Hořava-Lifshitz gravity: infrared dynamics and viability

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Outline of the talk

The framework

Various versions of Horava-Lifshitz gravity

Most general action with projectability

Degrees of freedom and infrared dynamics

Scalar mode and strong coupling

Conclusions and prospects

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Basic idea

Old problem: Can we make gravity renormalizable?

Possible solution: Add higher-order curvature invariants in order to make the coupling dimensionless

- Higher order term contain higher order time derivatives
- This introduces ghosts!

Simple solution: give up Lorentz invariance. Then

- Higher order spatial derivatives without higher order time derivatives, i.e. no ghosts
- Renormalizable theory (at the power counting level)

Well, maybe not that simple...

Splitting Spacetime into Space and time

If we want to add more spatial derivatives than time derivatives we have to choose a preferred foliation.

Remaining symmetry: "foliation preserving diffeomorphisms"

- Time reparametrization: $t \to \tilde{t}(t)$
- Spacetime-dependent 3-diffeos: $x^i \to \tilde{x}^i(t, x^i)$

Convenient to use the ADM form of the line element

$$ds^2 = -N^2 c^2 dt^2 + g_{ij}(dx^i - N^i dt)(dx^j - N^j dt)$$

The action

Consider the action

$$S = \int d^d x \, dt \, \sqrt{-g} \, N \left[\mathcal{T}(K) - \mathcal{V} \right]$$

where $\mathcal{T}(K)$ is to include the time derivatives and $\mathcal{V}$ the rest

The remaining symmetry requires that time derivatives come only in the combination

$$K_{ij} = \frac{1}{2N} \left\{ -\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i \right\}$$

and

$$\mathcal{T}(K) = g_K \left\{ \left(K^{ij} K_{ij} - K^2 \right) + \xi K^2 \right\}$$

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Anisotropic scaling

Choose the engineering dimensions

$$[dx] = [\kappa]^{-1} \qquad [dt] = [\kappa]^{-z}$$

Dimensional analysis then yields

$$[g_{ij}] = [N] = [1] \qquad [ds] = [\kappa]^{-1}$$

$$[N^i] = [c] = \frac{[dx]}{[dt]} = [\kappa]^{z-1} \qquad [K_{ij}] = \frac{[g_{ij}]}{[N][dt]} = [\kappa]^{z}$$

$$dV_{d+1} = \sqrt{g}\, N\ d^d x\ dt \qquad [dV_{d+1}] = [\kappa]^{-d-z}$$

$$[R^{ijkl}] = [\kappa]^2 \quad [\nabla R^{ijkl}] = [\kappa]^3 \quad [\nabla^2 R^{ijkl}] = [\kappa]^4$$

Dimensional analysis

Assuming that the action is dimensionless as usual yields

$$[g_K] = [\kappa]^{(d-z)} \qquad [\mathcal{V}] = [\kappa]^{d+z}$$

We can therefore make the coupling dimensionless if

$$d = z$$

In that case we might as well choose $g_K \to 1$

But now we have $[\mathcal{V}] = [\kappa]^{2d}$

so term of the following form will also have dimensionless couplings

$$\left\{(\text{Riemann})^d,\ [(\nabla\text{Riemann})]^2(\text{Riemann})^{d-3},\ \text{etc...}\right\}$$

P. Hořava, Phys. Rev. D 79, 084008 (2009)

The action - potential part

Invariance under foliation-preserving diffeos requires

$$\mathcal{V} = \mathcal{V}(g_{ij}, a_k)$$

where

$$a_i \equiv \partial_i N/N$$

The lowest order terms are

$$R \qquad\qquad a^i a_i$$

but there are also numerous higher order operators, such as

$$(\nabla_i R_{jk})(\nabla^i R^{jk}) \qquad a_i \nabla^2 a^i \qquad (\nabla^2 R)(\nabla^i a_i)$$

Note that the a_i terms have often been neglected

D. Blas, O. Pujolas and S. Sibiryakov, Phys. Rev. Let. 104, 181302 (2010)

Projectability

Symmetry: "foliation preserving diffeomorphisms"

- Time reparametrization: $t \to \tilde{t}(t)$
- Spacetime-dependent 3-diffeos: $x^i \to \tilde{x}^i(t, x^i)$

This mean that we have less gauge freedom than usual

$N = N(t, x^i)$ cannot be set to 1

We could match this by the restriction

$$N = N(t)$$

Caveats:

- Not all gauges will be available
- No super-hamiltonian constraint

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General action with projectability

Specializing to 3 spatial dimensions

$$[\mathcal{V}(g)] \to [\kappa]^6$$

and the Weyl tensor vanishes so our list reduces to

$$\left\{(\text{Ricci})^3, [\nabla(\text{Ricci})]^2, (\text{Ricci})\nabla^2(\text{Ricci}), \nabla^4(\text{Ricci})\right\}$$

To eliminate redundant terms we use

- Integration by part and discarding surface terms
- Commutator identities
- Bianchi identities
- Special relations appropriate to 3 dimensions

General action with projectability

Assembling all the pieces together

$$S = \int \left[\mathcal{T}(K) - \mathcal{V}(g)\right] \sqrt{g}\, N\; d^3x\; dt$$

with

$$\begin{aligned} \mathcal{V}(g) \quad &= \quad g_0\,\zeta^6 + g_1\,\zeta^4\,R + g_2\,\zeta^2\,R^2 + g_3\,\zeta^2\,R_{ij}R^{ij} \\ &\quad +g_4\,R^3 + g_5\,R(R_{ij}R^{ij}) + g_6\,R^i{}_j R^j{}_k R^k{}_i \\ &\quad +g_7\,R\nabla^2 R + g_8\,\nabla_i R_{jk}\,\nabla^i R^{jk} \end{aligned}$$

We still have the freedom to set $g_1 \to -1$

- 9 terms, 8 dimensionless couplings
- most general action in this setting (projectability)

T. P. Sotiriou, M. Visser and S. Weinfurtner, Phys. Rev. Lett. 102, 251601 (2009)

Changing back to relativistic scaling

We have chosen scalings so as to get a dimensionless couplings

- good for power counting
- bad for phenomenology

We cannot impose $\mathrm{c} \to 1$ as we have already set

$$\frac{[dx]^z}{[dt]} = [dx]^{z-1}\frac{[dx]}{[dt]} = \zeta^{-z+1}c \to 1$$

To set $\mathrm{c} \to 1$ consistently we also need to impose

$$dt \to \zeta^{-z+1}dt$$

We can now rewrite the action

Action in "physical" scaling

$$S = \zeta^2 \int dt\, d^3x \, \sqrt{g}\, N \Big\{ (K^{ij}K_{ij} - K^2) + R - g_0\,\zeta^2$$
$$+\xi\, K^2 - g_2\,\zeta^{-2}\, R^2 - g_3\,\zeta^{-2}\, R_{ij}R^{ij} - g_4\,\zeta^{-4}\, R^3$$
$$-g_5\,\zeta^{-4}\, R(R_{ij}R^{ij}) - g_6\,\zeta^{-4}\, R^i{}_j R^j{}_k R^k{}_i$$
$$-g_7\,\zeta^{-4}\, R\nabla^2 R - g_8\,\zeta^{-4}\, \nabla_i R_{jk}\,\nabla^i R^{jk} \Big\}$$

In this units it is easy to identify

$$(16\pi G_{\mathrm{Newton}})^{-1} = \zeta^2 \qquad\qquad \Lambda = \frac{g_0\,\zeta^2}{2}$$

- *So* ζ *is identified as the Planck scale*
- *We are free to choose the value of* Λ
- *Lorentz-violation scales controlled by the couplings*

Degrees of freedom

The graviton will satisfy a higher order dispersion relation, in the projectable case for example

$$\ddot{\tilde{H}}_{ij} = -\left[g_1\partial^2 + g_3\zeta^{-2}\partial^4 + g_8\zeta^{-4}\partial^6\right]\tilde{H}_{ij}$$

T. P. Sotiriou, M. Visser and S. Weinfurtner, JHEP 0910, 033 (2009)

There is also a scalar degree of freedom though

- Less symmetry leads to less gauge freedom and more actual degrees of freedom

The easiest way to discover it is to consider scalar perturbation around flat space for the lowest order action

The scalar mode

Consider the action

$$S = M_{\rm pl}^2 \int d^3x dt \sqrt{-g}\, N \left\{K^{ij}K_{ij} - K^2 + \xi K^2 + R + \eta\, a_i a^i\right\}$$

For scalar perturbations, the quadratic action is

$$S_2 = -M_{\rm pl}^2 \int d^3x dt \left[\frac{1}{c^2_{\rm spin\,0}}\dot{h}^2 - \frac{\eta-2}{\eta}(\partial h)^2\right]$$

where

$$c^2_{\rm spin\,0} = \frac{\xi}{2-3\xi}$$

For the scalar field to be stable and not a ghost one needs

$$c^2_{\rm spin\,0} < 0 \qquad\qquad 0 < \eta < 2$$

The scalar mode - projectable case

The projectable case corresponds to $\eta \to \infty$

$$S_2 = -M_{\rm pl}^2 \int d^3x dt \left[\frac{1}{c^2_{\rm spin\,0}}\dot{h}^2 - (\partial h)^2\right]$$

$c^2_{\rm spin\,0} = \dfrac{\xi}{2-3\xi}$ *becomes the low momentum phase velocity*

- *instability when* $\xi < 0 \quad \xi > 2/3$
- *a ghost when* $0 < \xi < 2/3$

T. P. Sotiriou, M. Visser and S. Weinfurtner, JHEP 0910, 033 (2009)

However, there is a claim that the mode is stable around de Sitter space

Y.Huang, A. Wang and Q. Wu, arXiv:1003.2003 [hep-th]

Strong coupling

C. Charmousis, G. Niz, A. Padilla and P. M. Saffin, JHEP 08, 070 (2009)
D. Blas, O. Pujolas and S. Sibiryakov, JHEP 10, 029 (2009)
K. Koyama and F. Arroja, JHEP 1003, 061 (2010)
A. Papazoglou and T. P. Sotiriou, Phys. Lett. B 685, 197 (2010)

Qubic lagrangian:

$$S_3 = \int dt d^3x \left\{ \left(1 - \frac{4(1-\eta)}{\eta^2}\right) h(\partial h)^2 - \frac{2}{c^4_{\rm spin\,0}} \dot{h}\partial_i h \frac{\partial^i}{\Delta}\dot{h} + \left(\frac{3}{2} + \frac{1}{\eta}\right)\left[\frac{1}{c^4_{\rm spin\,0}} h \left(\frac{\partial_i\partial_j}{\Delta}\dot{h}\right)^2 - \frac{(2c^2_{\rm spin\,0}+1)}{c^4_{\rm spin\,0}} h\dot{h}^2\right]\right\}$$

Normalizing as $\hat{h} = \dfrac{M_{\rm pl}\, h}{|c_{\rm spin\,0}|}$

- *all, but the first term scales as* $(|c_{\rm spin\,0}|M_{\rm pl})^{-1}$
- *strong coupling at* $\xi \to 0$ *at the scale* $|c_{\rm spin\,0}|M_{\rm pl}$

How bad is the problem then?

Strong coupling scale

- Recovering Lorentz invariance is out of the question...
- What about if we don't try to?

Absence of preferred frame effect in the Solar system is expected to provide constraints of the order

$$\xi, \eta \lesssim 10^{-7}$$

- The effective cutoff of the theory is lower than that of GR unless the renormalization group conspires
- This is supposed to be a UV complete theory!

Introducing an extra scale

D. Blas, O. Pujolas and S. Sibiryakov, arXiv:0912.0550

What if the higher order operators are suppressed by a scale

$$M_\star < |c_{\rm spin\,0}| M_{\rm pl}$$

then higher order operators take over before strong coupling

However

- tuning required
- L.V. constraints from kinetic term imply $\quad -\xi < \dots$
- L.V. constraints from potential term imply

$$M_\star > \dots \quad \rightarrow \quad |c_{\rm spin\,0}| > \dots \quad \rightarrow \quad -\xi > \dots$$

Potential problem?

Conclusions and Prospects

- An interesting possibility but not easy
- We wanted to avoid ghosts and instabilities and we encountered them again
- Avoiding strong coupling problems requires tuning
- Even if this is technically natural there might not be a window of opportunity left
- Numerous things to work out though, such as renormalizability, RG flow, coupling to matter, etc.