CONFORMAL DYNAMICAL EQUIVALENCE AND APPLICATIONS

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Recent Developments in Gravity (NEB XIV)
Ioannina, June 2010

ΣΥΜΜΟΡΦΗ ΔΥΝΑΜΙΚΗ ΙΣΟΔΥΝΑΜΙΑ ΚΑΙ ΕΦΑΡΜΟΓΕΣ

Ν.Κ ΣΠΥΡΟΥ

Νεώτερες Εξελίξεις στη Βαρύτητα (NEB XIV)
Ιωάννινα, Ιούνιος 2010

CONFORMAL DYNAMICAL EQUIVALENCE AND APPLICATIONS

Research Work since about 2000

Study Group

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NEB XIV,

Ioannina, 8-11 June 2010

BIBLIOGRAPHY

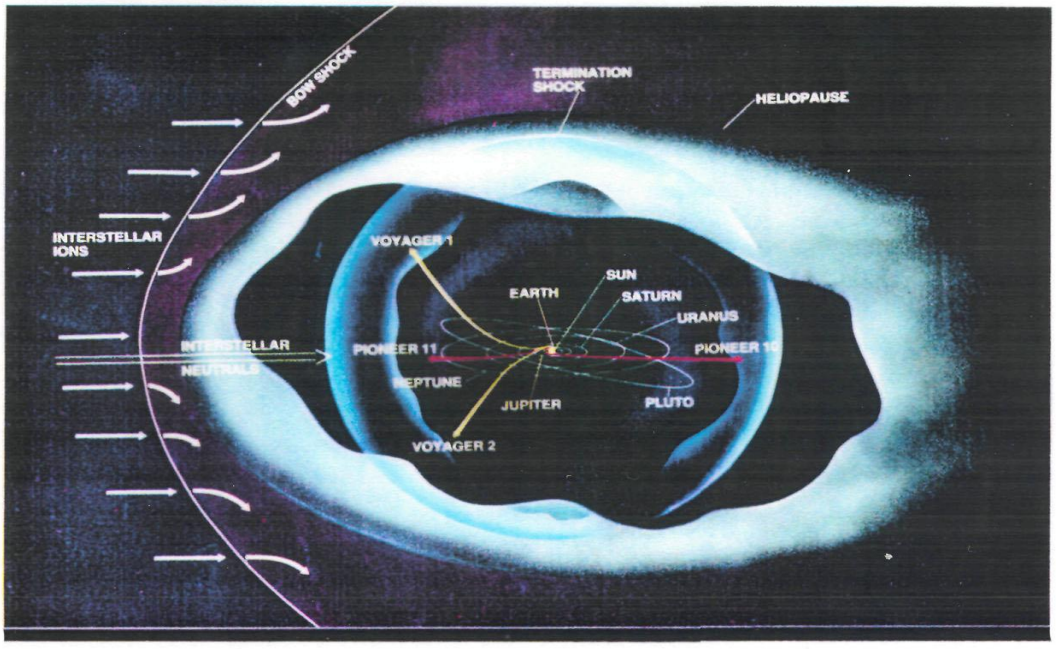
REFEREED JOURNALS

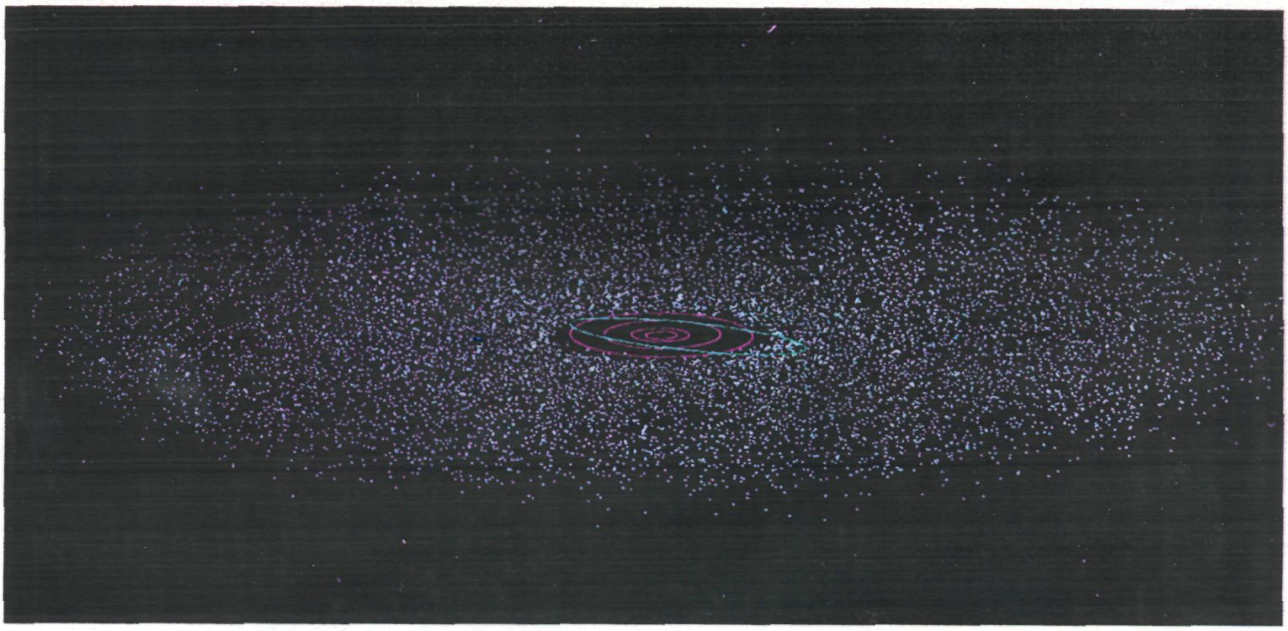
1. "Geodesic motion versus hydrodynamic flows in a gravitating perfect fluid: dynamical equivalence and consequences" (with K.Kleidis), ***Class. Quantum Grav***. 17, 2965-2982, 2000b.
2. "Covariant approach to the conformal dynamical equivalence in astrophysics" (with C.G.Tsagas), ***Class.Quantum Grav***.21,2435-2444,2004a (gr-qc/0403116).
3. "Newtonian non-linear hydrodynamics and magnetohydrodynamics" (with Ch. Tsagas*)* **Mon. Not. R. Astron. Soc**. 388, 187–196, 2008).
4. "Magnetohydrodynamic flows as Newtonian-type gravitational motions" (with C.Tsagas) to appear in the ***Central European Journal of Physics.***
5. "Collisional dark matter: A novel approach to the dark energy concept" (with K Kleidis), **in preparation (NEB-XIV).**
6. " A New Look of the Solar System" (with K. Zagkouris), **in preparation (NEB-XIV).**

PROCEEDINGS

1. Conformal Invariance and the Nature of Cosmological Structures", Invited Talk, Preceedings of The Conference on Applied Differential Geometry-General Relativity, and The Workshop on Global Analysis, Differential Geometry, Lie Groups, **Eds.G.Tsagas, C.Udriste, and D.Papadopoulos**, Thessaloniki, 27 June-1July, 2001.
2. "On the determination of the masses of cosmological structures", Invited Talk, Proceedings of the 2nd Hellenic Cosmology Workshop *Modern Theoretical nd Observational Cosmology,*Kluwer Academic Publishers, Dordrecht, **Eds.M. Plionis, and S.Cotsakis,** Vol.276,pp.35-43,2002.
3. "Conformal Invariance and the Cosmological Expansion of a Realistic Universe", In Proceedings of the International Conference Recent *Developments in Gravity-NEB X,* **Eds. K.D.Kokkotas and N.Stergioulas**, World Scientific Publishing Co. Pte.Ltd, Singapore,pp.90-96, 2003.
4. "A Classical Treatment of the Dark-Matter and Flat-Rotation-Curves Problems", In Proceedings of **the 6th Hellenic Astronomical Conference (Penteli, Athens),** September 2003.
5. "Conformal Invariance and the Cosmological Expansion of a Realistic Universe", In Proceedings of the International Conference Recent *Developments in Gravity-**NEB X**,* Eds. K.D.Kokkotas and N.Stergioulas, World Scientific Publishing Co. Pte.Ltd, Singapore,pp.90-96, 2003.
6. "A Classical Treatment of the Dark-Matter and Flat-Rotation-Curves Problems", In Proceedings of **the 6th Hellenic Astronomical Conference** (Penteli, Athens), September 2003 .
7. "A *Classical Treatment of the Problems of Dark Energy, Dark Matter, and Accelerating Expansion,* in Proceedings of the Conference *Recent Developments in Gravity **(NEB) XI** (Mytilini, Lesvos, Hellas) 2-6 June 2004.*
8. "Negative Mass and Repulsive Gravity in Newtonian Theory, and Consequences", In "Dynamics of Celestial Bodies", Proceeidings of the 2008 Interanational Conference in honour of **John Hadjidemetriou**, Litohoro, Greece**, Editors Harry Varvoglis and Zoran Knezevic**, p.p.181-184, Aristoteleion University of Thessaloniki, Greece, and Astronomical Observatory, Belgrade, Serbia (2009).

Diagram of the Solar System embedded in the solar wind: the Sun's extended atmosphere blowing out into interstellar space. Four operational spacecraft are racing out of the Solar System and have a chance of detecting the boundary between the wind from the Sun and the wind from the stars before their power fails: *Pioneers 10* and *11*, shown as red arrows, and *Voyagers 1* and *2*, shown as yellow arrows. The *Voyagers* are traveling faster and will retain transmitter power further into the future. From *EOS Transactions*, April 19, 1994. Courtesy American Geophysical Union.





Schematic diagram of the Kuiper Comet Belt: Millions of small, icy worlds are thought to orbit the Sun just beyond Neptune and Pluto. The orbits of Jupiter, Saturn, Uranus, and Neptune are shown in violet, the orbit of Pluto in green. Pluto's orbit is tilted with respect to the orbits of the other planets. (Bearing that in mind, you can see why Pluto is sometimes not the outermost planet.) Far beyond the planets, and also far beyond the Kuiper Belt, is an enormous spherical array of icy worlds orbiting the Sun called the Oort Comet Cloud. Diagram by Harold Levison, Southwest Research Institute.

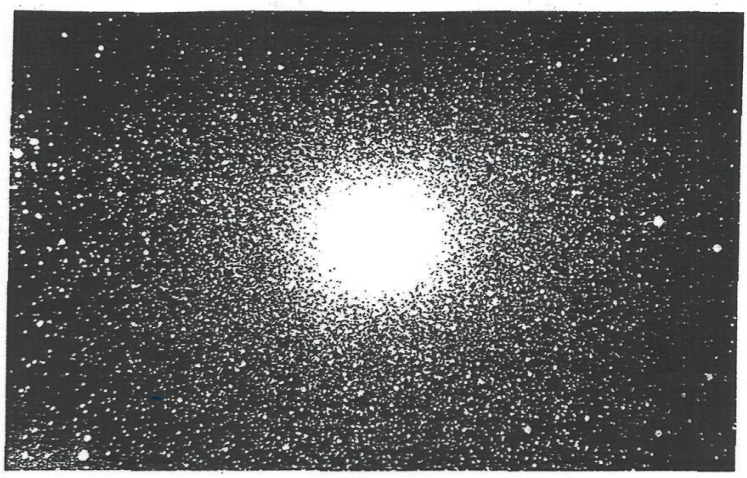


Figure 6: General structure of the Oort cloud, resembling that of a globular cluster.

blocks, known as 'planetesimals'. These bodies, which ended up with sizes in the $1–10^3$ km range, are nowadays associated either with observed comets and asteroids or the parent bodies of asteroids in the main belt. They were mostly gravitationally ejected from the solar system or scattered into long-lived orbits of various dynamical types, with lifetimes measured in billions of years, and eventually populated the orbits of so-called near-Earth objects (NEOs), which have dynamical lifetimes measured in millions, rather than billions, of years (e.g. Harris & Bailey 1998). The present NEOs, therefore, are a transient population, and by no means necessarily in a steady state.

2.1 Comets

Most observed long-period comets (those with $P > 200$ yr) originate in the Oort cloud, a nearly spherical reservoir containing orbits extending up to half-way to the nearest star. A good image of the Oort cloud is provided by a picture of a globular cluster, imagining the Sun at centre and the individual stars as comet nuclei (Figure 6). Oort cloud comets, sometimes called 'new' comets, have periods of revolution ranging up to several tens of Myr or more.

As reviewed by Bailey et al. (1990), Oort's primary reason for introducing the comet cloud appears to have been to provide a reservoir for long-period comets which was stable for the age of the solar system (Oort 1950). However, the discovery in the 1970s of massive molecular clouds in the Galaxy meant that the original cloud, as envisaged by Oort, was dynamically unstable (e.g. Napier & Staniucha 1982, Bailey

From:
Mark E. Bailey, "The Extraterrestrial Impact Hazard: Constraints on Impactor Populations",

Presented at the *Seminar on Current Issues of Astronomical and Planetary Environmental Concern,* 5-6 April 1998, Thessaloniki, Hellas (Greece), Editor: N.K.Spyrou, pp.37-55.

α - proxima Centauri
(The star closest to our Sun)

Annual Stellar Parallax: $\pi = 0''.763 \left(= \frac{1}{r_{(pc)}} \right)$

Distance: r=1.311 pc

$$\frac{1}{2} r = 0.655\, pc$$

$$= 0.655 \times 3.086 \times 10^{18}\, cm = 2.022 \times 10^{18}\, cm$$

$$= \frac{2.022 \times 10^{18}}{1.496 \times 10^{13}}\, AU = \mathbf{1.352 \times 10^{5}\ AU}$$

= Outer Boundary of the Oort Cloud !!!

Conclusion: The Solar System extends up to approximately half-way to the nearest star!!!

(Is this a general characteristic? Rather No !!!, However……)

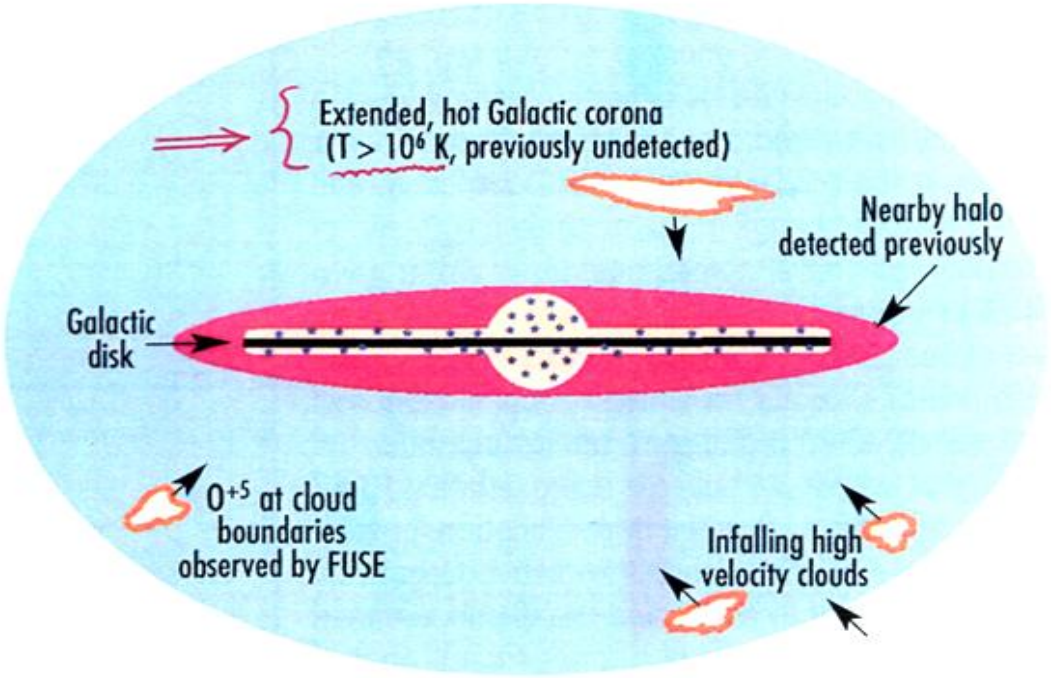
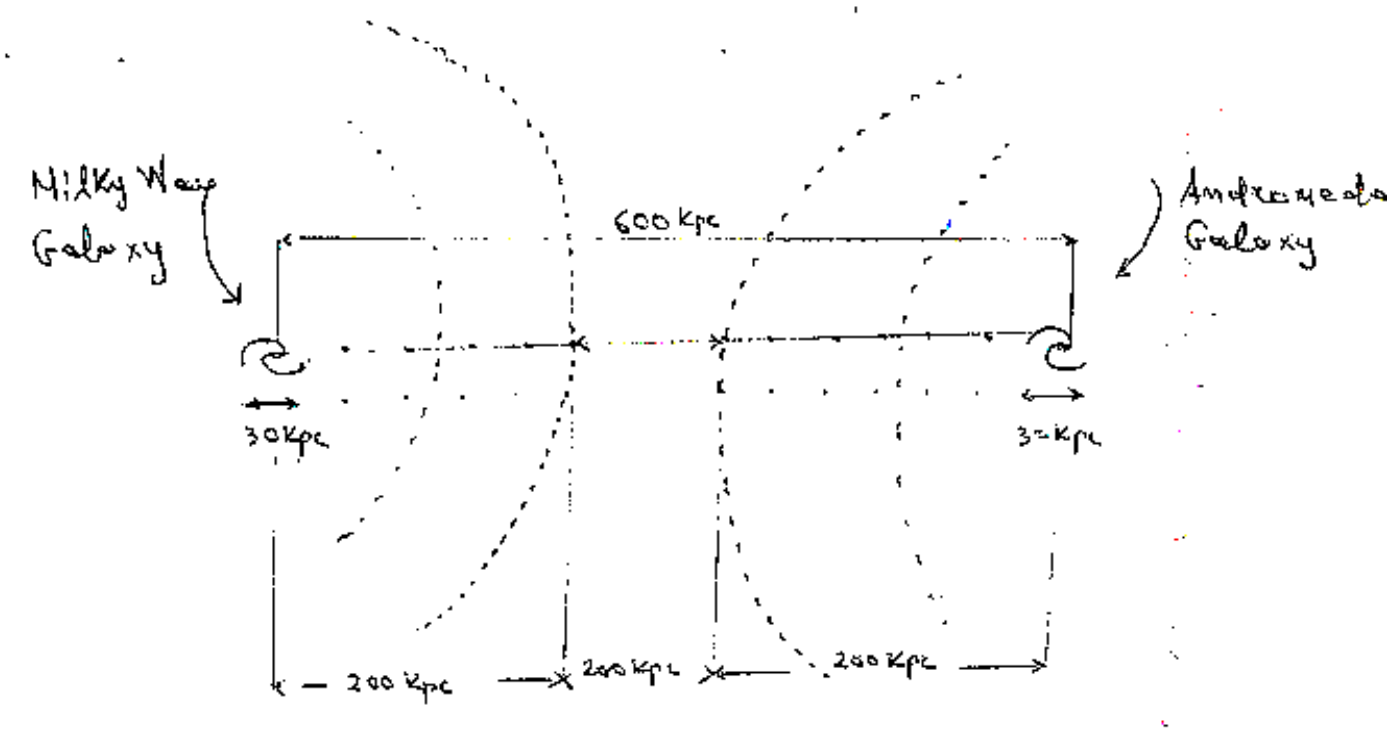


Figure 1. *A schematic showing high velocity clouds interacting with the Galactic corona.*

The Galactic Corona possibly extends out to the Magellanic Clouds or beyond (~200 Kpc!!!)

TWO NEIGHBOURING GALAXIES



OBSERVATIONAL DATA-INDICATIONS

According to many current observational data-indications:

The galaxies differ greatly from the simple picture of spiral or elliptical galaxies. They appear as almost spherically-symmetric, practically continuous, and of much larger linear dimensions cosmological structures.

The observable Universe differs greatly from the simple picture of a collection of galaxies (or higher-order cosmological structures) at mutual distances much larger than their linear dimensions.

As a consequence, the constituting elements of the Universe and the Universe, in its large extent, can plausibly be treated as **continuous gravitating perfect-fluid sources** (the physical description of which is well established in both the Newtonian and the relativistic regimes).

MOTIONS IN A GRAVITATING PERFECT-FLUID SOURCE

Two types of motion

Hydrodynamical Flows

$$\frac{d\vec{v}}{dt} = \vec{\nabla}U - \frac{1}{\rho}\vec{\nabla}p$$

$$\nabla^2 U = -4\pi G\rho$$

Geodesic Motions in the Fluid

$$\frac{d\vec{v}}{dt} = \vec{\nabla}U$$

$$\nabla^2 U = -4\pi G\rho$$

Recall: These geodesics are different from the geodesics in the vacuum $(\nabla^2 U = 0)$ (e.g. the motion of a planet around the Sun).

ISENTROPICITY OF FLOWS

For the isentropic flow motions the first thermodynamic axiom is assumed to hold,

$$d\Pi + pd\left(\frac{1}{\rho}\right) = 0$$

Π: internal thermodynamic energy (per unit mass)

p: isotropic pressure

$$d(...) = \frac{\partial(...)}{\partial x^i} dx^i$$

DYNAMICAL EQUIVALENCE BETWEEN FLOWS AND GEODESICS IN THE FLUID

The Euler Equations for the isentropic flows can be written in the form

$$\frac{d\vec{\upsilon}}{dt} = \vec{\nabla} V$$

$$V = U - \left(\Pi + \frac{p}{\rho} \right)$$

Results

The isentropic flows are **dynamically equivalent** to geodesic motions in the generalized potential V

Remarks

V **is not** the gravitational potential U "modified by hand"; instead, it is a natural generalization of U resulting from the assumption of isentropicity (for an isolated source).
Not a modified-gravity approach

In the generalized potential V, the volume element moves as a point mass, but, now, carrying along all the internal physical characteristic of the fluid source (not only its mass density!!!)

GENERALISED MASS DENSITY

The generalized gravitational potential, V, obeys the Poisson-type equation

$$\nabla^2 V = -4\pi G \rho_v$$

where

$$\rho_v = \rho + \rho_i \quad , \quad \rho_i = \frac{1}{4\pi G}\left[\vec{\nabla}\cdot\left(\frac{1}{\rho}\vec{\nabla}p\right)\right]$$

is the generalized mass density producing the generalized potential V (in analogy to the classical Poisson equation $\nabla^2 U = -4\pi G\rho$, with ρ producing the usual gravitational potential U)

Reasonable Necessity

The hydrodynamic flows, reduced to geodesic-like motions, are preferable than both the initial flows and the initial geodesics.

ATTRACTIVE GRAVITY OR/AND REPULSIVE GRAVITY

Attractive (usually) Gravity

$$\vec{\nabla}\cdot\frac{d\vec{\upsilon}}{dt}=\vec{\nabla}\cdot\left(\vec{\nabla}U\right)=-4\pi G\rho$$

$$\vec{\nabla}\cdot\frac{d\vec{\upsilon}}{dt}<0,\qquad \rho>0$$

Attractive gravitational acceleration decreases with increasing distance … a **usual** and **typical** property of attractive gravity!!!

Generalized Gravity

$$\vec{\nabla}\cdot\frac{d\vec{\upsilon}}{dt}=\vec{\nabla}\cdot\left(\vec{\nabla}V\right)=\nabla^2V=-4\pi G\rho_V$$

$$\vec{\nabla}\cdot\frac{d\vec{\upsilon}}{dt}\gtreqless 0,\qquad \rho_V=\rho+\rho_i\gtreqless 0$$

Consequence

Generalized Gravity can be attractive (through the gravitational potential **U**) or repulsive (through the thermodynamic potential $-\left(\Pi+\frac{p}{\rho}\right)$).
In the latter case, the acceleration **increases** with **increasing** distance

DYNAMICAL MASS IN ATTRACTIVE GRAVITY

For the source (baryonic and dark) $\frac{d\vec{\upsilon}}{dt} = \vec{\nabla} U, \quad \nabla^2 U = -4\pi G\rho,$ generally ρ is the total mass density

Circular equatorial velocity (in a spherically – symmetric source)

$$\upsilon_c^{\,2}(r) = \frac{Gm(r)}{r}$$

$$m(r) = \int \rho dV \rightarrow$$

Dynamical Mass

Is the 3-volume integral of the total mass density ρ manifesting itself in the usual Poisson equation.

This is the **usual meaning** of the dynamical mass in the case of the attractive gravity

DYNAMICAL MASS IN THE GENERALIZED GRAVITY

For the generalized source ($\frac{d\vec{u}}{dt} = \vec{\nabla} V, \quad \nabla^2 V = -4\pi G \rho_V$), the density ρ_V contains, additionally to the total density ρ, the part ρ_i , $\left(\rho_V = \rho + \rho_i\right)$, and so

$m_V = m + m_i \quad =$ Baryonic + Dark + m_i

Also, in this case, the dynamical mass is estimated, as before, through

$$\upsilon_c^{\,2}(r) = \frac{G m_V(r)}{r}$$

where, now,
Dynamical mass = m_V $\rightarrow$ 3-volume integral of ρ_V manifesting in the generalized Poisson equation,
is different than the 3-volume integral of ρ

Question
Which dynamical mass to use, **m** or m_V ?

Answer
In many cases we used m_V with encouraging results

APPLICATIONS OF NEWTONIAL DYNAMICAL EQUIVELANCE

Source: Spherically symmetric, gravitating, perfect – fluid source, with a given equation of state $p=k\rho$ $(k>0)$ and a given mass - density law $\rho(r)$, usually, a Plummer – type law $\rho(r)=\rho_0\left(1+\frac{r^2}{r_0^2}\right)^{-3/2}$ and with a central object of mass M_c.

Then, equations of motion:

$$\vec{\gamma}(r)=-\frac{G}{r^2}\left[M_c+m_V(r)\right]$$

$$=\vec{\gamma}_c+\vec{\gamma}_i$$

$$=-\frac{GM_c}{r^2}\hat{r}-\left[4\pi G\rho_0 r_0^2\,\frac{\ln\left(r/r_0\right)}{r/r_0}-3k\right]\frac{\hat{r}}{r}$$

$\vec{\gamma}_c$: Acceleration due to the point mass (spherically symmetric) central object

$\vec{\gamma}_i$: Extra internal acceleration

A Note on the Constants

The constants M_C, m, ρ_0 and r_0 depend on the source assumed
The density ρ(r) refers to both the baryonic matter and the dark matter of the source.
The equation of state p=kρ refers to the total pressure of both the baryonic and the dark matter, and k>0.
In some (not all) of the applications, the pressure is assumed thermal,

$$k = \frac{k_B T}{\mu m_H}$$

and, mostly, the pressure of the dark matter is ignored, and the density of dark matter largely exceeds that of the baryonic matter

Past Applications

1. Theoretical determination of the true linear dimensions of the Milky Way (HeL.A.S., 2003, Penteli, Greece)

   "… This implies that the Galactic Corona, and, hence, the Milky Way galaxy itself, possibly extends out to the Magellanic Clouds, and so the expected linear dimensions of the Milky Way galaxy are at least 200 Kpc, almost ten times larger than its optical linear dimensions (~30 Kpc) …"

2. Theoretical determinations of the true linear dimensions of superclusters of galaxies (Cosmology and Gravitational Physics Workshop, 2006, Thessaloniki)

   " … We conclude that only a few superclusters seem to be enough to comprise the whole observable Universe. Quite interestingly, this result is in accordance with and explains the fact that no third – order superclusters of galaxies are necessary (and so have not been observed up to now !!!) …"

Flat – Rotation Curves of Galaxies (Hel.A.S., 2003)

Circular Equatorial Velocity ($z=r/r_0$)

$$\upsilon_c = 2.989\times10^5\left\{\sqrt{2}\,\frac{1}{z}\ln\left(1+\sqrt{z^2+1}\right)-\frac{\sqrt{2}}{\sqrt{z^2+1}}-\frac{1}{z}\left[\ln\left(1+\sqrt{2}\right)-\frac{1}{\sqrt{2}}\right]-\frac{3k_BT}{\mu m_H c^2}\,\frac{z^2}{z^2-1}+\frac{1}{6z}\right\}^{1/2} km/s$$

Flat rotation curves appear naturally, and ρ_i (m_i) is the necessary additional ingredient.
A partial solution to the dark – matter problem???

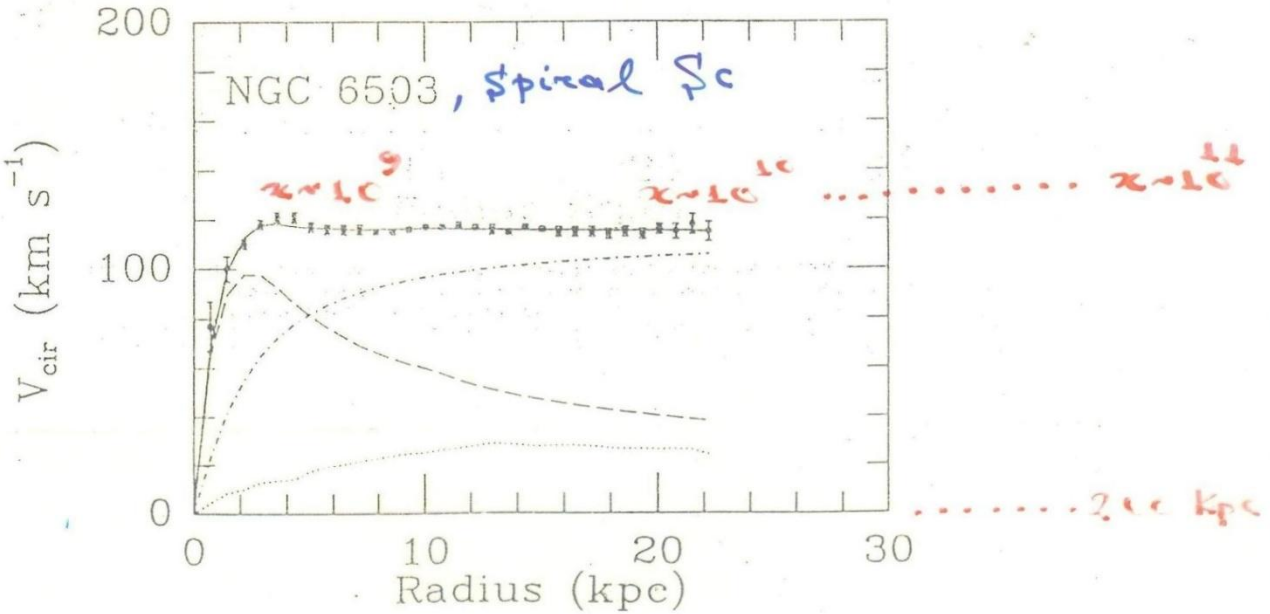
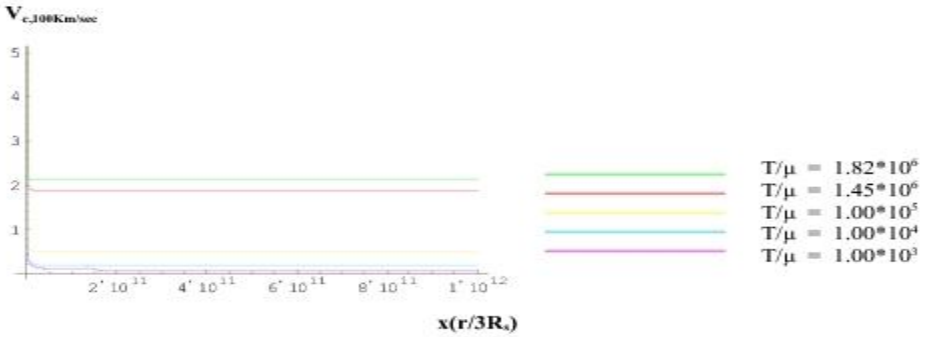
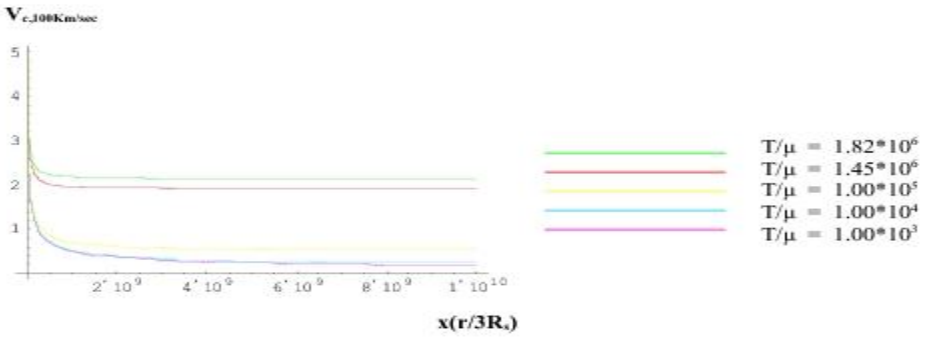
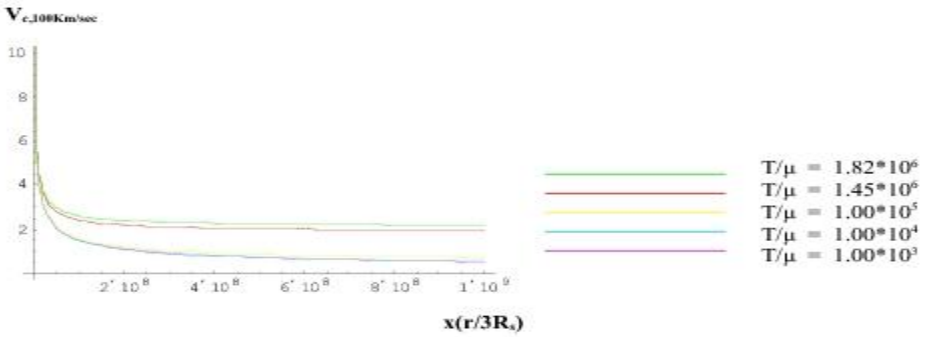


Figure 1 The observed rotation curve of NGC 6503 (Begeman, et. al 1991) and its contributing components. The dashed curve is for the visible light, the dotted curve is for the gas, and the dash-dot curve is for the dark halo.

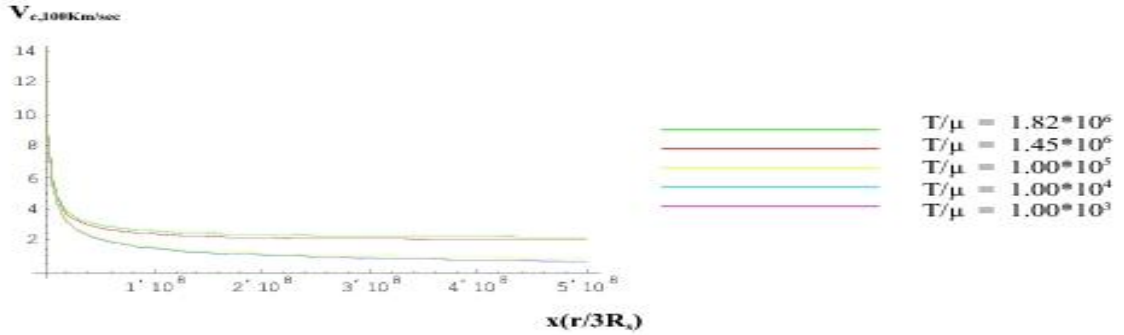
FLAT ROTATION CURVES

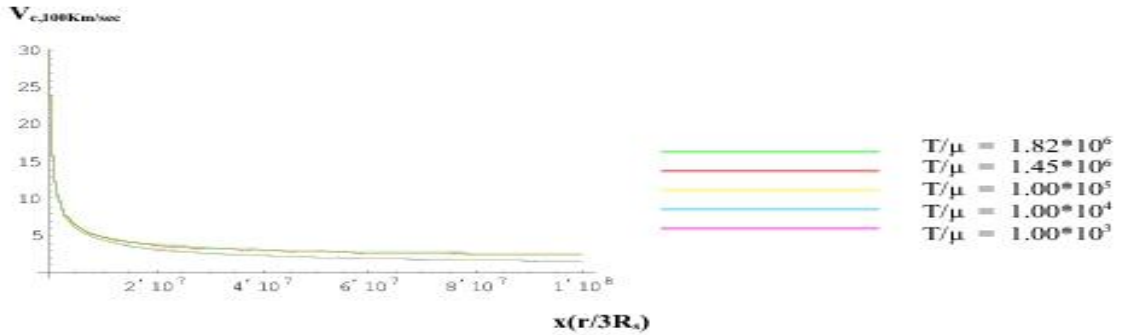


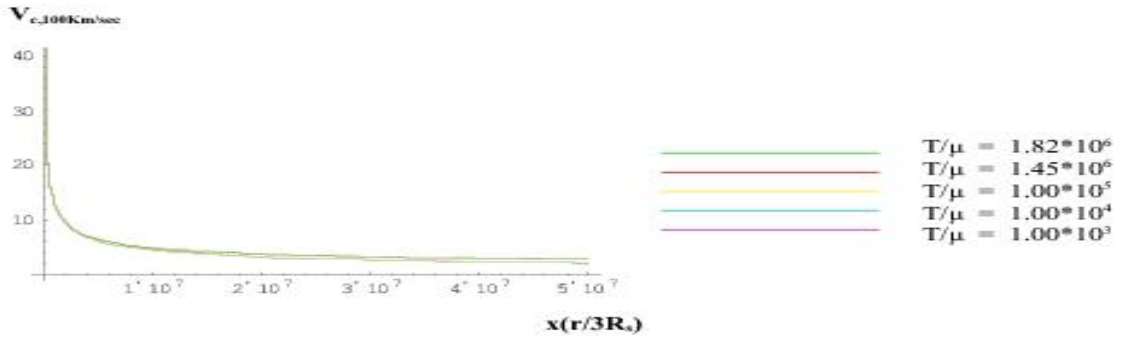




FLAT ROTATION CURVES







Further Applications (Zagkouris)

1. Theoretical explanation of the Pioneer – Anomaly Effect
2. Theoretical determination of the true linear dimensions of the Solar System
3. Evaluation of the internal thermodynamic energy in the near and far regions of the "new" Solar System
4. Further analogous predictions for other spaceprobes (e.g. New Horizons)

Total Acceleration in the Solar System

$$\vec{\gamma} = \vec{\gamma}_{\odot} + \vec{\gamma}_i$$

Acceleration due to Sun $\quad \vec{\gamma}_{\odot} = -\frac{GM_{\odot}}{r^2}\hat{r}$

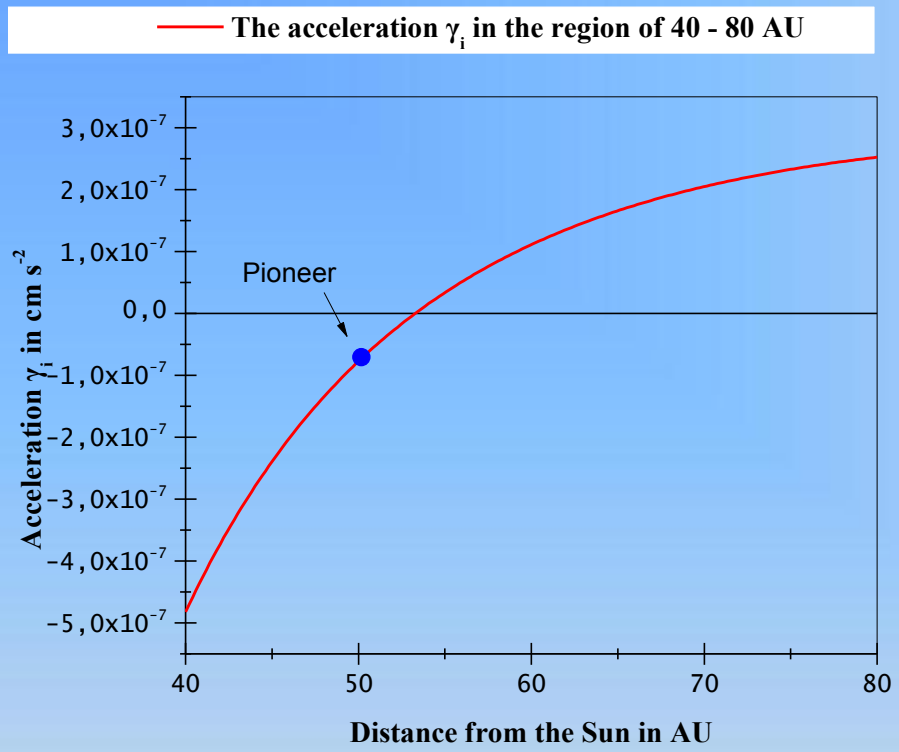
Extra Acceleration $\quad \vec{\gamma}_i = -\left(4\pi G\rho_0 r_0^2 \frac{\ln\left(r/r_0\right)}{r/r_0} - 3k \right)\frac{\hat{r}}{r} = \gamma_i \hat{r}$

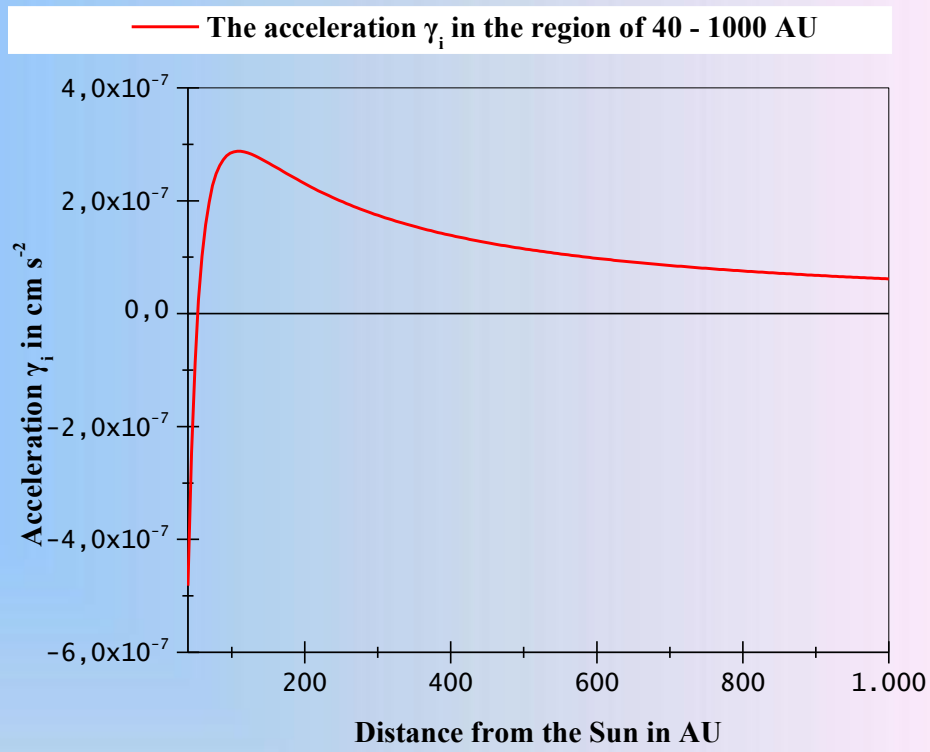
Plummer – type Density $\quad \rho(r) = \rho_0 \left(1 + \frac{r^2}{r_0^2}\right)^{-3/2}$

Equation of State $\quad p = k\rho$

(If simply baryonic, then $\quad k = \frac{k_B T}{\mu m_H}$)

The Behavior of the Extra Acceleration



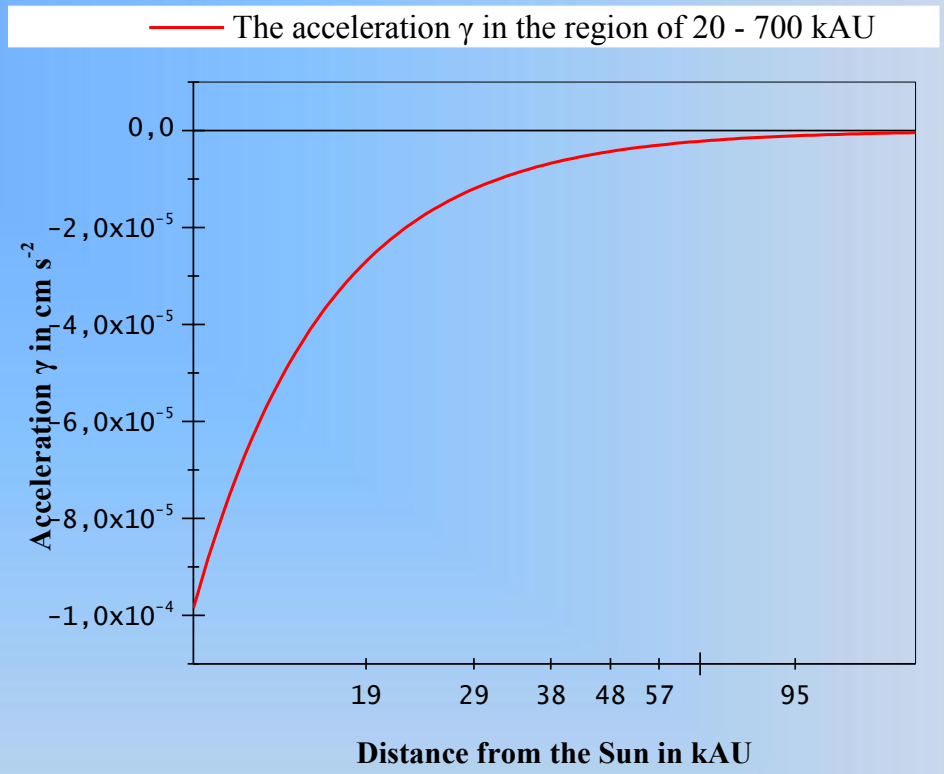


- After a certain distance, γ_i remains positive (outwards acceleration)
- It could explain the Pioneer – Anomaly Effect

Unexplained extra acceleration of Pioneer 10 : $-8.74 \pm 1.33 \cdot 10^{-8}\ cm\ s^{-2}$

Suggested – evaluated acceleration of Pioneer 10 : $-8.74311\ 10^{-8}\ cm\ s^{-2}$

The Total Acceleration in the Solar System



- γ is always negative inside the distribution, and it becomes zero at its boundary
- It sets a theoretically derived limit to the extent of the Solar System

Internal thermodynamic energy per unit mass

$$\Pi(r_0) = 4.01\times10^{9}\, cm^2\, s^{-2}$$

$$\Pi(R) = 1.43\cdot10^{8}\, cm^2\, s^{-2}$$

Solar System's Boundary $R_{\gamma=0} = 98.932\ \text{kAU} \simeq 100\,\text{kAU}$

~ 37% Distance to the A – Proxima Centauri

5. Theoretical explanation of the formation mechanism of winds and jets

Basic Idea

Generalized gravity can be repulsive if $\rho_V < 0$. Discrimination of the regions of prevalence of attractive and repulsive gravities is the vanishing of ρ_V.

The vanishing of the generalized density, ρ_V, is an algebraic equation for the distance, r, from the origin.

The solution is called inversion distance, r_{inv}.
For $r<r_{inv}$, attractive gravity prevails
For $r>r_{inv}$, repulsive gravity prevails

… hence the outwards acceleration necessary for the formation of the jet or winds

6. Conformal Dynamical Equivalence in Classical Magnetohydrodynamics (Tsagas)

The equations of motion of classical magnetohydrodynamics are put in the form of Newton's equation of motion

$$\frac{d\vec{\upsilon}}{dt} = \vec{\nabla} V, \quad \nabla^2 V = -4\pi G \rho_V, \quad \rho_V = \rho + \rho_{i(P)} + \rho_{i(B)}$$

This is accomplished in three steps:

1. By a suitable redefinition of the pressure, so as to include also the magnetic pressure, the MHD equations of motion are put in the form of the Euler's hydrodynamical equations of motion.
2. This is possible, only if the magnetic field is characterized by a "zero - curl Lorentz force "
$$\vec{\nabla} \times \left[\vec{B} \times \left(\vec{\nabla} \times \vec{B} \right) \right] = 0$$
3. Then, by a suitable redefinition of the internal thermodynamic energy of the magnetized fluid and of the condition of the adiabatic motions, the above Euler's equations are put in the form of Newton's gravitational equations of motion

The magnetic field's contribution to ρ_V becomes important in the central regions of a magnetized large – scale cosmological structure

7. Determination of the masses of clusters of galaxies (Plionis, Basilakos)

The determination, usually, is accomplished, by assuming a static source comprised of both baryonic matter and dark matter, in which the pressure contribution of dark matter is ignored, and the baryonic matter is used as a tracer of dark matter

The above method is generalized so as to include the total mass density (dark and baryonic) and total pressure of a non – static source, for which the notion of the spectral shift can also be used

METHOD

Assuming an equation of state for the total pressure, p, and the total density, ρ, of the form

$$p = w\rho c^2, \quad w > 0$$

we find

$$\rho_V = \rho + \frac{wc^2}{4\pi G}\left[\vec{\nabla}\cdot\left(\frac{\vec{\nabla}\rho}{\rho}\right)\right], \qquad m_V = \int \rho_V d^3x$$

For a given ρ(r) an w, the generalized density ρ_V is evaluated, and then, by its volume integration, the dynamical mass $m_V(r)$ is determined

Then, the assumption of circular motions leads to the spectral shift directly related to observations.

$$z^2 = \frac{Gm_V(r)}{c^2 r}$$

This is an interplay of w and the constants appearing in the law ρ(r)

Cosmology, Collisional Dark Matter and CDE (Kleidis)

A Mathematical Exercise

Applying a conformal transformation

$$\tilde{g}_{ik} = \Omega^2\left(x^l\right) g_{ik}$$

it can be proved that the equations of hydrodynamic flows in the metric g_{ik} $\left(T^{ik}_{;k} = 0\right)$ can be put in the form of the geodesic motions in the metric $\tilde{g}_{ik}$ with an energy – momentum tensor $\tilde{T}^{ik}$ different (and explicitly evaluated) than the original one T^{ik}

Astrophysical Significance of the CDE

The notion of the classical test particle is "replaced" by that of a "fluid volume element" moving as a test particle and carrying along all the physical characteristics of the fluid source.

The conformal factor is, essentially, the enthalpy of the fluid source

Future cosmological perspective

Up to now, the interpretation of the cosmological observational data was done with the aid of the "classical test particle" without considering the thermodynamic content of the fluid source.

The notion of the "more (astro)physical test particle" has to be used in interpreting correctly the cosmological observational data.

In this way the cosmological observations can be confronted successfully and be put in a more physical context.

In short :

Dark Matter, Yes ! (it appears enough !)

Dark Energy, No ! (it does not appear as necessary !)

Thank you for your patience and attention !!!