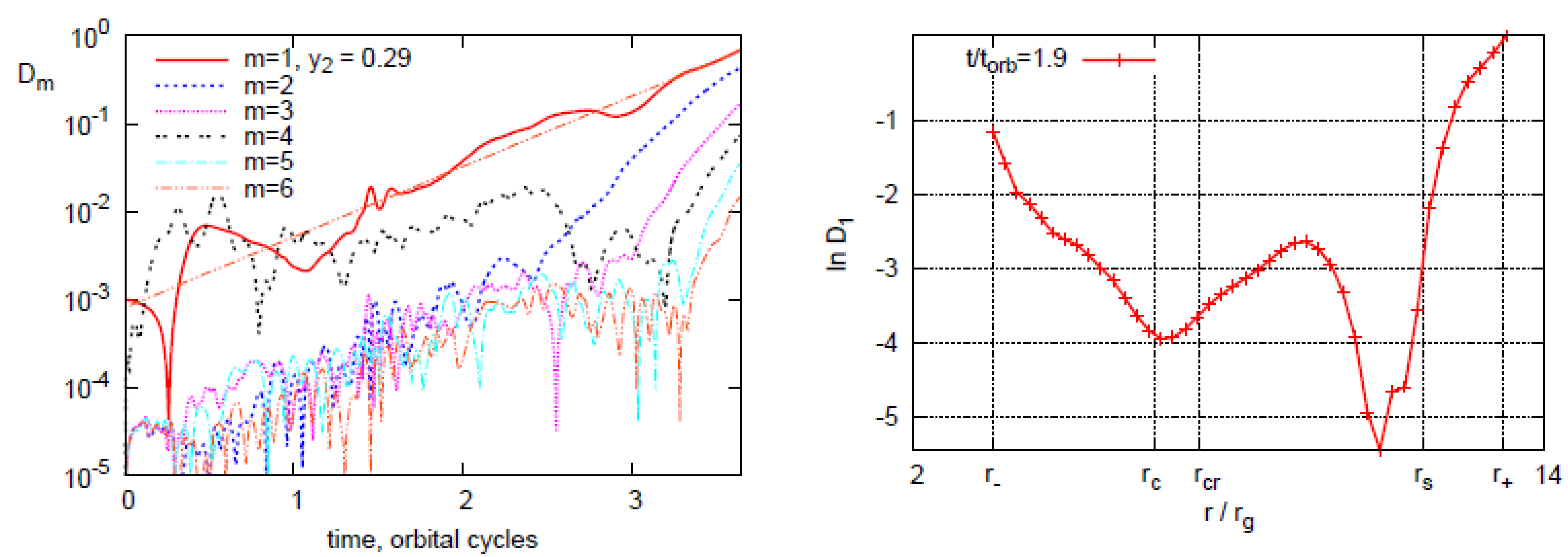


CONSTRAINT DAMPING

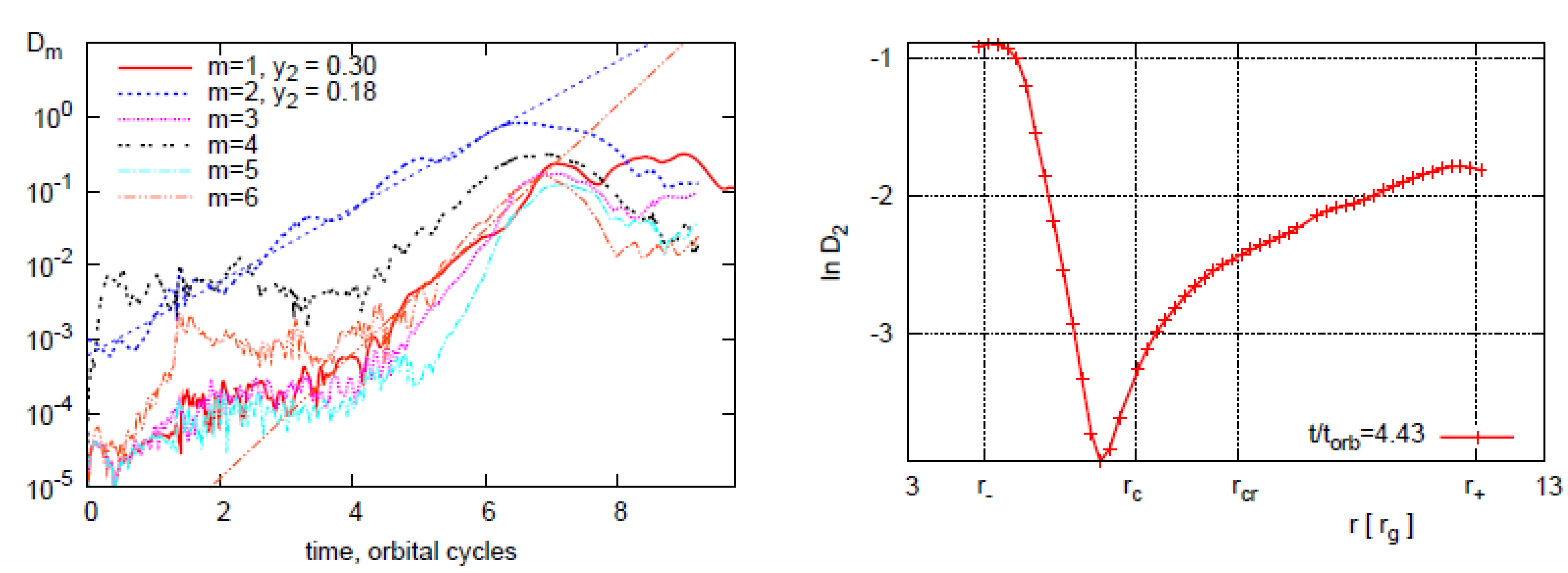



Contour
Z
X
Y
Time=0

NONAXISYMMETRIC INSTABILITIES

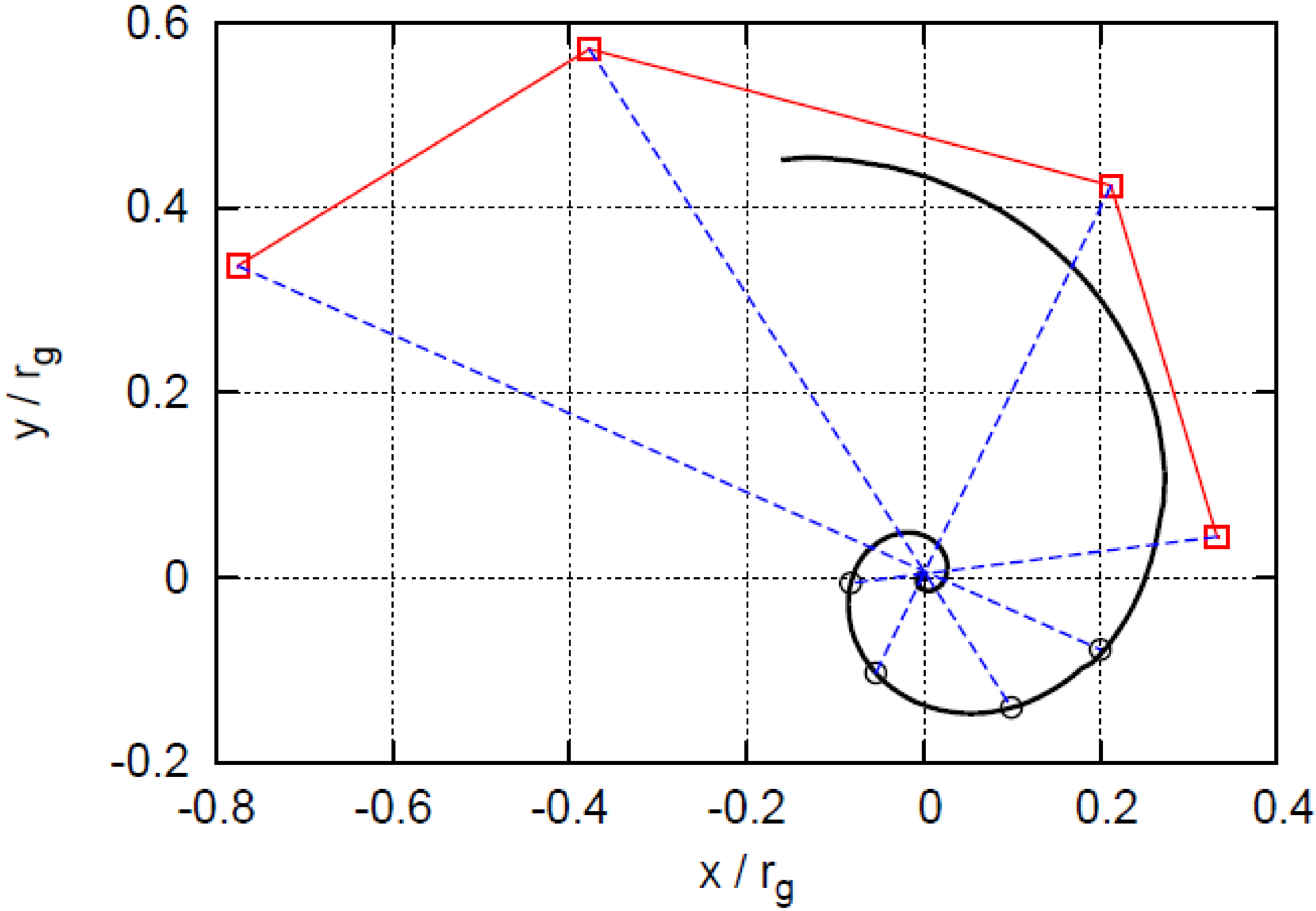


m=1
P-mode



m=2
I-mode

PRESERVATION OF CENTER OF MASS



Black: trajectory of black hole
Red: center of mass of disk

Summary:

- We constructed self-consistent models of BHs with massive accretion tori, including self-gravity.

- We evolved the BH-torus initial data with a 3D multipatch code in harmonic gauge.

- For l=const. disks we demonstrated the onset of several nonaxisymmetric instabilities.

- We demonstrated in full GR that including self-gravity converts m=2 P-modes to I-modes.

- The BH participates in the dynamics of the m=1 instability, so that the center of mass of the BH-torus system is preserved.