Affine gauge gravity and its reduction to the Riemann-Cartan geometry

Rodrigo F. Sobreiro

Universidade Federal Fluminense

Niterói - Brazil

NEB-14 : Recent Developments in Gravity

Ioannina, Greece : 08-11 June 2010

Gravity as a (affine) gauge theory

Gravity as a (affine) gauge theory

- Affine group: $A(d,\mathbb{R}) = GL(d,\mathbb{R}) \ltimes \mathbb{R}^d$:

$$[gl, gl] \subseteq gl \ , \ \ [gl, r] \subseteq r \ , \ \ [r, r] \subseteq 0 \ .$$

Gravity as a (affine) gauge theory

- Affine group: $A(d,\mathbb{R}) = GL(d,\mathbb{R}) \ltimes \mathbb{R}^d$:

$$[gl, gl] \subseteq gl \ , \ \ [gl, r] \subseteq r \ , \ \ [r, r] \subseteq 0 \ .$$

- General linear group: $GL(d,\mathbb{R}) = O(d) \otimes K(d)$:

$$[o, o] \subseteq o \ , \ \ [o, k] \subseteq k \ , \ \ [k, k] \subseteq o \ .$$

Gravity as a (affine) gauge theory

- Affine group: $A(d,\mathbb{R}) = GL(d,\mathbb{R}) \ltimes \mathbb{R}^d$:

$$[gl, gl] \subseteq gl\ ,\ \ [gl, r] \subseteq r\ ,\ \ [r, r] \subseteq 0\ .$$

- General linear group: $GL(d,\mathbb{R}) = O(d) \otimes K(d)$:

$$[o, o] \subseteq o\ ,\ \ [o, k] \subseteq k\ ,\ \ [k, k] \subseteq o\ .$$

- Fundamental 1-form fields (Utiyamma, Kibble, Zanelli, Hehl...): gauge-connection $Y = \omega + E$, the vielbein e; and the 0=form premetric g.

Gravity as a (affine) gauge theory

- Affine group: $A(d,\mathbb{R}) = GL(d,\mathbb{R}) \ltimes \mathbb{R}^d$:

$$[gl, gl] \subseteq gl \ , \ \ [gl, r] \subseteq r \ , \ \ [r, r] \subseteq 0 \ .$$

- General linear group: $GL(d,\mathbb{R}) = O(d) \otimes K(d)$:

$$[o, o] \subseteq o \ , \ \ [o, k] \subseteq k \ , \ \ [k, k] \subseteq o \ .$$

- Fundamental 1-form fields (Utiyamma, Kibble, Zanelli, Hehl...): gauge-connection $Y = \omega + E$, the vielbein e; and the 0=form premetric g.

- Gauge transformations:

$$\begin{aligned} \delta\omega^a{}_b &= D\alpha^a{}_b\,, \quad \delta E^a = D\xi^a + E^b\alpha^a{}_b\,, \\ \delta e^a &= -\alpha^a{}_b e^b\,, \quad \delta g_{ab} = \alpha^c{}_a g_{bc} + \alpha^c{}_b g_{ac} \end{aligned}$$

- Gauge transformations:

$$\begin{aligned} \delta\omega^a{}_b &= D\alpha^a{}_b\ , \quad \delta E^a = D\xi^a + E^b\alpha^a{}_b\ , \\ \delta e^a &= -\alpha^a{}_b e^b\ , \quad \delta g_{ab} = \alpha^c{}_a g_{bc} + \alpha^c{}_b g_{ac} \end{aligned}$$

- Relation with geometry:

$$\mathfrak{g} = g_{ab} e^a \otimes e^b \quad , \qquad\qquad \Gamma = \Gamma(\omega, e)\ .$$

- Gauge transformations:

$$\delta\omega^a{}_b = D\alpha^a{}_b \ , \quad \delta E^a = D\xi^a + E^b\alpha^a{}_b \ ,$$
$$\delta e^a = -\alpha^a{}_b e^b \ , \quad \delta g_{ab} = \alpha^c{}_a g_{bc} + \alpha^c{}_b g_{ac}$$

- Relation with geometry:

$$\mathfrak{g} = g_{ab} e^a \otimes e^b \ , \qquad \Gamma = \Gamma(\omega, e) \ .$$

- Gauge transformations are local and maintain spacetime fixed.

- Gauge transformations:

$$\delta\omega^a{}_b = D\alpha^a{}_b \,, \;\; \delta E^a = D\xi^a + E^b\alpha^a{}_b \,,$$
$$\delta e^a = -\alpha^a{}_b e^b \,, \;\; \delta g_{ab} = \alpha^c{}_a g_{bc} + \alpha^c{}_b g_{ac}$$

- Relation with geometry:

$$\mathfrak{g} = g_{ab} e^a \otimes e^b \;\;, \qquad\qquad \Gamma = \Gamma(\omega, e) \;.$$

- Gauge transformations are local and maintain spacetime fixed.
- e: gauge symmetry $\leftrightarrow$ diffeomorphisms through $A(d,\mathbb{R}) \leftrightarrow *\mathbb{T}^d(\mathbb{M})$.

- Gauge transformations:

$$\begin{aligned} \delta\omega^a{}_b &= D\alpha^a{}_b\ , \ \ \delta E^a \ = \ D\xi^a + E^b\alpha^a{}_b\ , \\ \delta e^a &= -\alpha^a{}_b e^b\ , \ \ \delta g_{ab} \ = \ \alpha^c{}_a g_{bc} + \alpha^c{}_b g_{ac} \end{aligned}$$

- Relation with geometry:

$$\mathfrak{g} = g_{ab} e^a \otimes e^b \quad , \qquad\qquad \Gamma = \Gamma(\omega, e)\ .$$

- Gauge transformations are local and maintain spacetime fixed.

- e: gauge symmetry $\leftrightarrow$ diffeomorphisms through $A(d, \mathbb{R}) \leftrightarrow *\mathbb{T}^d(\mathbb{M})$.

(Co)frame bundle

(Co)frame bundle

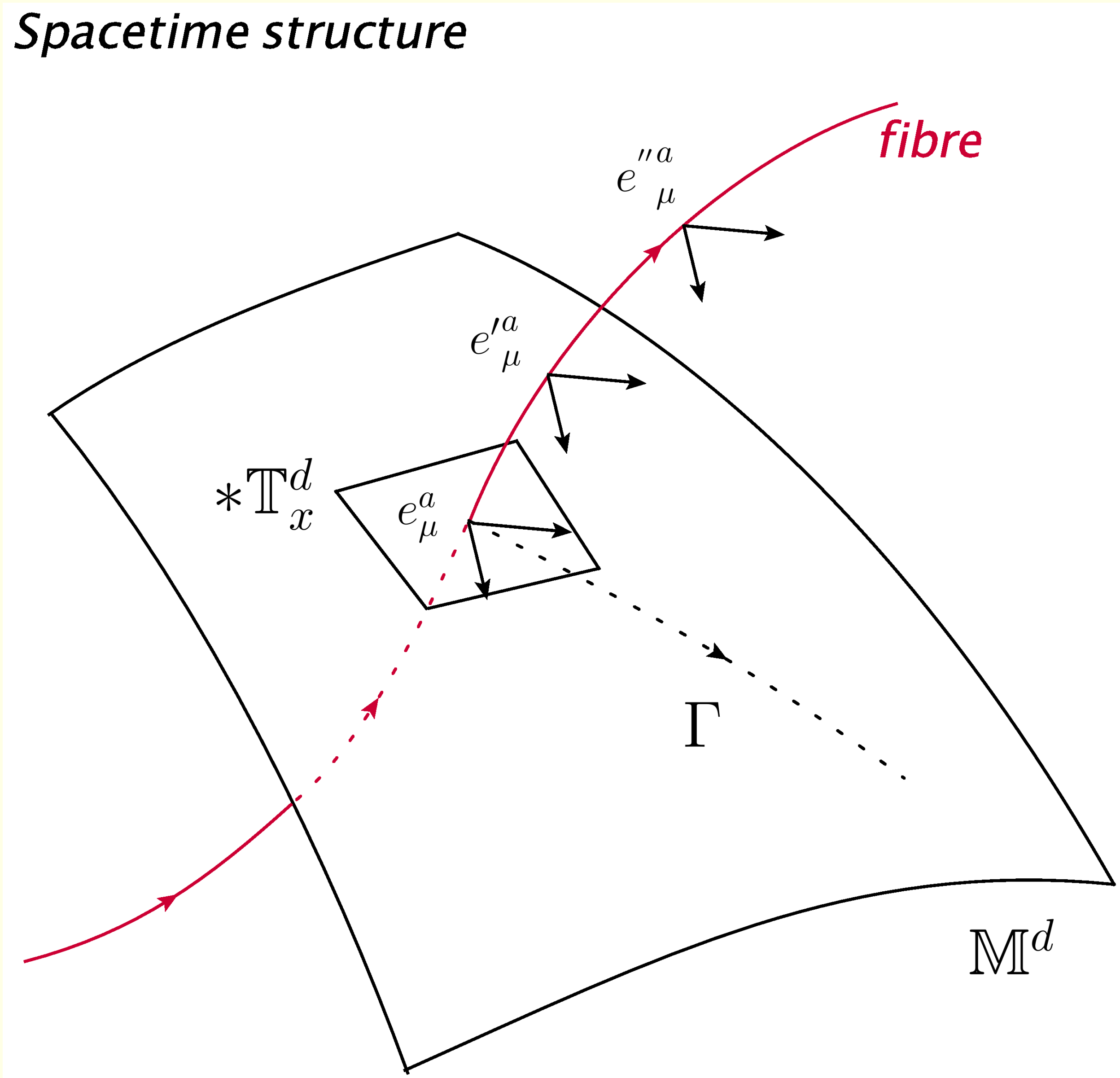
- 

Figura 1: First order geometry.

•

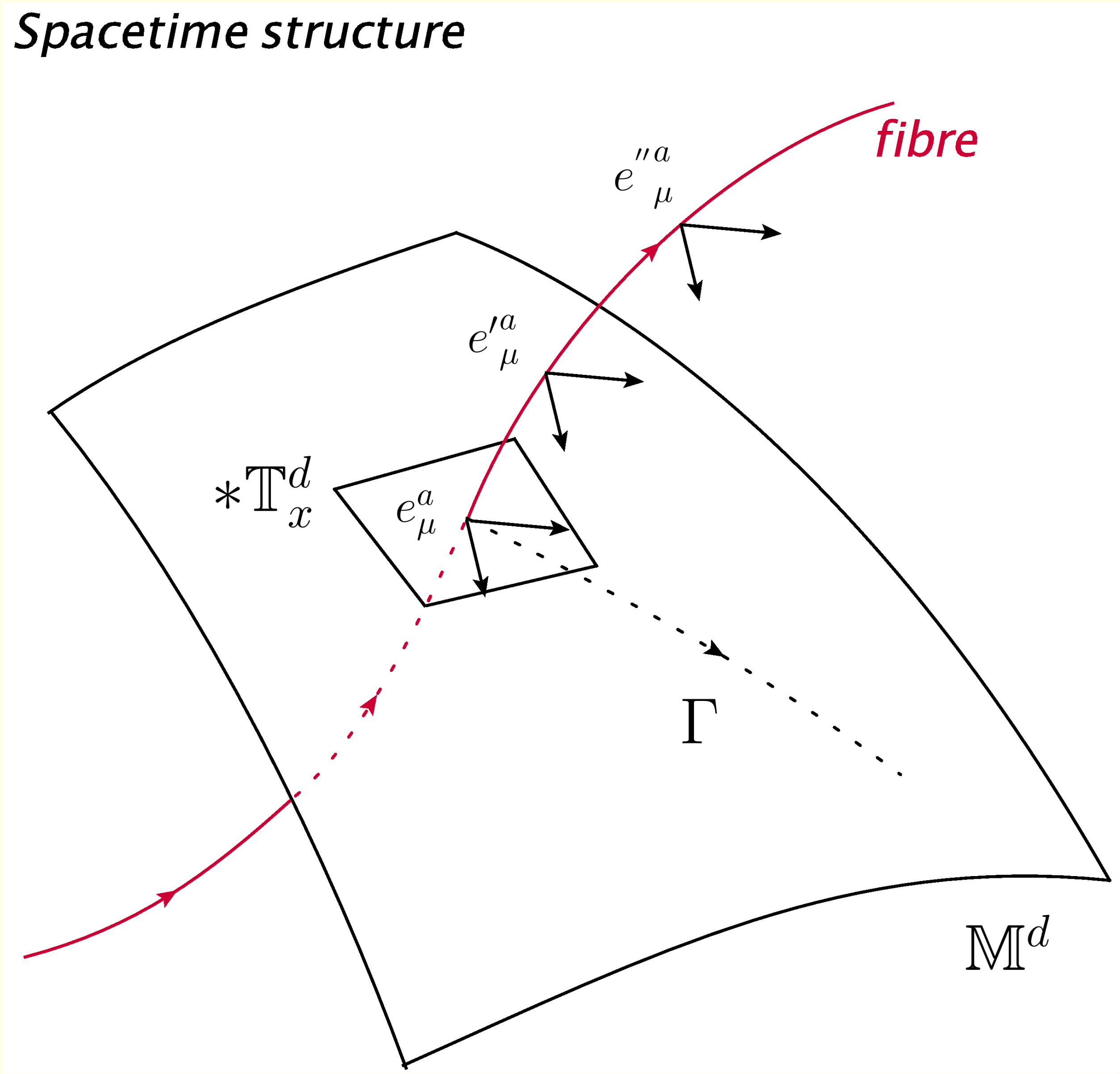
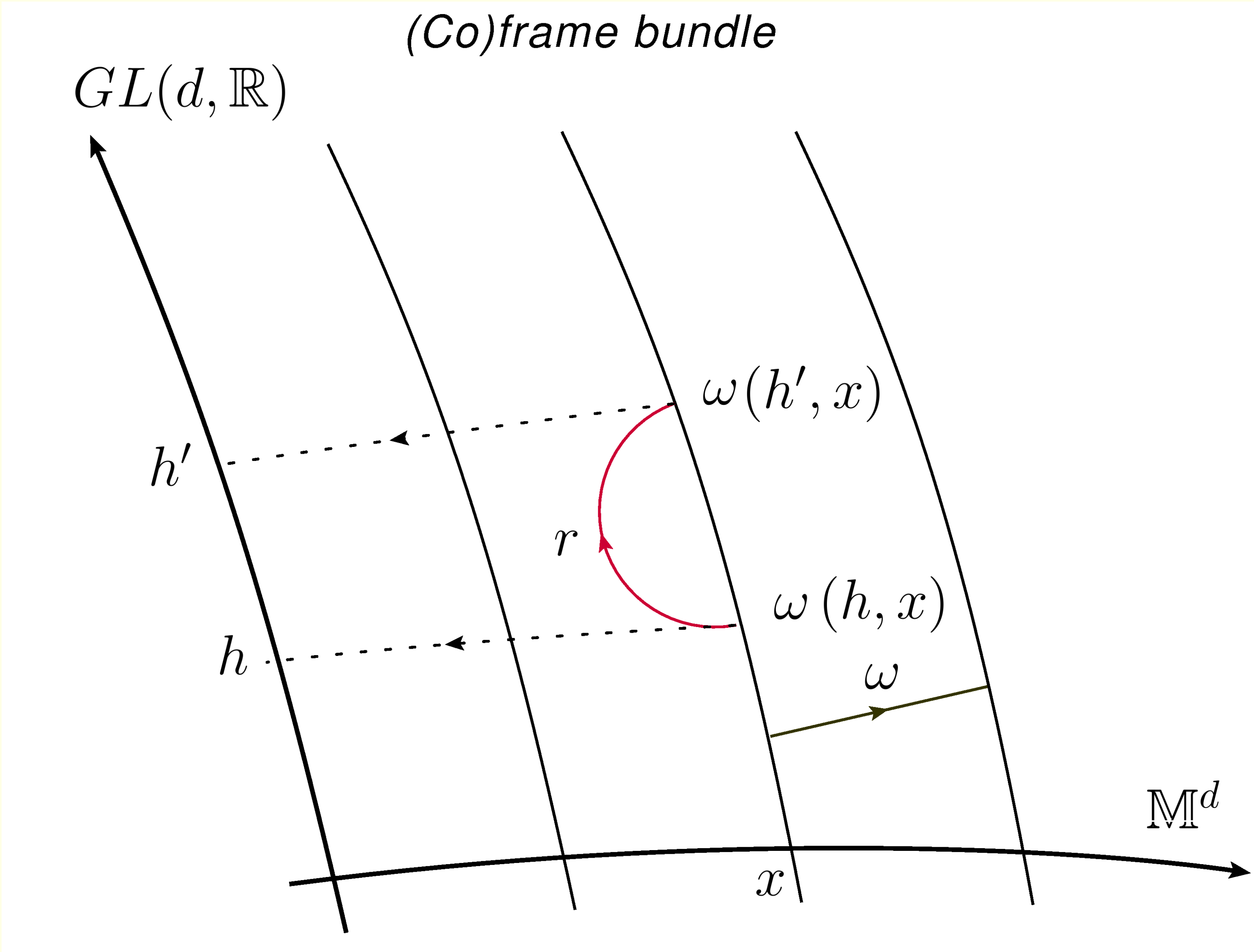


Figura 1: First order geometry.

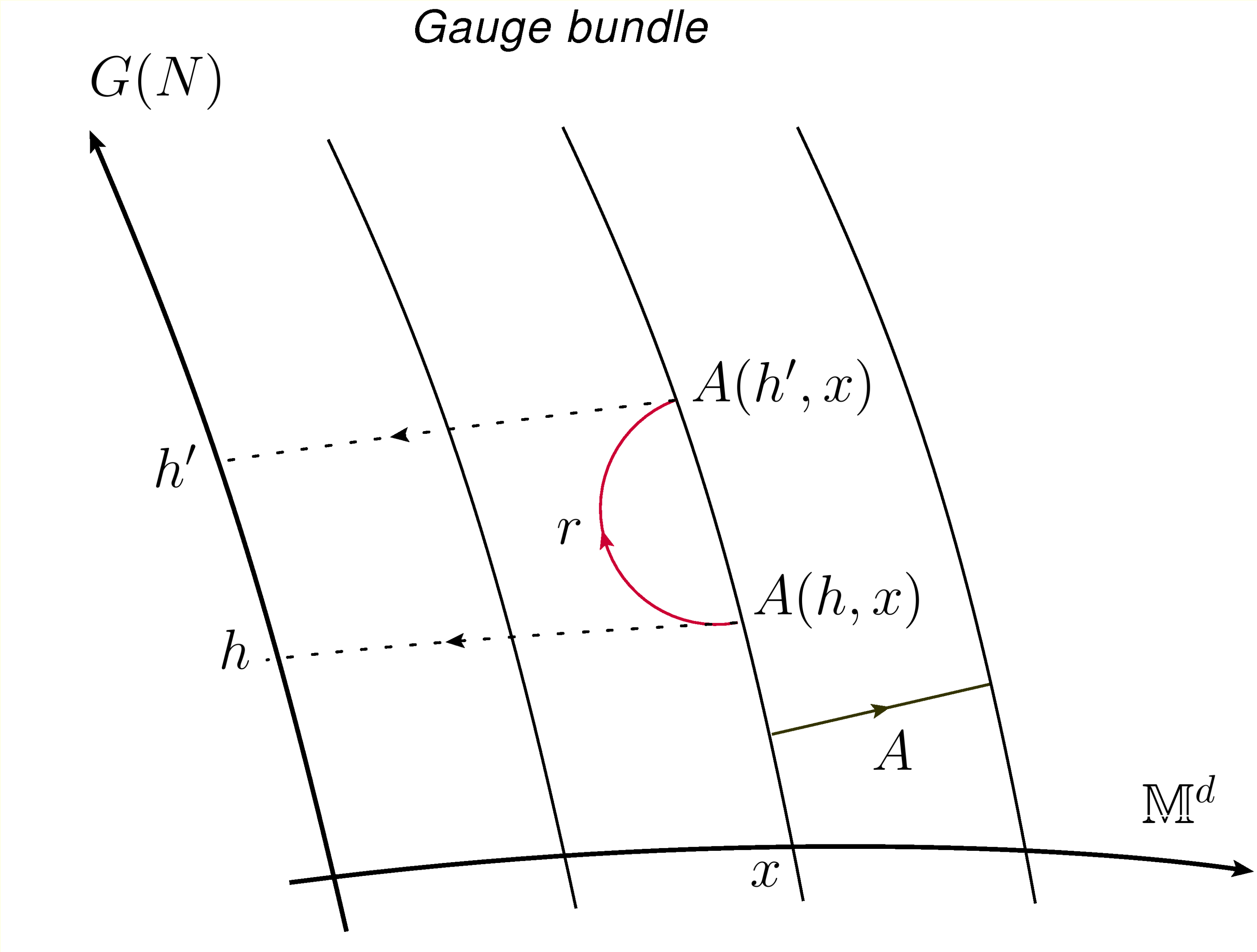


•

Figura 2: Gauge structure of gravity.

General gauge theories

General gauge theories



•

Figura 3: Basic principal bundle for gauge theories.

- Gauge space and spacetime are independent structures.

- Gauge space and spacetime are independent structures.
- The gauge bundle is static! So is the coframe bundle $\omega = \omega(e) \equiv$ second order gravity.

- Gauge space and spacetime are independent structures.
- The gauge bundle is static! So is the coframe bundle $\omega = \omega(e) \equiv$ second order gravity.
- Dynamical gauge bundle: Base space = all independent gauge connections $\mathcal{A}$ called moduli space; Structure group $G(N)$ = local group; Fibre: gauge orbits (group space copies):

$$A^h = h^{-1}(d + A)h \ .$$

- Gauge space and spacetime are independent structures.
- The gauge bundle is static! So is the coframe bundle $\omega = \omega(e) \equiv$ second order gravity.
- Dynamical gauge bundle: Base space = all independent gauge connections $\mathcal{A}$ called moduli space; Structure group $G(N)$ = local group; Fibre: gauge orbits (group space copies):

$$A^h = h^{-1}(d + A)h \; .$$

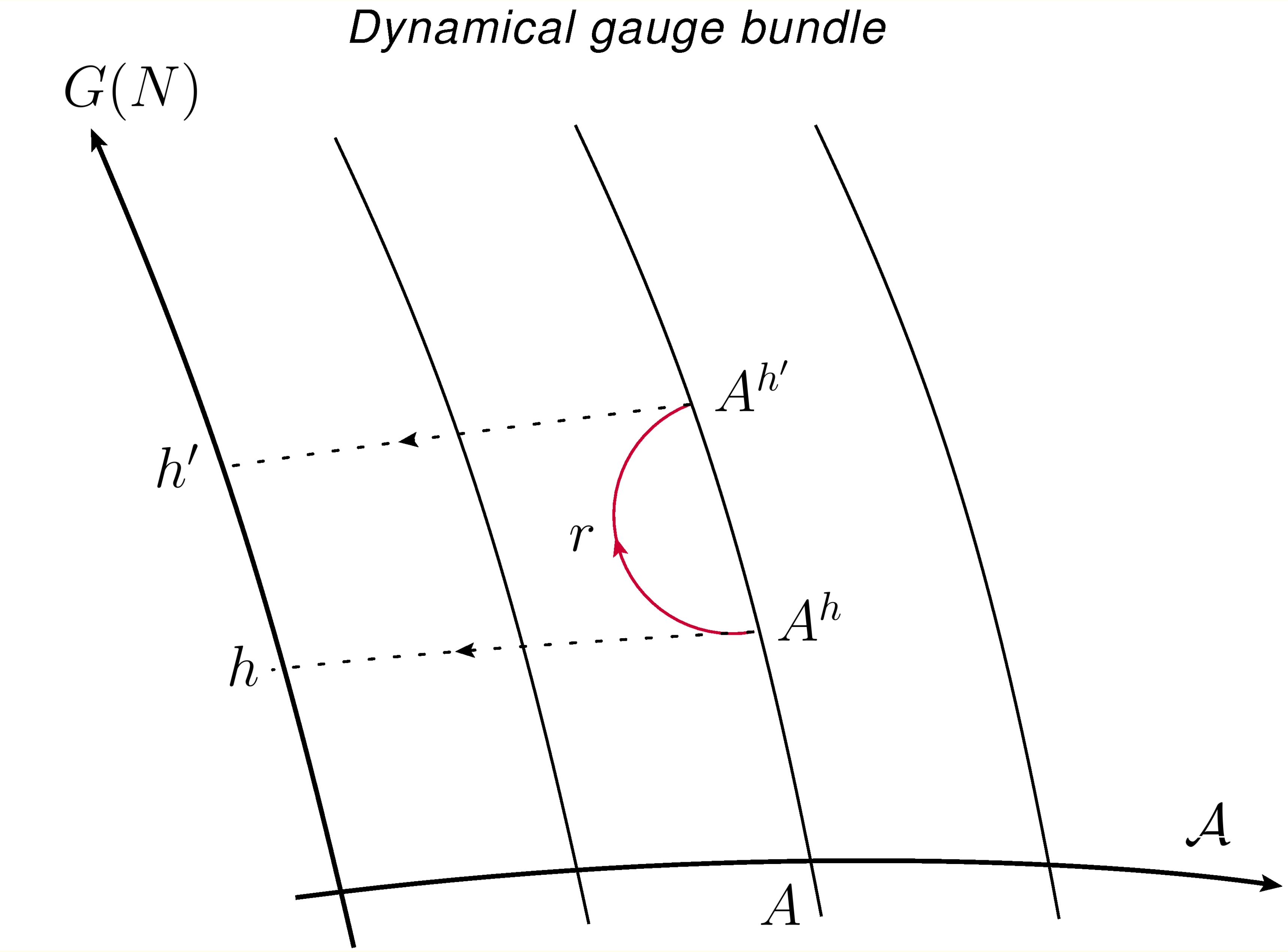
- 

Figura 4:

Gravity: QFT vs. Geometry

Gravity: QFT vs. Geometry

- Problems with the coframe bundle: e^a_μ and g_{ab} are dimensionless; e^a_μ induces a spacetime renormalization; EH action is a pure interacting term, there are no kinematical terms; Coframe bundle is static; ...

Gravity: QFT vs. Geometry

- Problems with the coframe bundle: e_μ^a and g_{ab} are dimensionless; e_μ^a induces a spacetime renormalization; EH action is a pure interacting term, there are no kinematical terms; Coframe bundle is static; ...

- Proposition for quantum gravity: start with independent gauge group $A(d, \mathbb{R})$ and spacetime. Gauge space is NOT identified with the tangent bundle.

Gravity: QFT vs. Geometry

- Problems with the coframe bundle: e^a_μ and g_{ab} are dimensionless; e^a_μ induces a spacetime renormalization; EH action is a pure interacting term, there are no kinematical terms; Coframe bundle is static; ...

- Proposition for quantum gravity: start with independent gauge group $A(d, \mathbb{R})$ and spacetime. Gauge space is NOT identified with the tangent bundle.

- At quantum regime: e and g are simple algebra-valued matter fields. They are not $\mathbb{M} \leftrightarrow \mathbb{T}$ mappings. Y is a gauge connection.

Gravity: QFT vs. Geometry

- Problems with the coframe bundle: e^a_μ and g_{ab} are dimensionless; e^a_μ induces a spacetime renormalization; EH action is a pure interacting term, there are no kinematical terms; Coframe bundle is static; ...

- Proposition for quantum gravity: start with independent gauge group $A(d, \mathbb{R})$ and spacetime. Gauge space is NOT identified with the tangent bundle.

- At quantum regime: e and g are simple algebra-valued matter fields. They are not $\mathbb{M} \leftrightarrow \mathbb{T}$ mappings. Y is a gauge connection.

- Such id. must emerge naturally in some regime. $\mathfrak{g} = g_{ab} e^a \otimes e^b$.

- UV gauge theory $\leftrightarrow$ IR geometry.

- UV gauge theory $\leftrightarrow$ IR geometry.
- $\Rightarrow$ Start with a MASSLESS dynamical gauge bundle for the affine group.

- UV gauge theory $\leftrightarrow$ IR geometry.
- $\Rightarrow$ Start with a MASSLESS dynamical gauge bundle for the affine group.
- Y: gauge field; e: matter field with dim=1; g: matter field with dim=1. Disables a possible gauge-geometry id.

- UV gauge theory $\leftrightarrow$ IR geometry.
- $\Rightarrow$ Start with a MASSLESS dynamical gauge bundle for the affine group.
- Y: gauge field; e: matter field with dim=1; g: matter field with dim=1. Disables a possible gauge-geometry id.
- **Transition to geometry:** dynamical effects induces mass parameters (dynamical masses, natural RG scales, Gribov pathologies...)

$$e_1 \longmapsto \mu e_0 \ , \quad g_1 \longmapsto \mu g_0 \ .$$

- UV gauge theory $\leftrightarrow$ IR geometry.
- $\Rightarrow$ Start with a MASSLESS dynamical gauge bundle for the affine group.
- Y: gauge field; e: matter field with dim=1; g: matter field with dim=1. Disables a possible gauge-geometry id.
- **Transition to geometry:** dynamical effects induces mass parameters (dynamical masses, natural RG scales, Gribov pathologies...)

$$e_1 \longmapsto \mu e_0 \ , \quad g_1 \longmapsto \mu g_0 \ .$$

- Thus, μ regulates UV-IR regimes.

- UV gauge theory $\leftrightarrow$ IR geometry.
- $\Rightarrow$ Start with a MASSLESS dynamical gauge bundle for the affine group.
- Y: gauge field; e: matter field with dim=1; g: matter field with dim=1. Disables a possible gauge-geometry id.
- **Transition to geometry:** dynamical effects induces mass parameters (dynamical masses, natural RG scales, Gribov pathologies...)

$$e_1 \longmapsto \mu e_0 \ , \quad g_1 \longmapsto \mu g_0 \ .$$

- Thus, μ regulates UV-IR regimes.

Contraction

Contraction

- Principal bundles might be reduced to smaller bundles if there are contractible sectors.

Contraction

- Principal bundles might be reduced to smaller bundles if there are contractible sectors.
- The connection is as well reduced if, and only if, the contracted coset space is symmetric (the coset space is a tensor space with respect the stability group).

Contraction

- Principal bundles might be reduced to smaller bundles if there are contractible sectors.
- The connection is as well reduced if, and only if, the contracted coset space is symmetric (the coset space is a tensor space with respect the stability group).
- Known results (McInnes (1984); Sobreiro & Vasquez (2010)):

$$
\begin{aligned}
A(d,\mathbb{R}) &\longmapsto GL(d,\mathbb{R}) \longmapsto O(d)\ , \\
A(d,\mathbb{R}) &\rightsquigarrow IO(d)\ .
\end{aligned}
$$

Contraction

- Principal bundles might be reduced to smaller bundles if there are contractible sectors.
- The connection is as well reduced if, and only if, the contracted coset space is symmetric (the coset space is a tensor space with respect the stability group).
- Known results (McInnes (1984); Sobreiro & Vasquez (2010)):

$$
\begin{aligned}
A(d,\mathbb{R}) &\longmapsto GL(d,\mathbb{R}) \longmapsto O(d)\ ,\\
A(d,\mathbb{R}) &\rightsquigarrow IO(d)\ .
\end{aligned}
$$

- So, all STATIC theories end up into the usual Riemann-Cartan orthogonal bundle.

Contraction

- Principal bundles might be reduced to smaller bundles if there are contractible sectors.

- The connection is as well reduced if, and only if, the contracted coset space is symmetric (the coset space is a tensor space with respect the stability group).

- Known results (McInnes (1984); Sobreiro & Vasquez (2010)):

$$
\begin{aligned}
A(d,\mathbb{R}) \;&\longmapsto\; GL(d,\mathbb{R}) \longmapsto O(d)\;, \\
A(d,\mathbb{R}) \;&\rightsquigarrow\; IO(d)\;.
\end{aligned}
$$

- So, all STATIC theories end up into the usual Riemann-Cartan orthogonal bundle.

- Dynamical bundle: reduces as well, but...

Contraction

- Principal bundles might be reduced to smaller bundles if there are contractible sectors.

- The connection is as well reduced if, and only if, the contracted coset space is symmetric (the coset space is a tensor space with respect the stability group).

- Known results (McInnes (1984); Sobreiro & Vasquez (2010)):

$$
\begin{aligned}
A(d,\mathbb{R}) &\longmapsto GL(d,\mathbb{R}) \longmapsto O(d)\;, \\
A(d,\mathbb{R}) &\rightsquigarrow IO(d)\;.
\end{aligned}
$$

- So, all STATIC theories end up into the usual Riemann-Cartan orthogonal bundle.

- Dynamical bundle: reduces as well, but...

- All original fields are dynamical: $Y = w + q + E$. The fibre (gauge orbit) decomposes under contraction:

$$\begin{aligned} w^L &= L^{-1}(d + w)L \ , \\ q^L &= L^{-1} q L \ , \\ E^L &= L^{-1} E \ , \end{aligned} \tag{1}$$

- All original fields are dynamical: $Y = w + q + E$. The fibre (gauge orbit) decomposes under contraction:

$$
\begin{aligned}
w^L &= L^{-1}(d+w)L \; , \\
q^L &= L^{-1} q L \; , \\
E^L &= L^{-1} E \; ,
\end{aligned}
\tag{1}
$$

- The original matter fields:

$$
\begin{aligned}
e &= L^{-1} e \; , \\
g^L &= L^{-1} g (L^{-1})^T \; ,
\end{aligned}
\tag{2}
$$

- All original fields are dynamical: $Y = w + q + E$. The fibre (gauge orbit) decomposes under contraction:

$$\begin{aligned} w^L &= L^{-1}(d+w)L \ , \\ q^L &= L^{-1}qL \ , \\ E^L &= L^{-1}E \ , \end{aligned} \tag{1}$$

- The original matter fields:

$$\begin{aligned} e &= L^{-1}e \ , \\ g^L &= L^{-1}g(L^{-1})^T \ , \end{aligned} \tag{2}$$

- So q and E are now additional matter fields. Also, e and E obey the same transformation rule!

- All original fields are dynamical: $Y = w + q + E$. The fibre (gauge orbit) decomposes under contraction:

$$
\begin{aligned}
w^L &= L^{-1}(d+w)L \,, \\
q^L &= L^{-1}qL \,, \\
E^L &= L^{-1}E \,,
\end{aligned}
\tag{1}
$$

- The original matter fields:

$$
\begin{aligned}
e &= L^{-1}e \,, \\
g^L &= L^{-1}g(L^{-1})^T \,,
\end{aligned}
\tag{2}
$$

- So q and E are now additional matter fields. Also, e and E obey the same transformation rule!

- Finally (Sobreiro & Vasquez (2010)): $\mathbb{A} \longmapsto \mathbb{O} \oplus \mathcal{E} \oplus \mathcal{Q}$. An Affine gauge theory reduces to the usual orthogonal one with additional matter fields.

- Finally (Sobreiro & Vasquez (2010)): $\mathbb{A} \longmapsto \mathbb{O} \oplus \mathcal{E} \oplus \mathcal{Q}$. An Affine gauge theory reduces to the usual orthogonal one with additional matter fields.

Identifying geometry

- First option: identify the affine gauge theory with the cotangent bundle of spacetime $\equiv$ **Metric-Affine Gravity** (Hehl (1976))

- Finally (Sobreiro & Vasquez (2010)): $\mathbb{A} \longmapsto \mathbb{O} \oplus \mathcal{E} \oplus \mathcal{Q}$. An Affine gauge theory reduces to the usual orthogonal one with additional matter fields.

Identifying geometry

- First option: identify the affine gauge theory with the cotangent bundle of spacetime $\equiv$ **Metric-Affine Gravity** (Hehl (1976))
- Second option: Reduce to the Riemann-Cartan bundle and then identify it with the cotangent bundle. Careful to keep all degrees of freedom.

- Finally (Sobreiro & Vasquez (2010)): $\mathbb{A} \longmapsto \mathbb{O} \oplus \mathcal{E} \oplus \mathcal{Q}$. An Affine gauge theory reduces to the usual orthogonal one with additional matter fields.

Identifying geometry

- First option: identify the affine gauge theory with the cotangent bundle of spacetime $\equiv$ **Metric-Affine Gravity** (Hehl (1976))
- Second option: Reduce to the Riemann-Cartan bundle and then identify it with the cotangent bundle. Careful to keep all degrees of freedom.
- So, contracting the affine curvature:

$$
\begin{aligned}
\Omega^a{}_b &= R^a{}_b + V^a{}_b \, , \\
\Xi^a &= DE^a - q^a{}_b E^b \, ,
\end{aligned}
$$

where

$$
\begin{aligned}
R^a{}_b &= dw^a{}_b + w^a{}_c w^c{}_b \ , \\
V^a{}_b &= Dq^a{}_b - q^a{}_c q^c{}_b \ ,
\end{aligned}
$$

And the minimal couplings:

$$
\begin{aligned}
T^a &= De^a - q^a{}_b e^b \ , \\
Q_{ab} &= Dg_{ab} + q^c{}_a g_{cb} + q^c{}_b g_{ca} \ ,
\end{aligned}
$$

where

$$
\begin{aligned}
R^a{}_b &= dw^a{}_b + w^a{}_c w^c{}_b \ , \\
V^a{}_b &= Dq^a{}_b - q^a{}_c q^c{}_b \ ,
\end{aligned}
$$

And the minimal couplings:

$$
\begin{aligned}
T^a &= De^a - q^a{}_b e^b \ , \\
Q_{ab} &= Dg_{ab} + q^c{}_a g_{cb} + q^c{}_b g_{ca} \ ,
\end{aligned}
$$

- Then, identifying with geometry ($e \longmapsto \mu$ coframes):

$$
\begin{aligned}
g_{ab} &= \delta_{ab} + \gamma_{ab} \ , \\
\mathfrak{g} &= \delta_{ab} e^a \otimes e^b \ .
\end{aligned}
$$

Thus

$$T^a = De^a \; ,$$

$$Q_{ab} = D\gamma_{ab} + 2q_{ab} + q^c{}_a\gamma_{bc} + q^c{}_b\gamma_{ac} \; .$$

Thus

$$T^a = De^a ,$$
$$Q_{ab} = D\gamma_{ab} + 2q_{ab} + q^c{}_a \gamma_{bc} + q^c{}_b \gamma_{ac} .$$

- Conclusion: START- gauge conn.Y and matter.(e, g). END- gauge conn.w, vielbein.e, matter.$(E, q, \gamma) \equiv$ Riemann-Cartan geometry plus matter.

Conclusions and perspectives

Thus

$$T^a = De^a \,,$$
$$Q_{ab} = D\gamma_{ab} + 2q_{ab} + q^c{}_a\gamma_{bc} + q^c{}_b\gamma_{ac} \,.$$

- Conclusion: START- gauge conn.Y and matter.(e, g). END- gauge conn.w, vielbein.e, matter.$(E, q, \gamma) \equiv$ Riemann-Cartan geometry plus matter.

Conclusions and perspectives

- Independently of the starting "action": any affine gauge theory reduces to the Riemann-Cartan one with additional matter fields.

Thus

$$T^a = De^a ,$$
$$Q_{ab} = D\gamma_{ab} + 2q_{ab} + q^c{}_a\gamma_{bc} + q^c{}_b\gamma_{ac} .$$

- Conclusion: START- gauge conn.Y and matter.(e, g). END- gauge conn.w, vielbein.e, matter.$(E, q, \gamma) \equiv$ Riemann-Cartan geometry plus matter.

Conclusions and perspectives

- Independently of the starting "action": any affine gauge theory reduces to the Riemann-Cartan one with additional matter fields.
- Gauge-spacetime independence $\Rightarrow$ Euclidean spacetime at quantum level. So, we know what to do!

- Are those fields related with dark issues?

- Are those fields related with dark issues?
- Which is the correct mechanism of mass generation?

- Are those fields related with dark issues?
- Which is the correct mechanism of mass generation?
- Quantum sector (Spin-1 theory) vs. Geometric sector (Spin-2?)

- Are those fields related with dark issues?
- Which is the correct mechanism of mass generation?
- Quantum sector (Spin-1 theory) vs. Geometric sector (Spin-2?)
- Inclusion of (fermionic and gauge) matter.

- Are those fields related with dark issues?
- Which is the correct mechanism of mass generation?
- Quantum sector (Spin-1 theory) vs. Geometric sector (Spin-2?)
- Inclusion of (fermionic and gauge) matter.
- Formalization of a theory!

- Are those fields related with dark issues?
- Which is the correct mechanism of mass generation?
- Quantum sector (Spin-1 theory) vs. Geometric sector (Spin-2?)
- Inclusion of (fermionic and gauge) matter.
- Formalization of a theory!
- To be continued...