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Cosmologies with a time-dependent vacuum

Phenomenology of a possible dynamical Λ term in Einstein's equations

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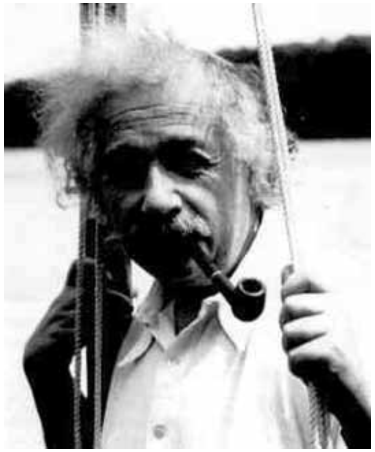
Guidelines of the Talk

- Dark Energy and the CC problems
- The fine-tuning problem in QFT
- Dynamical CC term in Einstein's equations
- The LXCDM models: the new "cosmon"
- Attempting the old CC problem: the "relaxed" universe with a non-fine-tuned Λ term
- Conclusions

Cosmological Sum Rule



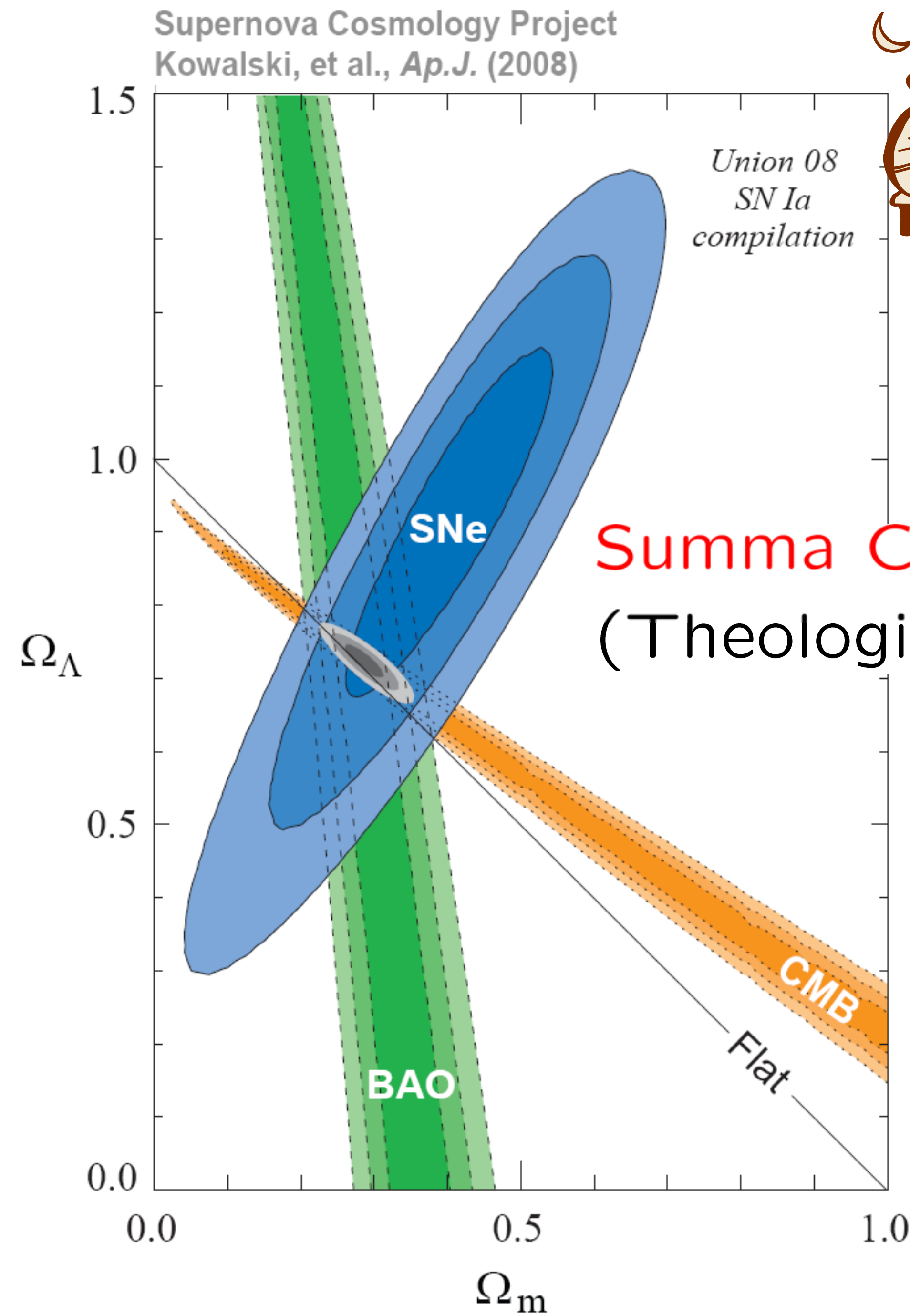
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda\, g_{\mu\nu} = 8\pi G_N\, T_{\mu\nu}$$

In the FLRW metric,

$$\rho_c = \frac{3H^2}{8\pi G_N}$$

$$\Omega_M = \frac{\rho_M}{\rho_c} \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} \qquad \Omega_K = -\frac{K}{H^2}$$

$$\Omega_M + \Omega_\Lambda + \Omega_K = 1$$

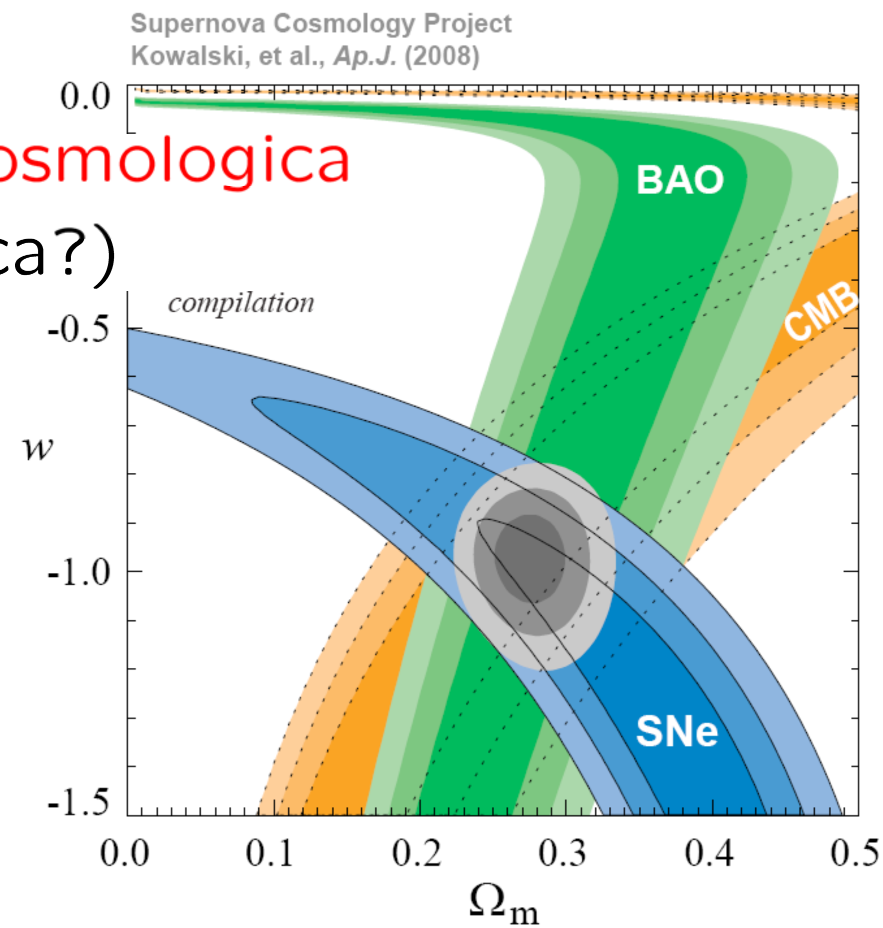


$$\Omega_M + \Omega_\Lambda + \Omega_K = 1$$

$$\Omega_K \simeq 0$$

Summa Cosmologica

(Theologica?)



Cosmology in history

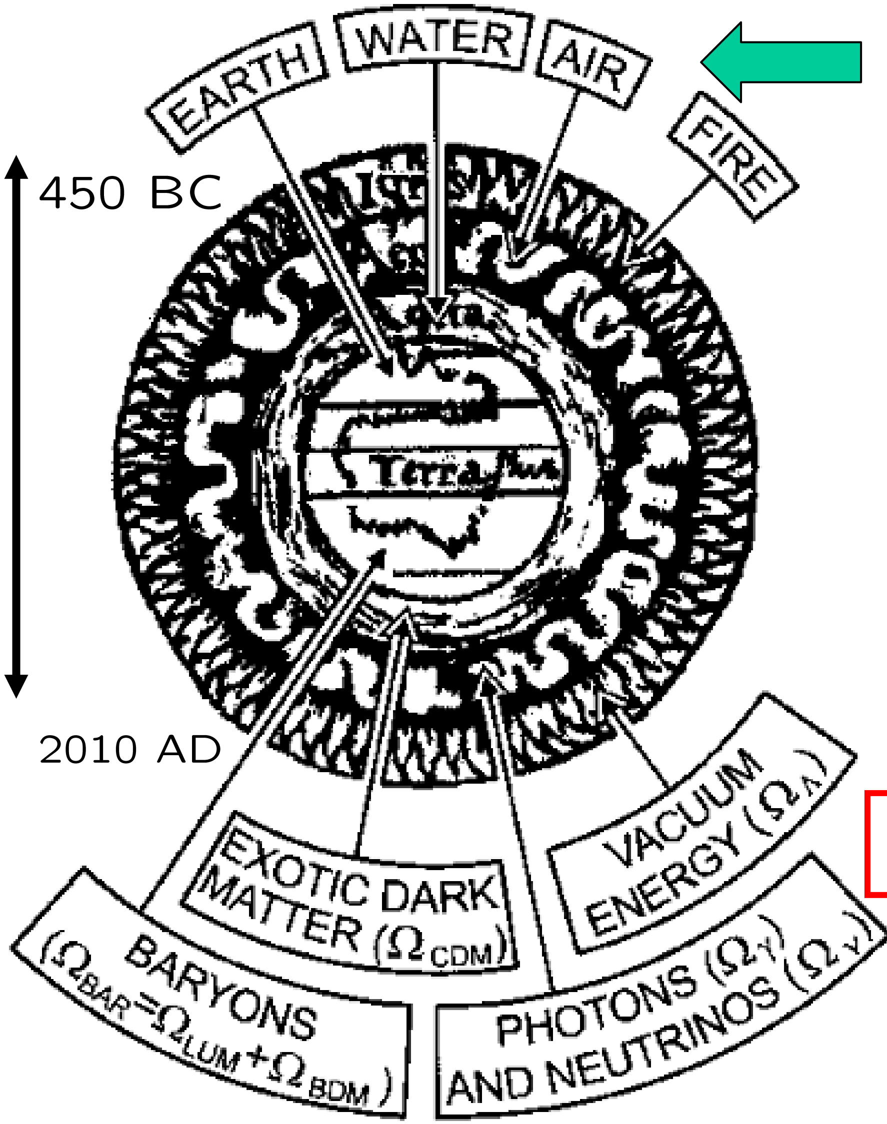
Thales
Heraclitus
Empedocles
Leucippus
Democritus
Anaximenes
Xenophanes

. . .

450 BC

2450 yr.

2010 AD



Before

Now!...

(96% still unknown !!)

Is it VERY different after all?...

Λ in QFT: the Vacuum Energy

A bit of QFT stuff...

◇ Action integral for a scalar QFT:

$$S[\phi] = \int d^4x\,\sqrt{-g}\,\mathcal{L}(\phi, \partial_\mu\phi)$$

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi - V_{eff}(\phi)$$

◇ Matter field energy-momentum tensor:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\,\frac{\delta\,S}{\delta\,g^{\mu\nu}} = 2\,\frac{\partial\mathcal{L}}{\partial\,g^{\mu\nu}} - g_{\mu\nu}\,\mathcal{L}$$

$$= \left[\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\;\partial_\alpha\phi\,\partial^\alpha\phi\right] + g_{\mu\nu}\,V_{eff}$$

◇ For static equilibrium configurations $\Rightarrow$

$$< T_{\mu\nu} >= g_{\mu\nu}\;< V_{eff} >$$

◇ The Cosmological Constant Problem in modern QFT is the realization that the Physical Cosmological Constant is an Effective Cosmological Constant!! ...a **HUGE** one !!! (Zeldovich 1967; Linde 1974, Veltman 1975):

$$\rho_{\Lambda \text{phys}} = \rho_{\Lambda \text{vac}} + \rho_{\Lambda \text{ind}}$$

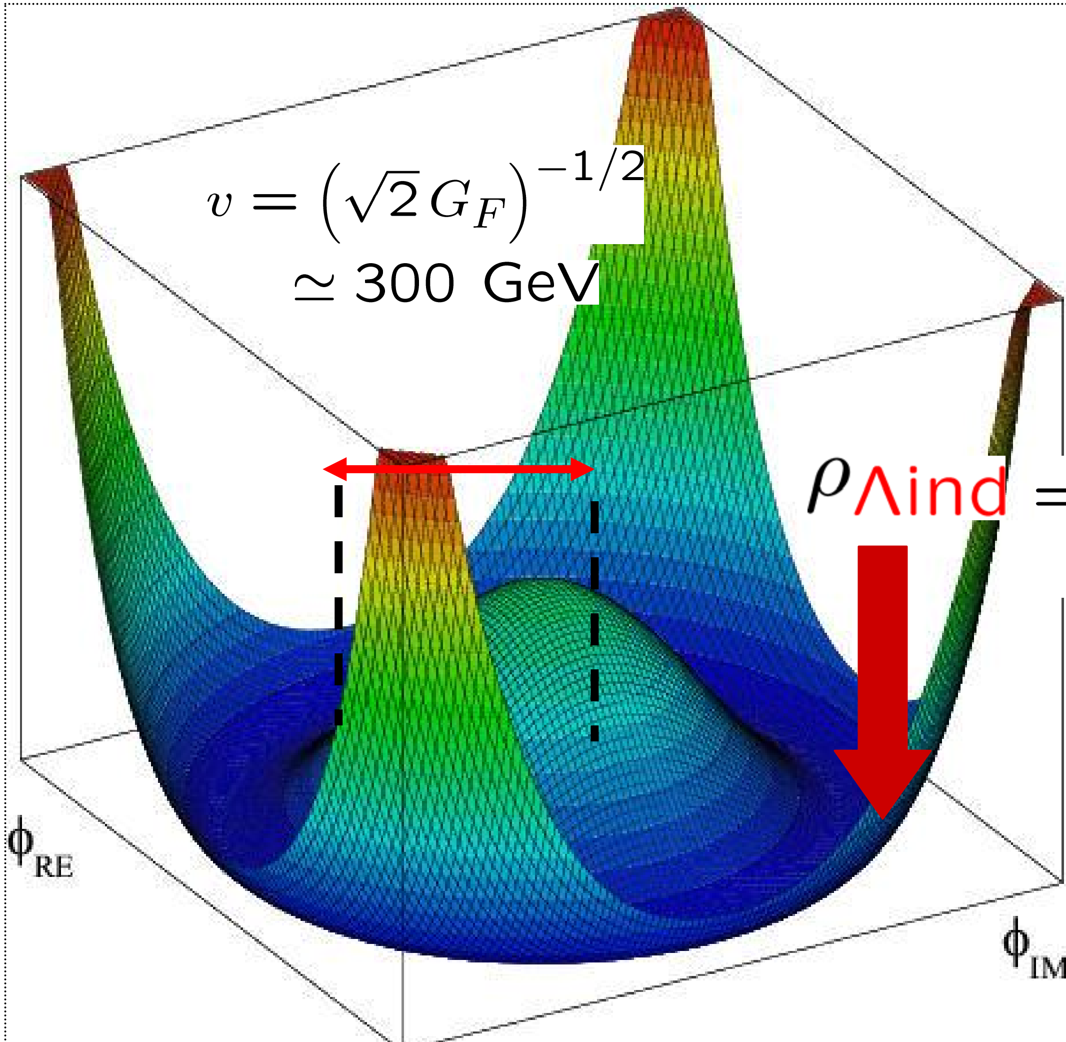
$$S_{EH} = \frac{1}{16\pi\, G_N} \int d^4x \sqrt{|g|}\, (\, R - 2\, \Lambda_{\text{vac}}\,) = \int d^4x \sqrt{|g|} \left(\frac{1}{16\pi\, G_N} R - \rho_{\Lambda \text{vac}} \right)$$

Vacuum bare term in Einstein eqs.

$$R_{ab} - \frac{1}{2} g_{ab} R = -8\pi\, G_N \left(\langle \tilde{T}^{\varphi}_{ab} \rangle + T_{ab} \right) = -8\pi\, G_N\, g_{ab} \, (\rho_{\Lambda \text{vac}} + \rho_{\Lambda \text{ind}} + T_{ab})$$

Quantum effects $\Rightarrow$ $\rho_{\Lambda \text{ind}} = \langle V(\varphi) \rangle +$ ZPE

Higgs Potential → Vacuum Energy



$$v = \left(\sqrt{2}\,G_F\right)^{-1/2} \simeq 300 \text{ GeV}$$

$$V(\varphi) = \frac{1}{2} m^2 \varphi^2 + \frac{1}{4!} \lambda \varphi^4$$

$$m^2 < 0 \;\Rightarrow$$

$$v \equiv \langle \varphi \rangle = \sqrt{\frac{-6\,m^2}{\lambda}}$$

$$\rho_{\Lambda \mathrm{ind}} = \langle V(\varphi) \rangle = -\tfrac{1}{8} M_{\mathcal{H}}^2 v^2 \sim -10^8 \; GeV^4 \text{ !!}$$

$$M_W = \tfrac{1}{2} g v$$

$$M_Z = \tfrac{1}{2} v \sqrt{g^2 + g'^2}$$

$$m_e = \lambda_e \frac{v}{\sqrt{2}}$$

$$m_u = \lambda_u \frac{v}{\sqrt{2}}$$

$$m_d = \lambda_d \frac{v}{\sqrt{2}}$$

...

$\mu \to \nu_\mu \bar{\nu}_e\, e \quad \Rightarrow G_F \qquad M_{\mathcal{H}} > 114$ GeV

Theory:

$$\rho_{\Lambda \mathrm{ind}} = \langle V(\varphi)\rangle = -\frac{1}{8}\, M_{\mathcal{H}}^2\, v^2 \sim -10^8\ GeV^4$$

versus observation:

$$\Omega_{\Lambda} \simeq 0.7 \quad \Leftrightarrow \quad \rho_{\Lambda \mathrm{ind}} \simeq 10^{-47}\ \mathrm{GeV}^4$$

$$\frac{\rho_{\Lambda \mathrm{ind}}}{\rho_{\Lambda \mathrm{phys}}} \simeq \frac{10^8}{10^{-47}} \simeq 10^{55}$$

!!

➢ The old CC problem as a fine tuning problem

A little bit more of QFT stuff...

F. Bauer, JS, H. Stefancic
PLB 678:427-433,2009
+ in preparation

Take a scalar QFT with effective potential

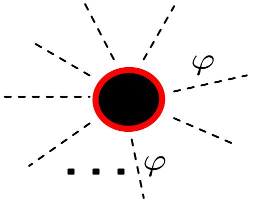
$$V_{\rm eff} = V + \hbar\, V_1 + \hbar^2\, V_2 + \hbar^2\, V_3 + \ldots$$

where

$$V_1 = V_P^{(1)} + V_{\rm scal}^{(1)}(\varphi)\,, \qquad V_2 = V_P^{(2)} + V_{\rm scal}^{(2)}(\varphi)\,, \qquad V_3 = V_P^{(3)} + V_{\rm scal}^{(3)}(\varphi)\ldots.$$

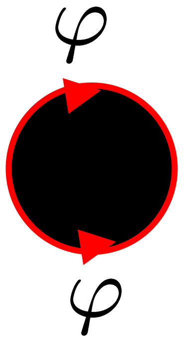
Thus,

$$\boxed{V_{\rm eff}(\varphi) = V_{\rm ZPE} + V_{\rm scal}(\varphi)}$$



with

$$V_{\rm ZPE} = \hbar\, V_P^{(1)} + \hbar^2\, V_P^{(2)} + \hbar^3\, V_P^{(3)} + \ldots.$$



Putting everything together:

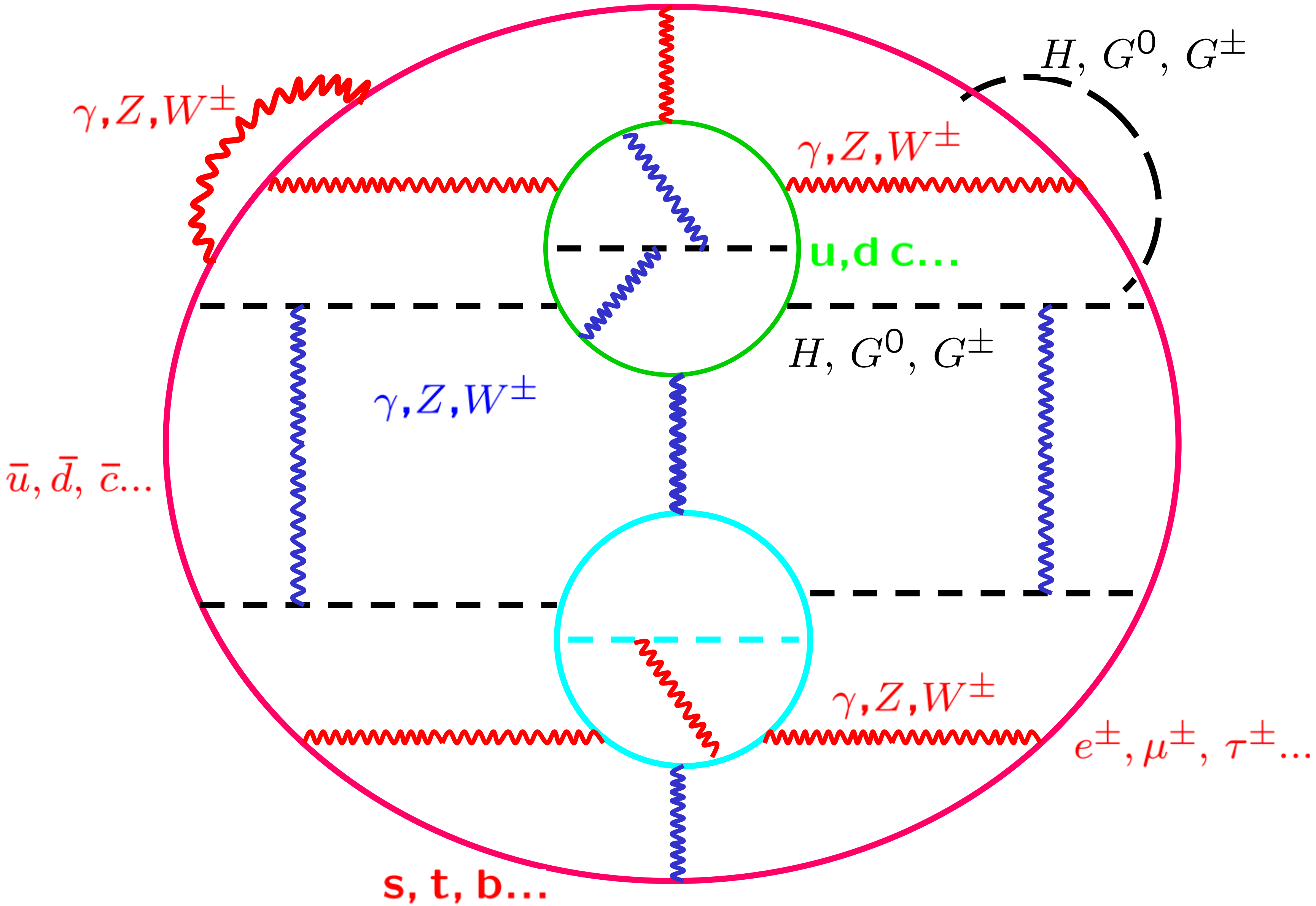
$$\rho_{\Lambda \rm ph} = \rho_{\Lambda \rm vac} + \rho_{\Lambda \rm ind} = \rho_{\Lambda \rm vac}^{\rm ren} + \langle V_{\rm eff}^{\rm ren}(\varphi)\rangle$$
$$= \rho_{\Lambda \rm vac}^{\rm ren} + V_{\rm ZPE}^{\rm ren} + \langle V_{\rm scal}^{\rm ren}(\varphi)\rangle$$

$$10^{-47}\,{\rm GeV}^4 = \rho_{\Lambda \rm vac} - 10^8\,{\rm GeV}^4 + \hbar\,V_P^{(1)} + \hbar^2\,V_P^{(2)} + \hbar^3\,V_P^{(3)} \ldots.$$
$$+ \hbar\,V_{\rm scal}^{(1)}(\varphi) + \hbar^2\,V_{\rm scal}^{(2)}(\varphi) + \hbar^3 V_{\rm scal}^{(3)}(\varphi) \ldots$$

With $v \sim 100$ GeV, which is the highest loop involved?:

$$\left(\frac{g^2}{16\,\pi^2}\right)^n v^4 = 10^{-47}\,GeV^4 \qquad \Rightarrow \boxed{n \simeq 21} \; !!$$

21th loop (one among many thousands...)



The many Cosmological Constant Problems

S. Weinberg, Rev.Mod.Phys.61 (1989) 1

In the SM, $\Lambda_{\rm ph} = \Lambda_v + \Lambda_{SM}$

- **Problem I**:

The "Classic" CC Problem:

$$\left(\frac{\Lambda_{SM}}{\Lambda_{\rm ph}} \simeq \frac{10^8}{10^{-47}} \simeq 10^{55}\right)$$

Why the induced and vacuum counterparts of the CC cancel each other with such a huge precision?

F. Bauer, JS, H. Stefancic
PLB 678:427-433,2009.

- **Problem II**:

The (first) "Coincidence" CC Problem:

J.Grande, A. Pelinson, J. Solà,
Phys.Rev.D79:043006,2009.
JCAP 0712:007,2007.
JCAP 0608:011,2006.

Why the observed CC in the present-day Universe is so close to the matter density ρ?

coincidence ratio now:

$$r \equiv \frac{\rho^0_\Lambda}{\rho^0_M} = \frac{\Omega^0_\Lambda}{\Omega^0_M} \simeq \frac{7}{3} = \mathcal{O}(1)$$

Λ in the SM and beyond

Source	Effect (GeV^4)	Λ/Λ_{exp}
electron 0-point	10^{-16}	10^{31}
QCD chiral	10^{-4}	10^{43}
QCD gluon	10^{-2}	10^{45}
Electroweak SM	10^{+9}	10^{56}
typical GUT	10^{+64}	10^{111}
Quantum Gravity	10^{+76}	10^{123} !!

• **Problem III**:

The "nature" of the the CC Problem:

In more recent times the notion of Λ has been superseded by that of the DE. The latter is more general and involves a variety of models leading to an accelerated expansion of the universe in which the DE itself is a time-evolving entity. These models include dynamical scalar fields (quintessence... and the like), phantom fields, braneworld models, Chaplygin gas, holographic dark energy, cosmic strings, domain walls...

What is, then, the true dynamical cause responsible for the DE?

- **Problem IV**:

The (second) “Coincidence” Problem:

Present observations seem to indicate an evolving DE with a potential phantom phase near our time.

If the dark energy behaves phantom-like, why just now?

➢ ''Canonical'' definition of Dynamical Dark Energy

One popular possibility is the idea of quintessence, where there is no "true" Λ

The total energy-momentum tensor on the *r.h.s.* of Einstein eqs. is the sum

$$\tilde{T}_{\mu\nu} \equiv T^{M}_{\mu\nu} + T^{D}_{\mu\nu} \,.$$

One assumes that both tensors are separately conserved, and so $\bigtriangledown^{\mu}\tilde{T}_{\mu\nu} = 0$ is equivalent to

$$\bigtriangledown^{\mu} T^{M}_{\mu\nu} = 0 \Longleftrightarrow \frac{d\rho}{dt} + 3\,H\,(\rho + p) = 0,$$

and (<u>unmixed</u> conservation laws)

$$\bigtriangledown^{\mu} T^{D}_{\mu\nu} = 0 \Longleftrightarrow \frac{d\rho_D}{dt} + 3\,H\,(\rho_D + p_D) = 0$$

Nice feature of quintessence field:

$$\omega_D = \frac{p_D}{\rho_D} = \frac{\frac{1}{2}\xi\dot{\chi}^2 - V(\chi)}{\frac{1}{2}\xi\dot{\chi}^2 + V(\chi)} \simeq -1 + \xi\,\dot{\chi}^2/V(\chi)$$

Problems with quintessence field:

Even taking the simplest form $V(\chi) = (1/2)\, m_\chi^2\, \chi^2$

$$\rho_\Lambda^0 = V(\chi)$$

$$\chi \simeq M_X \simeq 10^{16}\, GeV \quad \Rightarrow \quad m_\chi \simeq H_0 \simeq 10^{-33}\, eV$$

$$\chi \simeq M_F = G_F^{-1/2} \quad \Rightarrow \quad m_\chi \simeq 10^{-12}\, eV$$

(Recall that $m_\Lambda \sim$ meV $\Rightarrow$ billion times) **!**

Question:

Can a dynamical DE still be Λ?...

$$\Downarrow$$

Need $\Lambda = \Lambda(t)$!!

But still $w = -1$...!!

$$T_{\mu\nu} = \rho_\Lambda(t) - p\, g_{\mu\nu} + (\rho + p)\, U_\mu U_\nu \quad = -\tilde{p}\, g_{\mu\nu} + (\tilde{\rho} + \tilde{p})\, U_\mu U_\nu$$

$$\tilde{p} = p - \rho_\Lambda(t) \qquad \tilde{\rho} = \rho + \rho_\Lambda(t) \qquad \boxed{\Rightarrow p_\Lambda(t) = -\rho_\Lambda(t)}$$

DE picture of it: $\dot{\rho}_D + 3\, H_D(1 + \omega_D)\rho_D = 0$

Generic time-varying CC models versus observation

S. Basilakos, M. Plionis, JS, Phys.Rev.D80 (2009)
(see talk by S. Basilakos)

1) Quantum field vacuum (Λ_{RG})

$$\Lambda(H) = n_0 + n_2 H^2$$

I.L.Shapiro, JS.
JHEP 0202 (2002) 6
Phys.Lett.B475 (2000) 236.

2) Power series model (Λ_{PS1})

$$\Lambda(H) = n_1 H + n_2 H^2$$

S. Basilakos
MNRAS 395 (2009) 2347

3) Linear model (Λ_{PS2})

$$\Lambda(H) \propto H$$

R. Schutzhold, PRL 89 (2002)
S. Carneiro et al. (2008), F. Klinkhammer and G.E. Volovik (2009) etc

4) Quadratic model

$$\Lambda(H) \propto H^2 \propto \rho_T$$

J.C. Carvalho et al (1992),
R.C. Arcuri and I. Waga (1994) etc.

5) Power law model (Λ_n)

$$\Lambda(H) \propto a^{-n}$$

M. Ozer and O. Taha (1987),
W. Chen and Y.S. Wu (1990)

Variable Λ

- For variable Λ, the conserved quantity is not the matter energy-momentum tensor $T_{\mu\nu}$, but the sum

$$\tilde{T}_{\mu\nu} \equiv T_{\mu\nu} + g_{\mu\nu}\,\rho_\Lambda(t)\,, \qquad \nabla^\mu \tilde{T}_{\mu\nu} = 0\,.$$

By the Bianchi identities, Λ is constant $\Longleftrightarrow$ the matter $T_{\mu\nu}$ is individually conserved ($\nabla^\mu T_{\mu\nu} = 0$)– in particular, $\rho_\Lambda = \text{const.}$ if $T_{\mu\nu} = 0$ (e.g. during inflation).

- From FLRW metric

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1 - k\,r^2} + r^2\,d\theta^2 + r^2\,\sin^2\theta\,d\phi^2\right)\,,$$

we may compute explicitly the local energy-conservation law $\nabla^\mu \tilde{T}_{\mu\nu} = 0$. The result is an equation allowing transfer of energy between ordinary matter and the dark energy associated to the Λ term :

$$\frac{d\rho_\Lambda}{dt} + \frac{d\rho}{dt} + 3\,H\,(\rho + p) = 0,$$

(<u>mixed</u> conservation law!)

Running Λ from Planck Scale Physics

- One may expect that the **RGE** of Λ is totally dominated by sub-Planckian masses:

$$\frac{d\Lambda}{d\ln\mu} = \frac{1}{(4\pi)^2}\sum_i c_i\,\mu^2 M_i^2 + \ldots = \frac{1}{(4\pi)^2}\sum_i c_i\,H^2 M_i^2 + \ldots$$

$$(\mu = H)$$

$$= \frac{1}{(4\pi)^2}\,\sigma\,H^2 M^2 + \ldots$$

I.L.Shapiro, J.S.
JHEP 0202 (2002) 6

with

$$M \equiv \sqrt{\sum_i c_i\,M_i^2}\,.$$

- Provides a natural explanation for the geometric mean puzzle:

$$\Lambda \simeq \sqrt{\rho_P\,\rho_H} = \sqrt{{M_P}^4\,H^4} = {M_P}^2\,H^2$$

A semiclassical FLRW with running Λ

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho + \Lambda) + H_0^2\,\Omega_K^0(1+z)^2$$

$$\frac{d\Lambda}{dt} + \frac{d\rho}{dt} + 3H(\rho + p) = 0\,.$$

$$\frac{d\Lambda}{d\ln H} = \frac{1}{(4\pi)^2}\sum_i c_i\, M_i^2\, H^2 + \ldots = \frac{3\nu}{4\pi} M_P{}^2 H^2.$$

$$\nu = \frac{\sigma\,\mathrm{M}^2}{12\pi M_P{}^2}$$

$$\rho_\Lambda \equiv \Lambda = C_1 + C_2\, H^2\,.$$

Running both... G and Λ ?

I.L. Shapiro, J.S., H. Stefancic
JCAP 0501 (2005)

J. S., J.Phys.A41 (2008)

Bianchi identity leads to $\nabla^{\mu}\left[G\left(T_{\mu\nu}+g_{\mu\nu}\,\rho_{\Lambda}\right)\right]=0$

$$\frac{d}{dt}\left[G(\rho_m+\rho_{\Lambda})\right]+3\,G\,H\,(\rho_m+p_m)=0\,.$$

Possible scenario:

$$\dot{G}\neq 0 \;\; and \;\; \dot{\rho_{\Lambda}}\neq 0 \Rightarrow \qquad \dot{\rho}_m+3\,H(\rho_m+p_m)=0$$

$$(\rho+\rho_{\Lambda})\dot{G}+G\dot{\rho_{\Lambda}}=0$$

Astrophysical implications?...

Basic set of equations:
(k=0)

$$\begin{cases} \rho + \rho_\Lambda = \dfrac{3H^2}{8\pi G}\,, \\ \rho_\Lambda = C_1 + C_2 H^2\,, \\ (\rho + \rho_\Lambda)\, dG + G\, d\rho_\Lambda = 0 \end{cases}$$

$$C_1 = \rho_\Lambda^0 - \frac{3\nu}{8\pi} M_P^2 H_0^2\,, \qquad C_2 = \frac{3\nu}{8\pi} M_P^2$$

$$G(H;\nu) = \frac{G_0}{1 + \nu \ln\left(H^2/H_0^2\right)}$$

$H \to a$ in J. S., J.Phys.A41 (2008)

"DE picture" versus "CC picture"

- Observations leading to the **EOS** of the **DE** are sensitive to the function $H = H(z)$.
- We can describe a variable CC model with mixed energy-conservation law as if it would be a dynamical DE model with unmixed EC-law.
- Let us assume there is an underlying fundamental dynamics

$$\rho_\Lambda(z) = \rho_\Lambda(\rho_M(z), H(z), ...)\,, \quad G(z) = G(\rho_M(z), H(z), ...)$$

⇓

$$H_\Lambda^2 = \frac{8\pi G}{3}(\rho_M + \rho_\Lambda)$$

In the CC picture

$$H_\Lambda^2(z) = H_0^2 \left[\Omega_M^0 f_M(z;r)(1+z)^\alpha + \Omega_\Lambda^0 f_\Lambda(z;r)\right]$$

$$\alpha = 3(1+\omega_m) \;\; (\omega_m = 1/3 \text{ or } \omega_m = 0)$$

$$\xi_M(z) \equiv \frac{G(z)}{G_0}\rho_M(z)$$

$$f_M(z) \equiv \frac{\xi_M(z)}{\rho_M^0\,(1+z)^\alpha}$$

$$\xi_\Lambda(z) \equiv \frac{G(z)}{G_0}\rho_\Lambda(z)$$

$$f_\Lambda(z;r) = \frac{\xi_\Lambda(z)}{\rho_\Lambda^0}$$

Whatever it be their form, these functions must satisfy $f_M(0;r) = f_\Lambda(0;r) = 1$ in order that the cosmic sum rule $\Omega_M^0 + \Omega_\Lambda^0 = 1$ is fulfilled.

In the DE picture

$$H_D^2(z) = \frac{8\pi G}{3}\left[\rho_M^0\,(1+z)^\alpha + \rho_D(z)\right]$$

$$\dot{\rho_D} + 3\,H_D(1+\omega_D)\rho_D = 0 \quad \Rightarrow \rho_D(z) = \rho_D(0)\,\zeta(z)$$

$$\zeta(z) \equiv \exp\left\{3\int_0^z dz' \frac{1+\omega_D(z')}{1+z'}\right\}$$

$$\omega_D(z) = -1 + \frac{1}{3}\,\frac{1+z}{\zeta}\,\frac{d\zeta}{dz}$$

$$H_D^2(z) = H_0^2\left[\tilde{\Omega}_M^0\,(1+z)^\alpha + (1-\tilde{\Omega}_M^0)\,\zeta(z)\right]$$

$(\Delta\Omega_M \equiv \Omega_M^0 - \tilde{\Omega}_M^0)$

⇒ "Matching condition" of the two pictures:

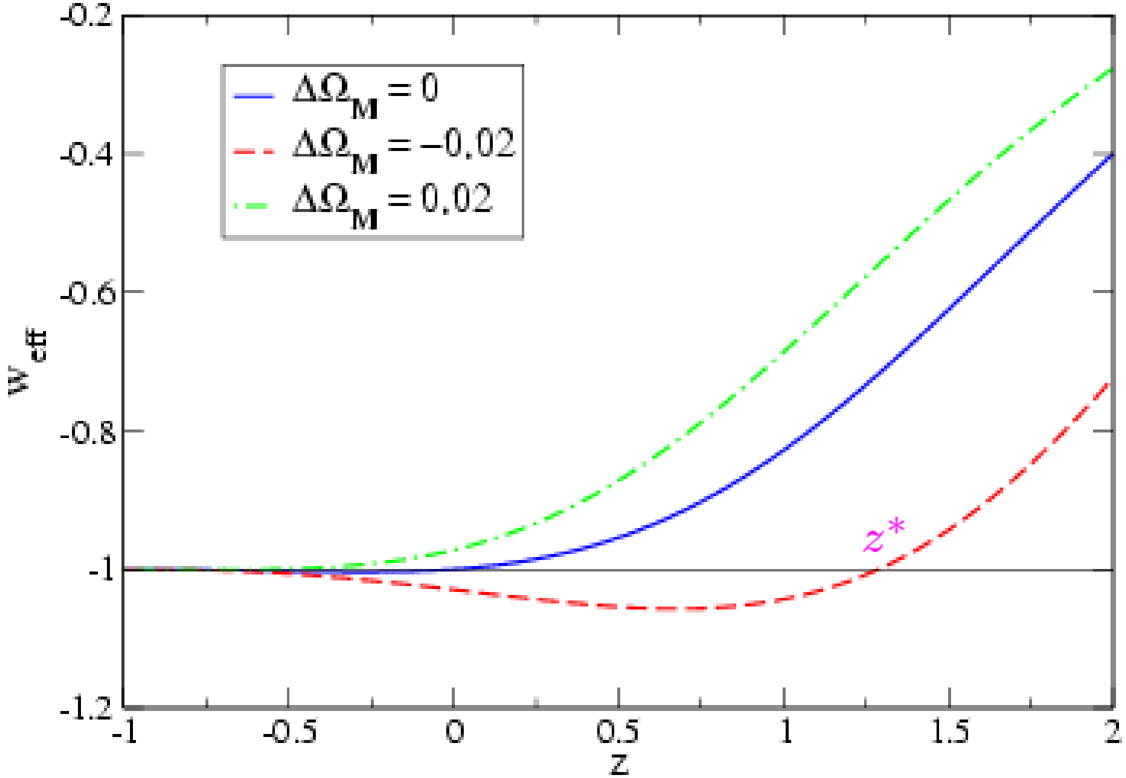
$$H_D^2(z) = H_\Lambda^2(z)$$

$$H_0^2 \left[\tilde{\Omega}_M^0 \, (1+z)^\alpha + (1-\tilde{\Omega}_M^0)\, \zeta(z)\right] = H_\Lambda^2(z)$$

Matching generates an "effective **EOS**" for Λ:

$$\omega_{\rm eff}(z) = -1 + \frac{1}{3}\,\frac{1+z}{\rho_D}\,\frac{d\rho_D}{dz} = -1 + \frac{1}{3}\,\frac{1+z}{\zeta}\,\frac{d\zeta}{dz} \quad \Rightarrow \quad \omega_{\rm eff} = \omega_{\rm eff}(z)$$

(case $\nu = -\nu_0 < 0$)



J.Solà, H. Stefancic
Phys. Lett. B 624 (2005) 147

Using phantom scalar fields:
R.R. Caldwell, PLB 545 (2002) 23
A. Melchiorri et al.
PRD 68 (2003) 043509
B. Feng, X.L. Wang, X.M. Zhang,
PLB 607 (2005) 35

$$\nu = \frac{1}{12\pi}\frac{M^2}{M_P^2} \quad \Rightarrow \quad \nu_0 = \frac{1}{12\pi} \simeq 0.026$$

J.Solà, H. Stefancic.
Mod. Phys. Lett. A21 (2006) 479

Effective equation of state for the variable Λ as a function of the redshift: $\omega_{\rm eff} = \omega_{\rm eff}(z;\nu)$

H. Stefancic, J.S.
Phys. Lett. B624 (2005) 147

$$\omega_{\rm eff}(z) = -1+(1-\nu)\,\frac{\Omega_M^0\,(1+z)^3\,\left[(1+z)^{-3\nu}-1\right]}{1-\nu-\Omega_M^0+\Omega_M^0\,(1+z)^3\,\left[(1+z)^{-3\nu}-1+\nu\right]}\,.$$

$$\simeq -1-3\,\nu\frac{\Omega_M^0}{\Omega_\Lambda^0}\,(1+z)^3\,\ln(1+z)\,.$$

$$\omega_{\rm eff}(z)\,|_{\Delta\Omega\neq 0} = -1+(1-\nu)\,\frac{\Omega_M^0\,(1+z)^{3(1-\nu)}-\tilde{\Omega}_M^0\,(1+z)^3}{\Omega_M^0\,[(1+z)^{3(1-\nu)}-1]-(1-\nu)\,[\tilde{\Omega}_M^0\,(1+z)^3-1]}$$

NEXT STEP?...........

Running cosmological parameters
and dynamical dark energy component?

↓

The "$\Lambda XCDM$" model

J. Grande, H. Stefancic, J.S. ,
JCAP 0608:011,2006
Phys.Lett.B645:236-245,2007

The road to the: "ΛXCDM" model

"M" Model: First Cosmological model

A. Einstein, 1915

"ΛM" Model: Second Cosmological model

A. Einstein, 1917

"CDM" Model: Old Standard Cosmological model

A. Einstein-de Sitter, 1932

"ΛCDM" Model: Standard Cosmological model

Peebles, 1984

"XCDM": Quintessence Cosmological model

Turner-White 1997

"ΛXCDM"

"X" → "Cosmon"

R.Peccei, J.S. and C. Wetterich
Phys. Lett. B 195 (April 1987) 183

Effective EOS of a mixture of cosmic fluids:

$$w_e = \frac{p_D}{\rho_D} = \frac{\omega_1\,\rho_1 + \omega_2\,\rho_2 + \ldots}{\rho_1 + \rho_2 + \ldots}$$

General Bianchi identity. Defining $\alpha_i \equiv 3(1+\omega_i)$

$$\frac{d}{dt}\left[G\left(\sum_i \rho_i\right)\right] + G\,H\,\sum_i \alpha_i\,\rho_i = 0$$

In the ΛXCDM case, with $G = const.$ and separate conservation of matter and DE:

$$\dot{\rho}_m + \alpha_m \rho_m H = 0 \,, \qquad \alpha_m = 3(1 + \omega_m)$$

$$\dot{\rho}_D + \alpha_D \rho_D H = 0 \,, \qquad \alpha_D = 3(1 + \omega_e)$$

with $$\rho_D = \rho_\Lambda + \rho_X$$

cosmological "constant" contribution   cosmon contribution

$$\omega_e = \frac{p_\Lambda + p_X}{\rho_\Lambda + \rho_X} = -1 + \frac{1}{3}\,\frac{\alpha_X \rho_X}{\rho_D}$$

Equivalently, we have

$$\dot{\rho}_\Lambda + \dot{\rho}_X + \alpha_X \rho_X H = 0 \,, \qquad \alpha_X = 3(1 + \omega_X)$$

Among the many possibilities (cf gr-qc/0604057), consider

$0 < \alpha_X < 2\,,$ $\nu = 0 \Leftrightarrow$ quintessence and $\Lambda = const.$

$(-1 < \omega_X < -1/3)$

$$\Omega_D(z) = \Omega_\Lambda^0 + \Omega_X^0\,(1+z)^{\alpha_X}$$

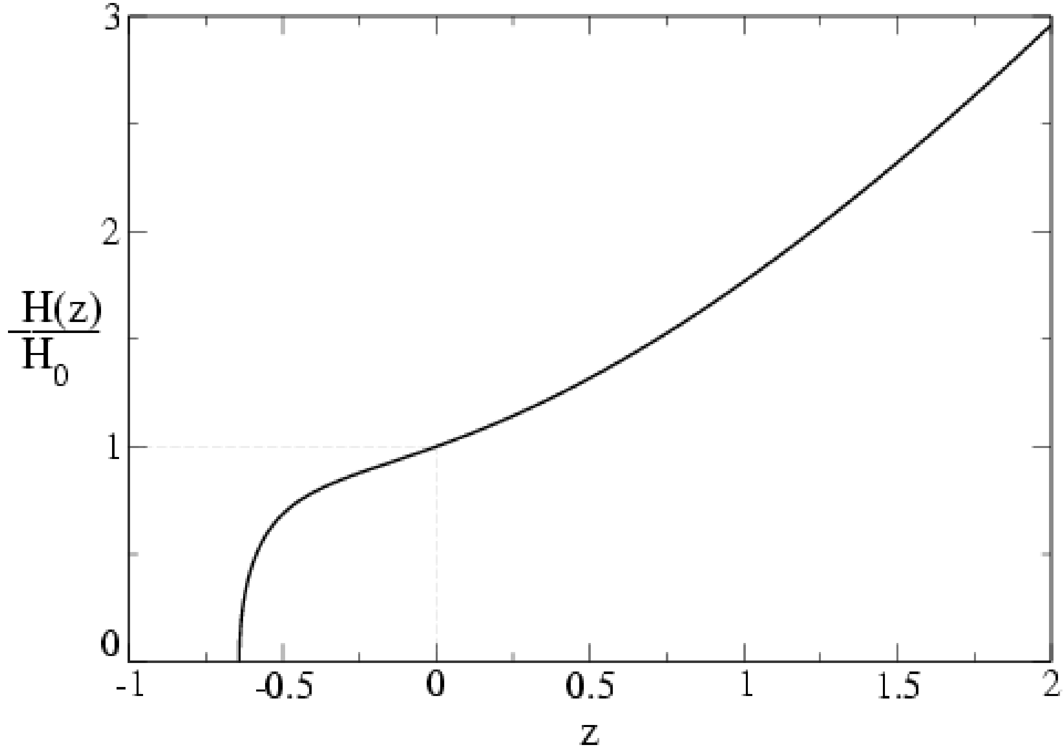
Since $\Omega_m^0 + \Omega_\Lambda^0 + \Omega_X^0 = 1 \;\Rightarrow\; \Omega_\Lambda^0 < 0$ is possible!

There can be stopping (turning point) of the evolution in the future!

However...

in the ΛXCDM model there are many other stopping possibilities !!

Example of Stopping



$$\Omega_{\Lambda} = 0.75,\ \omega_X = -1.85,\ \nu = -\nu_0$$

The next to simplest one is $-\delta < \alpha_X < 0 \,, \quad \nu = 0 \quad (\delta > 0)$

The cosmon has $\omega_X < -1 \;\Rightarrow$ phantom behavior!
(“standard” type)

Big Rip ?

Yes... except if $\Omega_X^0 < 0$ phantom matter!
(non-standard !)

$$\Omega_D(z) = \Omega_\Lambda^0 + \Omega_X^0\,(1+z)^{\alpha_X} \quad \Rightarrow$$ stopping with $\Omega_\Lambda^0 > 0$

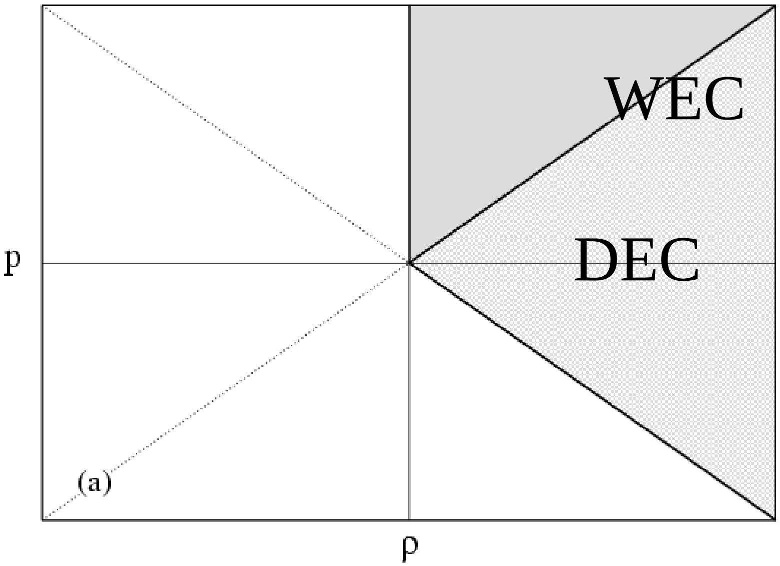
But in the ΛXCDM still many other stopping possibilities ...

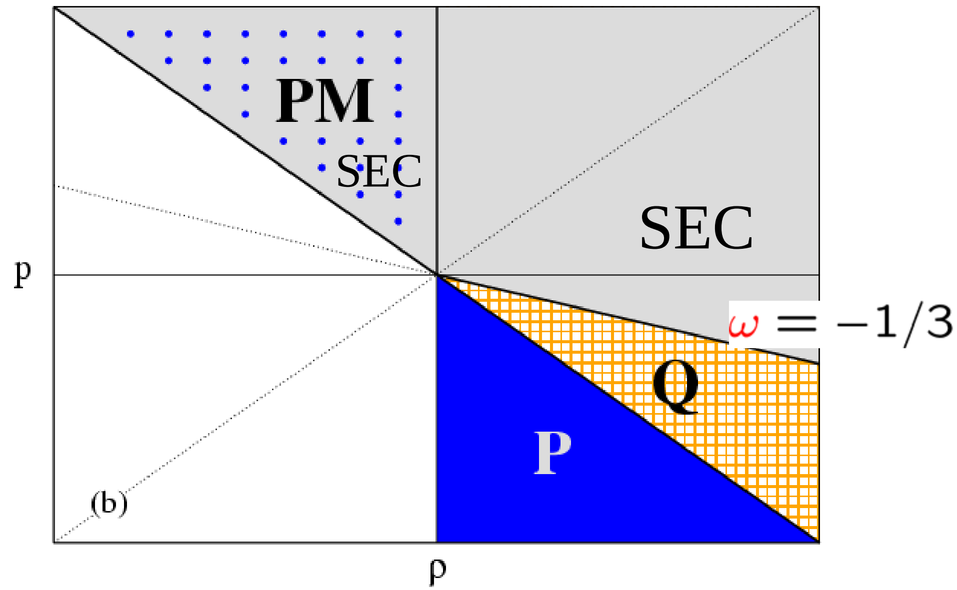
$\nu \neq 0$!!

Energy conditions

$$p = \omega \rho$$

$\omega < -1 \, (\rho < 0)$ **Phantom matter**





$$\rho + p \geq 0; \rho + 3p \geq 0 \ \text{(SEC)}$$

SEC ⇨ gravitational attraction

$$\omega < -1 \ (\rho > 0)$$

Phantom DE

Parameter Space

$$(\Omega_m, \Omega_\Lambda, \Omega_X, \omega_X, \nu)$$

Priors and constraints
$$\begin{cases} \Omega_m = 0.3\,, \quad \Omega_K^0 = 0 \\ \Omega_m^0 + \Omega_\Lambda^0 + \Omega_X^0 = 1 \end{cases}$$

$$\Downarrow$$

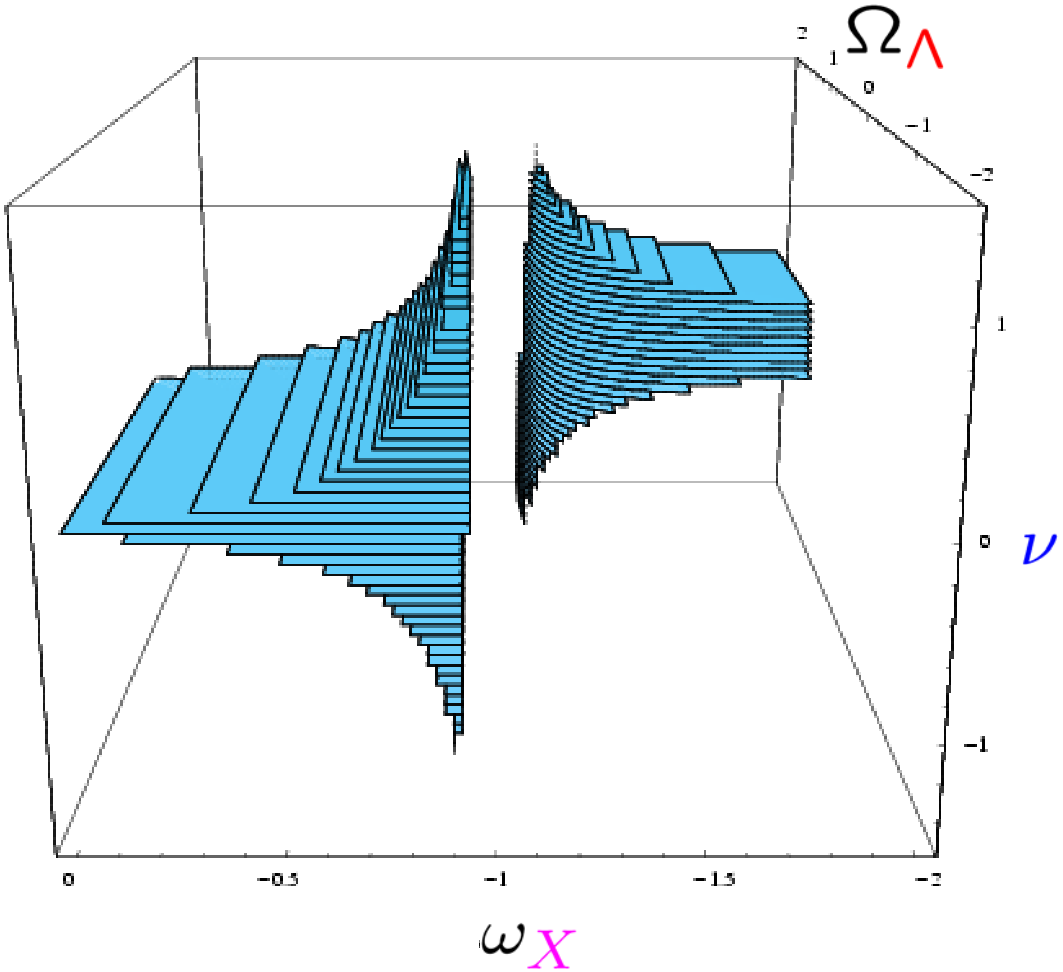
$$(\Omega_\Lambda, \omega_X, \nu)$$

Physical region?

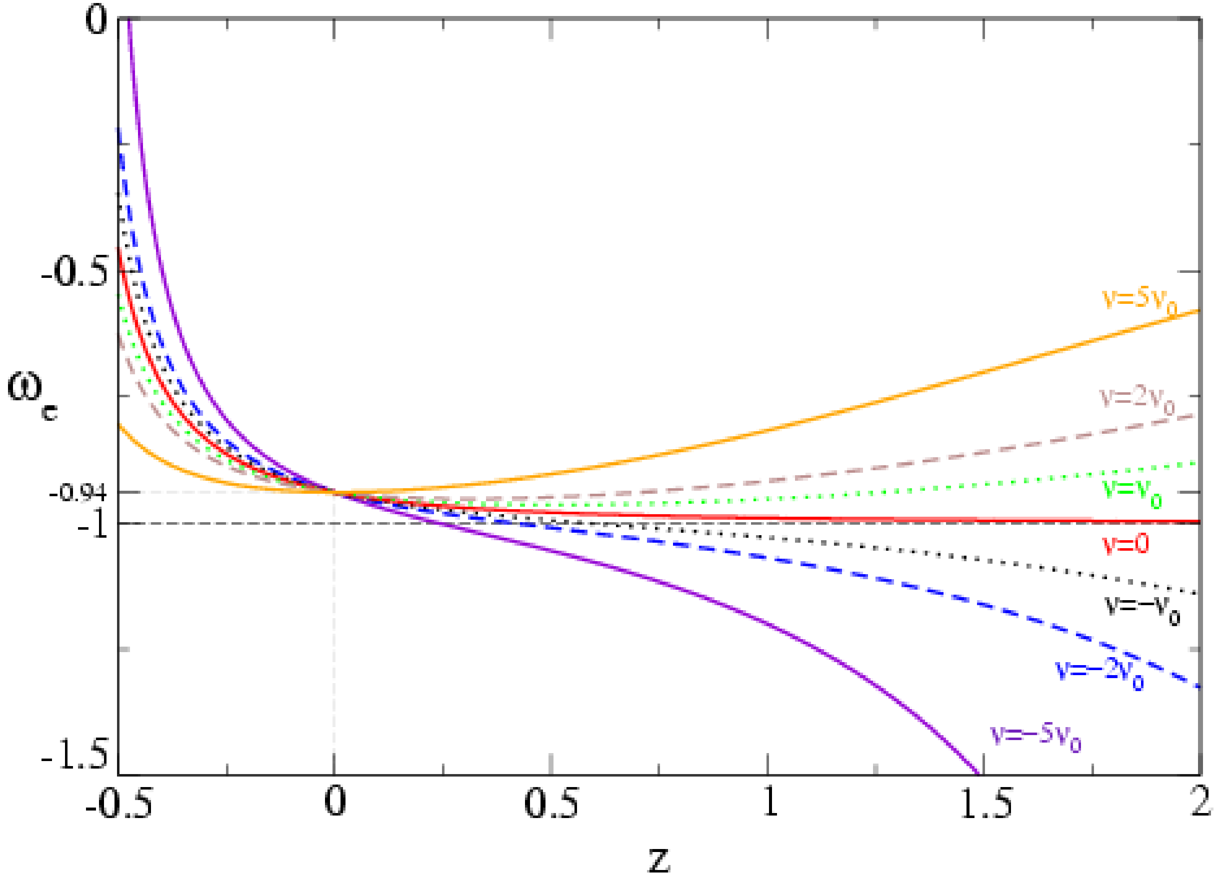
Subspace satisfying:

- i) Nucleosynthesis bound
- ii) Stopping condition
- iii) $r \equiv \rho_D/\rho_m < 10$

Physical subregion of $(\Omega_{\Lambda}, \omega_X, \nu)$



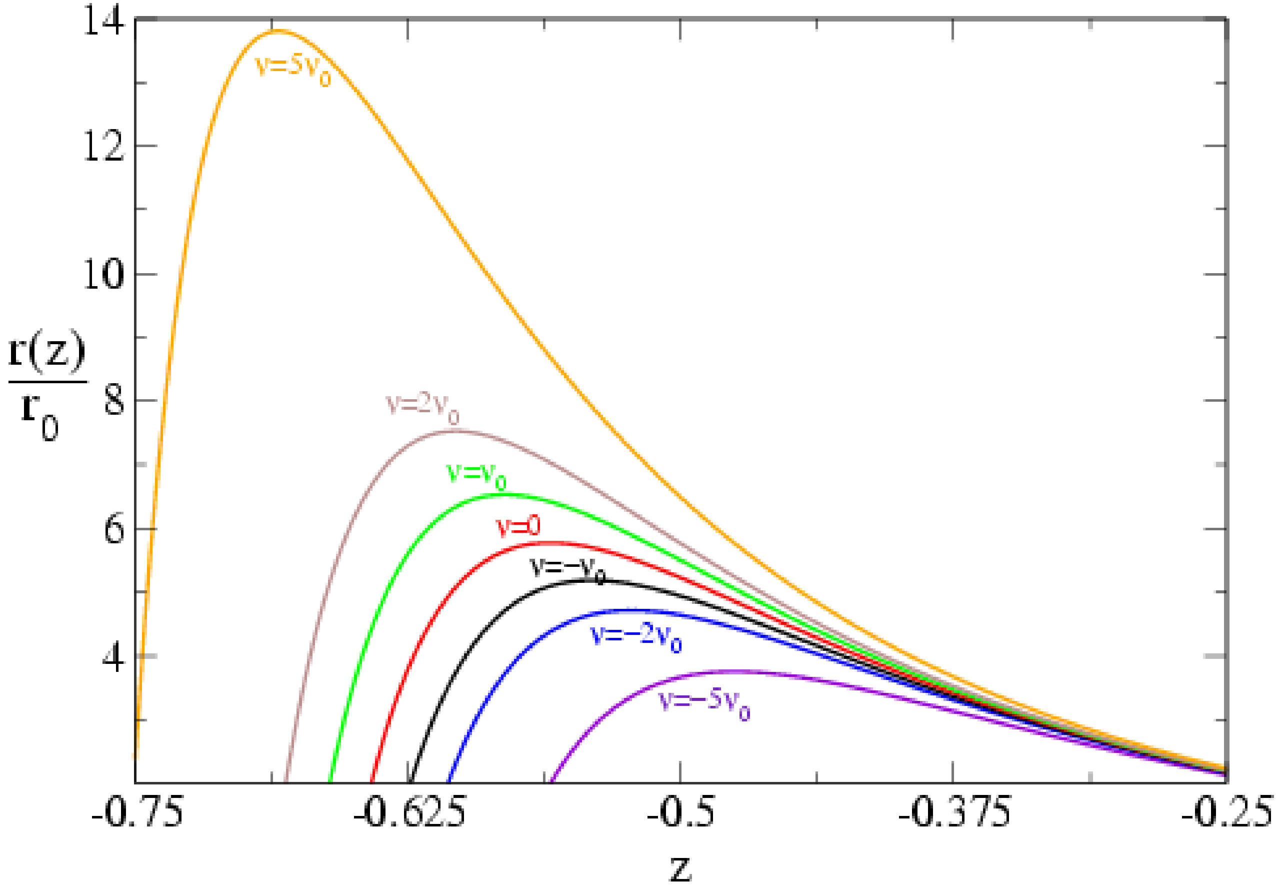
Effective EOS



Comparison of the effective EOS parameter of the ΛXCDM model, ω_e, for fixed values $\omega_X = -1.85$, $\Omega_\Lambda^0 = 0.75$, and different values of ν in units of ν_0. All curves give $\omega_e(0) = -0.94$ at the present time.

$$\left(\nu_0 = \frac{1}{12\pi} \simeq 0.026\right)$$

Evolution of the Ratio $r = \rho_D/\rho_m$



Evolution of r for $\omega_X = -1.85$, $\Omega_\Lambda = 0.75$ and different ν

➤ A ΛXCDM model without fine-tuning

The "Relaxed Universe"

F. Bauer, JS, H. Stefancic
PLB 678 (2009) ;B688 (2010)
+ in preparation

$$\mathcal{S} = \int d^4x \sqrt{|g|} \left[\frac{1}{16\pi G_N} R - \rho_\Lambda^i - \mathcal{F}(R, \mathcal{G}) + \mathcal{L}_\varphi \right]$$

Arbitrarily large

Field equations:

$$G^a_{\ b} = -8\pi G_N \left[\rho_\Lambda^i \delta^a_{\ b} + 2E^a_{\ b} + T^a_{\ b} \right]$$

$$E^0_{\ 0} = \frac{1}{2}\left[\mathcal{F}(R,\mathcal{G}) - 6(\dot{H} + H^2)\mathcal{F}^R + 6H\dot{\mathcal{F}}^R - 24H^2(\dot{H} + H^2)\mathcal{F}^\mathcal{G} + 24H^3\dot{\mathcal{F}}^\mathcal{G} \right]$$

$$E^i_{\ j} = \frac{1}{2}\delta^i_{\ j}\left[\mathcal{F}(R,\mathcal{G}) - 2(\dot{H} + 3H^2)\mathcal{F}^R + 4H\dot{\mathcal{F}}^R + 2\ddot{\mathcal{F}}^R - 24H^2(\dot{H} + H^2)\mathcal{F}^\mathcal{G} + 16H(\dot{H} + H^2)\dot{\mathcal{F}}^\mathcal{G} + 8H^2\ddot{\mathcal{F}}^\mathcal{G} \right],$$

To counterbalance ρ_Λ^i dynamically $\Rightarrow$

$$\mathcal{F}(R,\mathcal{G}) = \beta\, F(R,\mathcal{G}) + A(R)$$

$$F(R,\mathcal{G}) = \frac{1}{B(R,\mathcal{G})}$$

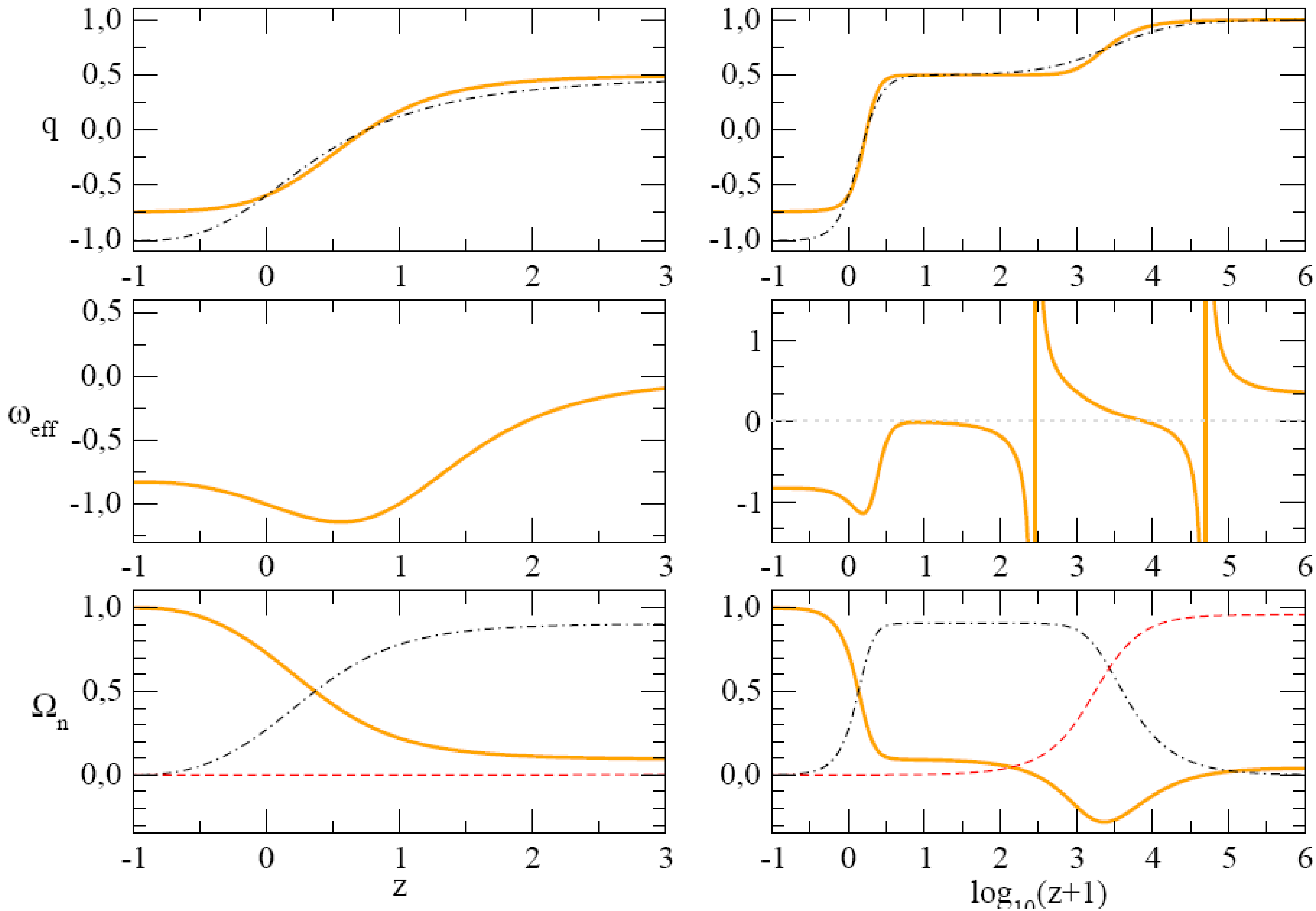
$$\Downarrow$$

$$B(R,\mathcal{G}) = \frac{2}{3}R^2 + \frac{1}{2}\mathcal{G} + (y\,R)^n$$

$$\Downarrow$$

$$= 24H^4(q - \tfrac{1}{2})(q-2) + \left[6yH^2(1-q)\right]^n$$

Taking now the full model ⇒ all the eras right!



Conclusions :

- **Dynamical dark energy** $\rho_{\rm D}$ can be mimicked by a variable Λ;
- In QFT we generally expect Λ and G to be variable: their value should have run in the course of the Universe evolution due to **quantum effects**;
- In general the total DE density can be the sum of a variable Λ and other dynamical entities (the **cosmon**: X): **ΛXCDM** models. In them, matter is separately conserved;
- **ΛXCDM** models may have **effective EOS** $\boxed{\omega_{\rm eff} = \omega_{\rm eff}(z)}$ which can be of **quintessence** and **phantom** type and even exhibit a transition from one regime to the other;
- The dynamics of the **ΛXCDM** models can be such that their **effective EOS** is arbitrarily close to $\omega_{\rm eff} = -1$ (!) around our present time, thus mimicking an almost perfect standard model **ΛCDM** cosmology;
- However, the "hidden" dynamics of **ΛXCDM** can manifest in the future, where the ratio

$$1 \lesssim \frac{\rho_{\rm D}}{\rho_{\rm m}} \lesssim 10 \quad \Rightarrow \textbf{no cosmic coincidence!}$$

- **Observable effect** :**renormalization** of $\Omega_{\rm m}$ when comparing intermediate redshift data (from supernovae) and high-z (from CMB);

$$\Downarrow$$

- **Moral**: high precision cosmology experiments in the near future, like **SNAP** and **PLANCK**, should bear in mind this possibility!!

$\Rightarrow$ A large class of ΛXCDM models can solve the fine-tuning problem and hence could have great impact on the old CC problem **!!**

Maybe in NEB 15...

Σας ευχαριστώ πάρα πολύ!