Cosmology beyond the Standard Model

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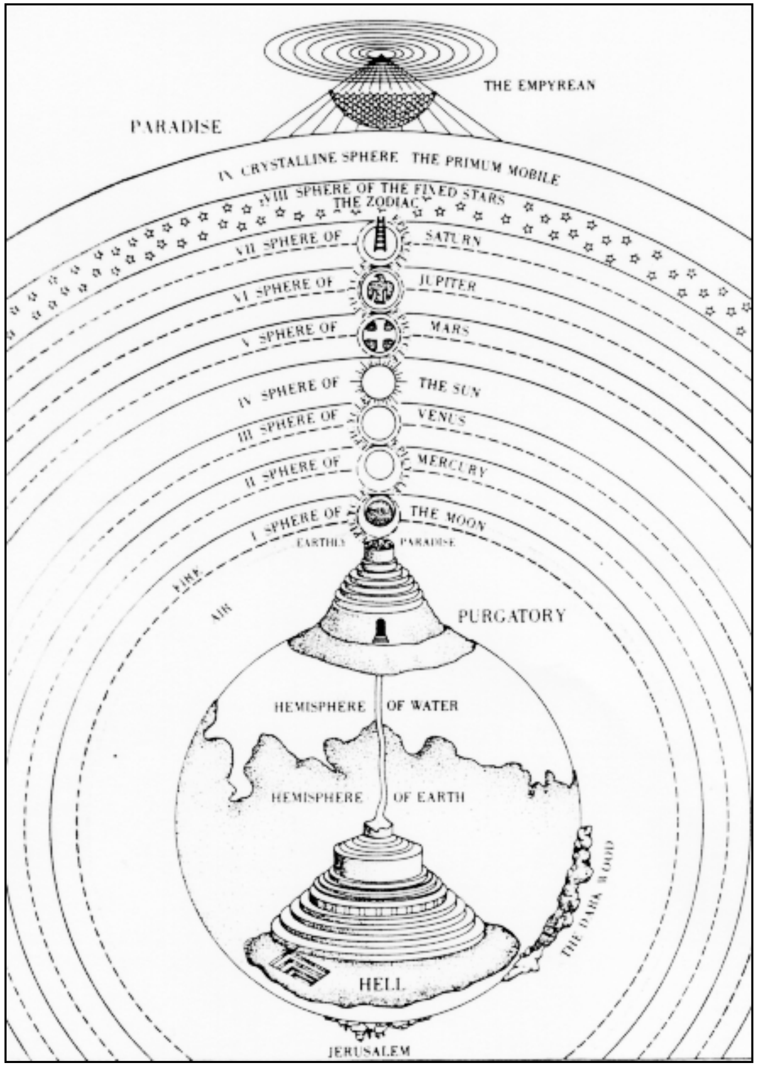
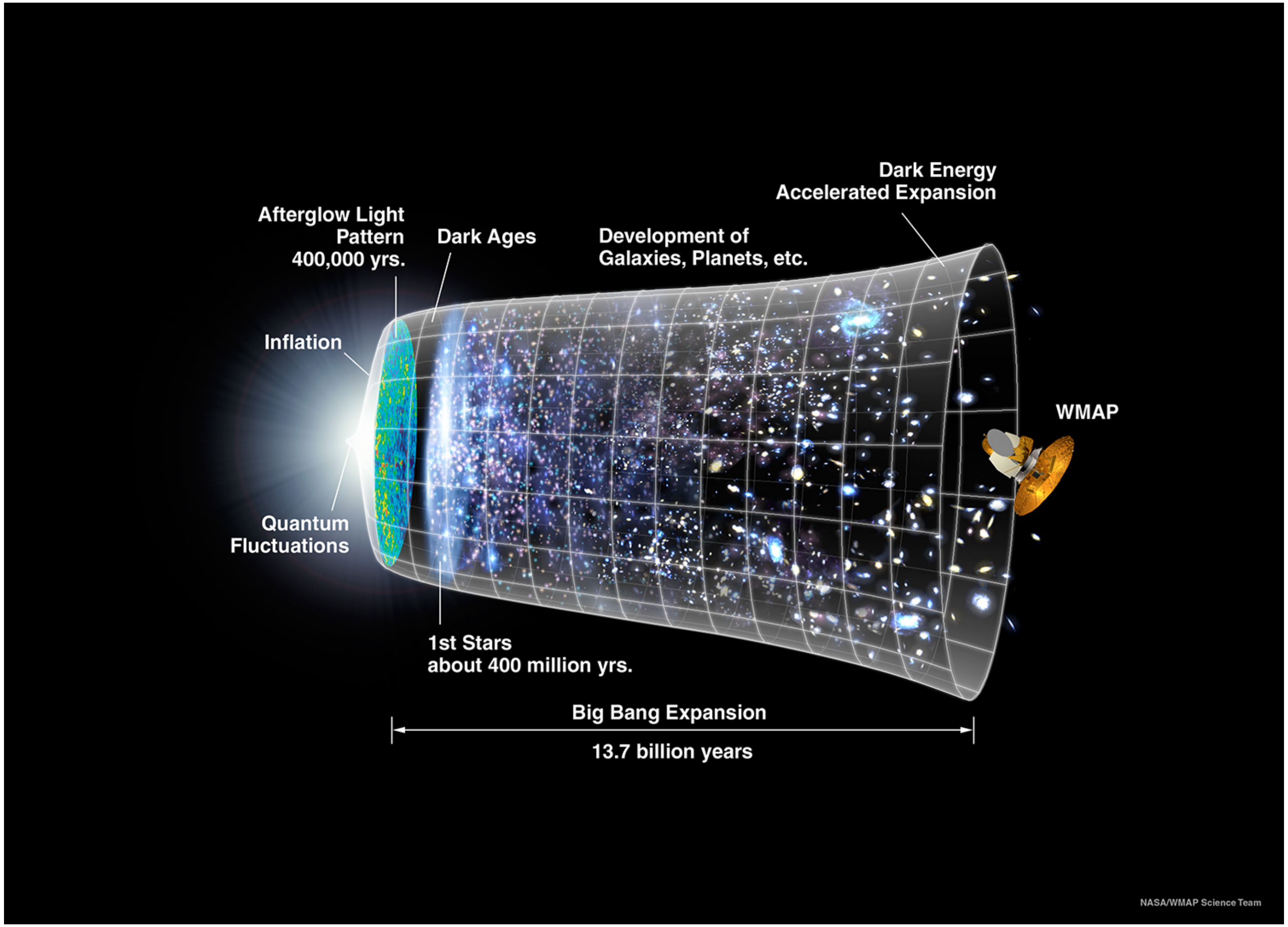
In the Aristotlean 'standard model' of cosmology (circa 350 BC) the universe was *static* and *finite* and *centred on the Earth*



The Divine Comedy, Dante Alligheri (1321)

This was a 'simple' model and fitted all the observational data
… but the underlying principle was unphysical

Today we have a new standard model of the universe …
dominated by dark energy and undergoing accelerated expansion



Courtesy: NASA/WMAP Science Team (2007)

It too is 'simple' and fits all the observational data
but lacks an underlying physical basis

The **Standard $SU(3)_c$ x $SU(2)_L$ x $U(1)_Y$ Model** provides an exact description of all *microphysics* (up to some high energy cut-off scale M)

Cosmological constant *Higgs mass divergence*

$$\mathcal{L}_{\text{eff}} = M^4 + M^2\Phi^2 \quad \textbf{super-renormalisable}$$
$$+ \quad (D\Phi)^2 + \bar{\Psi}\, \not{D}\Psi + F^2 + \bar{\Psi}\Psi\Phi + \Phi^2 \quad \textbf{renormalisable}$$
$$+ \quad \frac{\bar{\Psi}\Psi\Phi\Phi}{M} + \frac{\bar{\Psi}\Psi\bar{\Psi}\Psi}{M^2} + \ldots \quad \textbf{non-renormalisable}$$

The effects of *new* physics beyond the SM (neutrino mass, nucleon decay, FCNC ...) → **non-renormalisable operators** suppressed by M^n … which 'decouple' as $M \to M_P$

But as M is raised, the effects of the **super-renormalisable operators** are *exacerbated*

Solution for 2nd term → 'softly broken' supersymmetry at M ~ 1 TeV (10^2 new parameters)

This suggests possible mechanisms for **baryogenesis**, candidates for **dark matter**, … (as do other proposed extensions of the SM, e.g. new dimensions @ TeV scale)

The 1st term **couples to gravity** so the *natural* expectation is $\Omega_\Lambda \sim (1\ \text{TeV})^4$ *i.e.* the universe should have been inflating since (or collapsed at) $t \sim 10^{-12}$ s

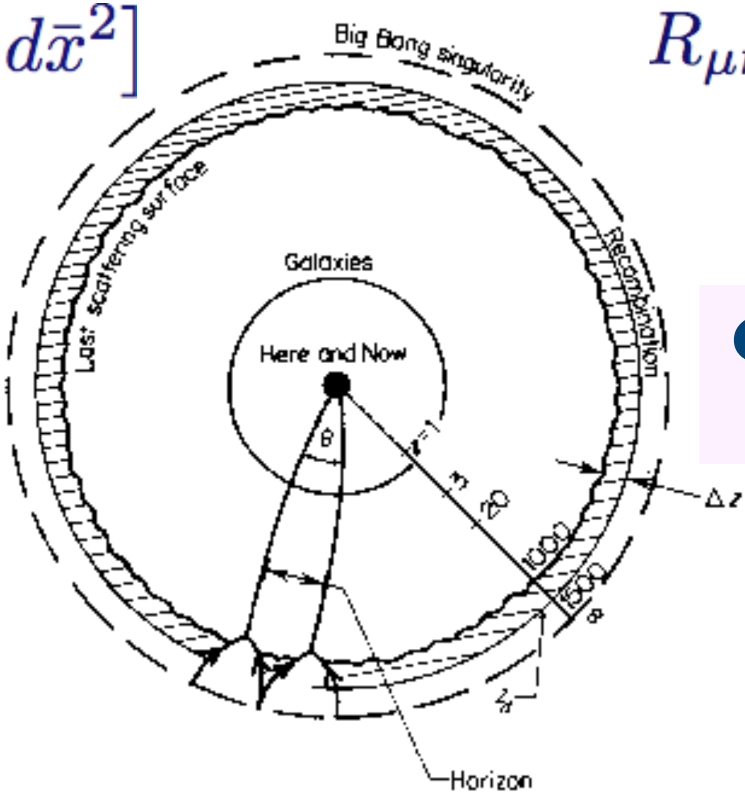
There *must* be some reason why this did *not* happen ($\Lambda \to 0$?)

The **standard cosmological model** is based on several key assumptions:
maximally symmetric space-time + general relativity + *ideal fluids*

$$ds^2 = a^2(\eta)\left[d\eta^2 - d\bar{x}^2\right]$$

$$a^2(\eta)d\eta^2 \equiv dt^2$$

Space-time metric
Robertson-Walker



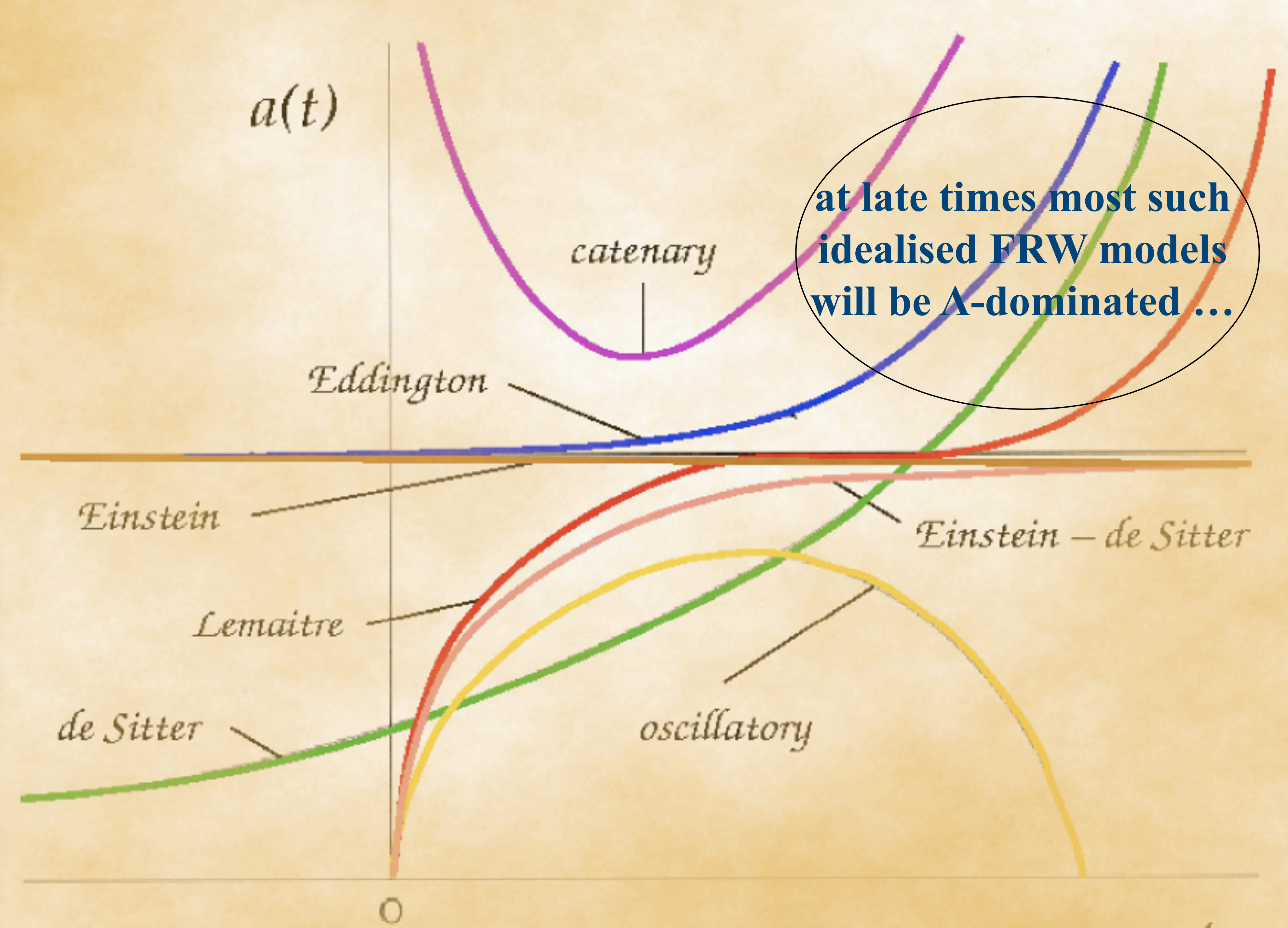
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_{\mathrm{N}}T_{\mu\nu}$$

Geometrodynamics
Einstein

$$\Rightarrow H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_{\mathrm{N}}\rho_{\mathrm{m}}}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

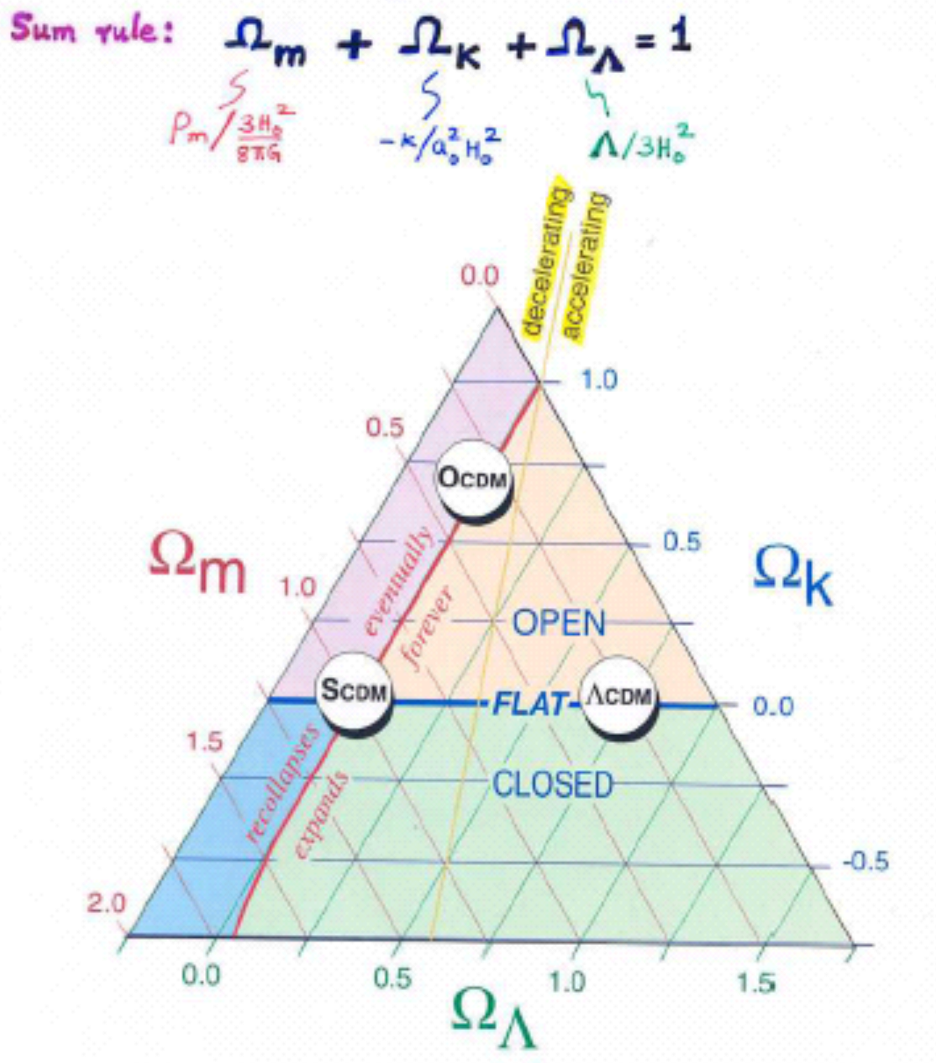
$$\equiv {H_0}^2\left[\Omega_{\mathrm{m}}(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda\right]$$

… and *naturally* exhibits 'dark energy' as $z \to 0$!



(Courtesy: Thomas Buchert)

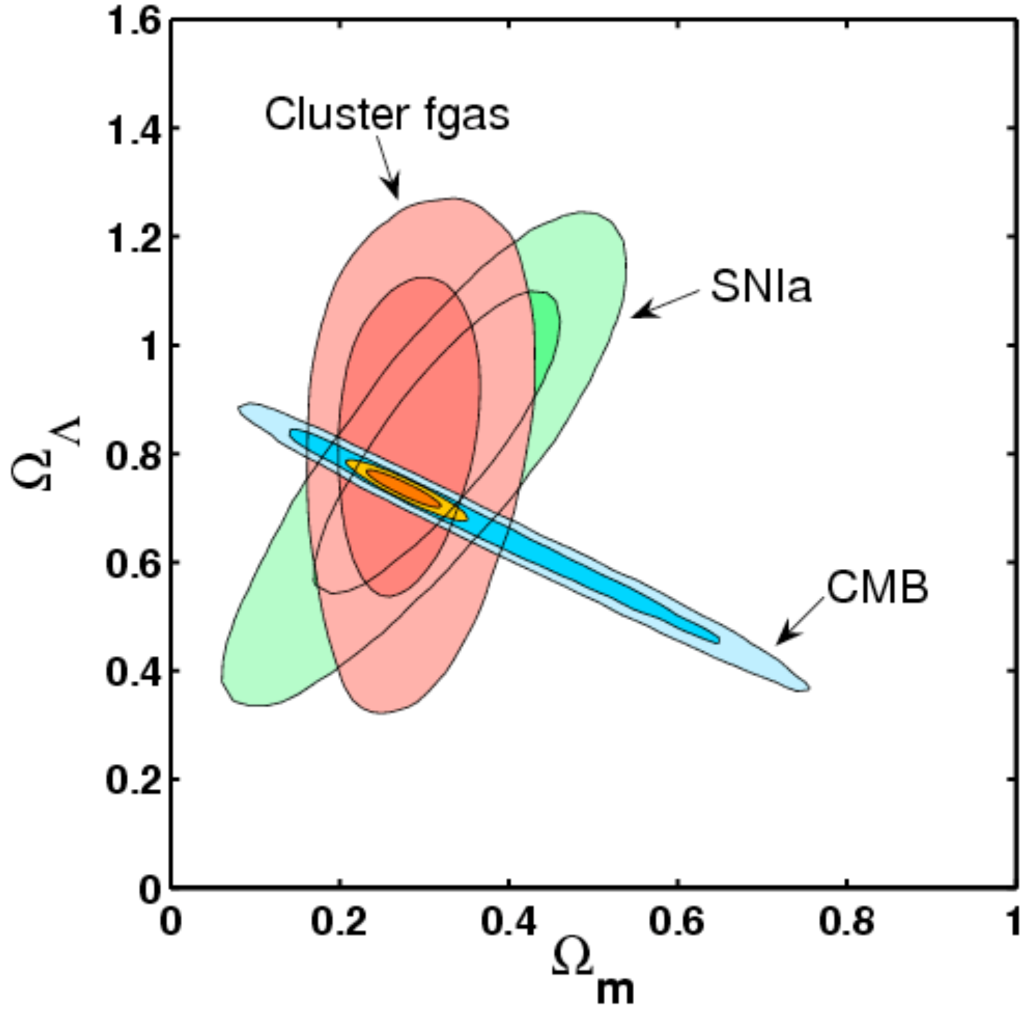
It is *natural* for data interpreted in this idealised model to yield $\Lambda \sim H_0^2$



Bahcall, Ostriker, Perlmutter & Steinhardt (1999)

… so *not* surprising that we infer Ω_Λ $(= \Lambda/3H_0^2)$ to be of $O(1)$ from the **cosmic sum rule**, given the uncertainties in measuring Ω_m and Ω_k and the possibility of other components (Ω_x) which are *not accounted* for

Observations indicate $\Omega_k \approx 0$ so the FRW model is simplified further, leaving only two free parameters (Ω_Λ and Ω_m) to be fitted to data



Allen *et al* (2008)

If we *underestimate* Ω_m, or if there is a Ω_x (e.g. "back reaction") which the FRW model does not include, then we will *necessarily* infer $\Omega_\Lambda \neq 0$

Quantities averaged over a domain $\mathcal{D}$ obey modified Friedmann equations Buchert 1999:

$$
\begin{aligned}
3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} &= -4\pi G\langle\rho\rangle_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}} \;, \\
3\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 &= 8\pi G\langle\rho\rangle_{\mathcal{D}} - \frac{1}{2}\langle{}^{(3)}R\rangle_{\mathcal{D}} - \frac{1}{2}\mathcal{Q}_{\mathcal{D}} \;,
\end{aligned}
$$

where $\mathcal{Q}_{\mathcal{D}}$ is the backreaction term,

$$
\mathcal{Q}_{\mathcal{D}} = \frac{2}{3}\left(\langle\theta^2\rangle_{\mathcal{D}} - \langle\theta\rangle^2_{\mathcal{D}}\right) - \langle\sigma^{\mu\nu}\sigma_{\mu\nu}\rangle_{\mathcal{D}} \;.
$$

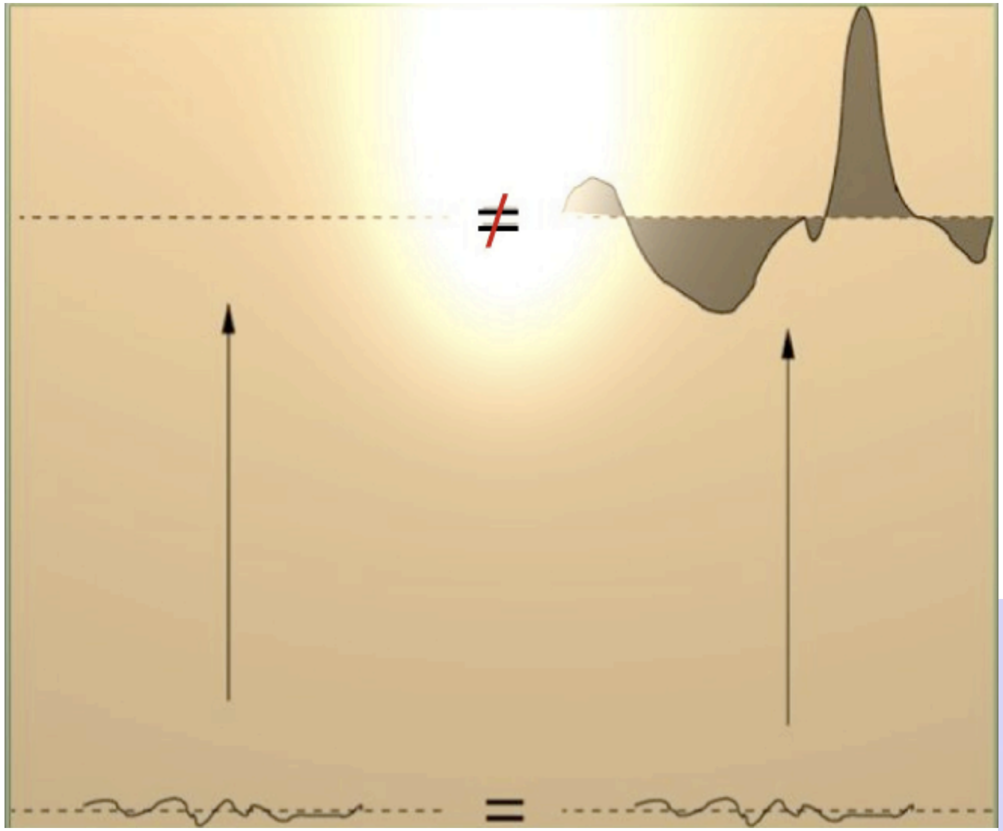
Variance of the expansion rate.

Average shear.

If $\mathcal{Q}_{\mathcal{D}} > 4\pi G\langle\rho\rangle_{\mathcal{D}}$ then $a_{\mathcal{D}}$ accelerates.

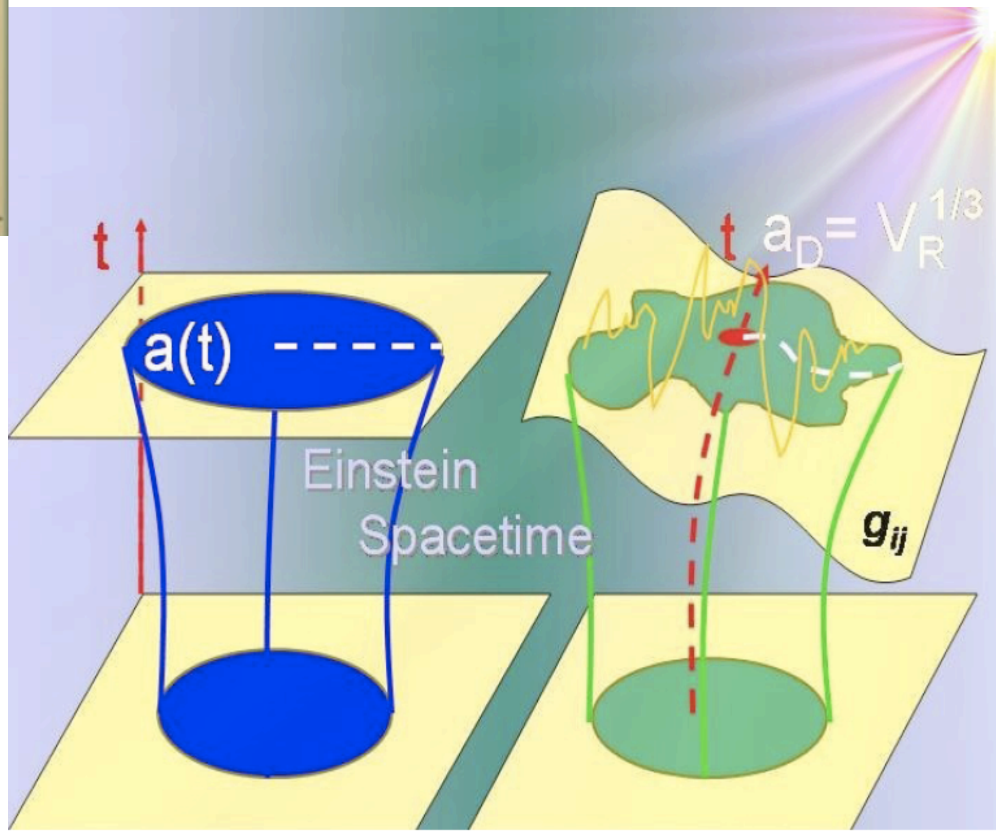
Can mimic a cosmological constant if $\mathcal{Q}_{\mathcal{D}} = -\frac{1}{3}\langle{}^{(3)}R\rangle_{\mathcal{D}} = \Lambda_{\rm eff}$.

Whether the backreaction can be sufficiently large is an ***open question***



‘Back reaction’ is hard to compute because spatial averaging and time evolution (along our past light cone) do *not* commute (Ellis 1982)

Due to structure formation, the homogeneous solution of Einstein’s eqs. is distorted - its average must be taken over the *actual* geometry … the result is *different* from the standard FRW model



Courtesy: Thomas Buchert

Interpreting Λ as vacuum energy raises the coincidence problem:

why is $\Omega_\Lambda \approx \Omega_m$ *today*?

An evolving ultralight scalar field ('**quintessence**') can display 'tracking' behaviour: this requires $V(\varphi)^{1/4} \sim 10^{-12}$ GeV but $\sqrt{d^2V/d\varphi^2} \sim H_0 \sim 10^{-42}$ GeV to ensure slow-roll

… i.e. just as much fine-tuning as a bare cosmological constant

A similar comment applies to models (e.g. '**DGP brane-world**') wherein gravity is modified on the scale of the present Hubble radius so as to mimic vacuum energy

… this scale is unnatural in a fundamental theory and is simply put in by hand

The only *natural* option is if $\Lambda \sim H^2$ *always*, but this is just a renormalisation of G_N – recall: $H^2 = 8\pi G_N/3 + \Lambda/3$ (and in any case this will not yield accelerated expansion)

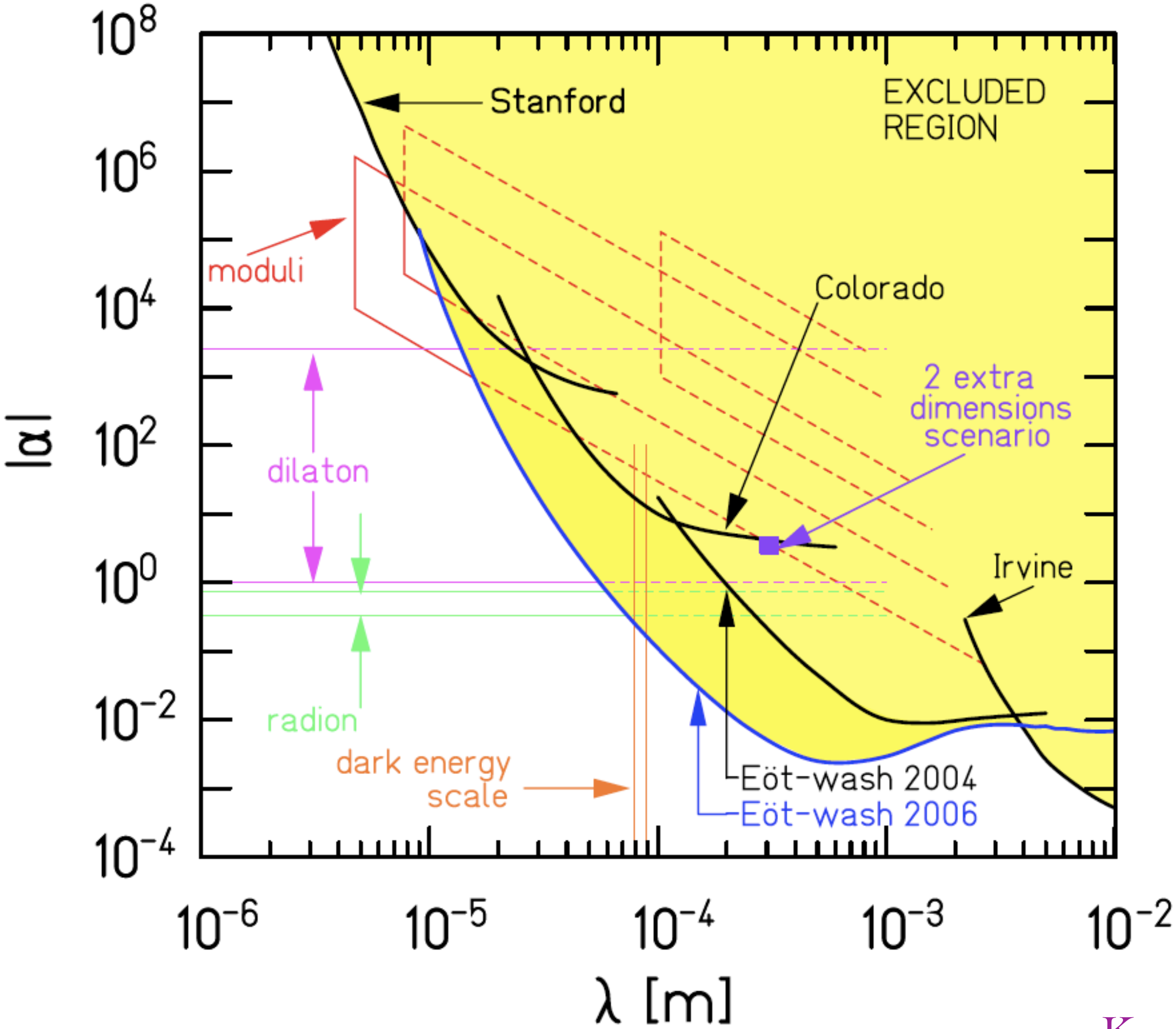
… *ruled out* by Big Bang nucleosynthesis (requires G_N to be within 5% of lab value)

There cannot be a *natural* explanation for the coincidence problem

Do we see $\Lambda \sim H_0^2$ because that is just the **observational sensitivity?**

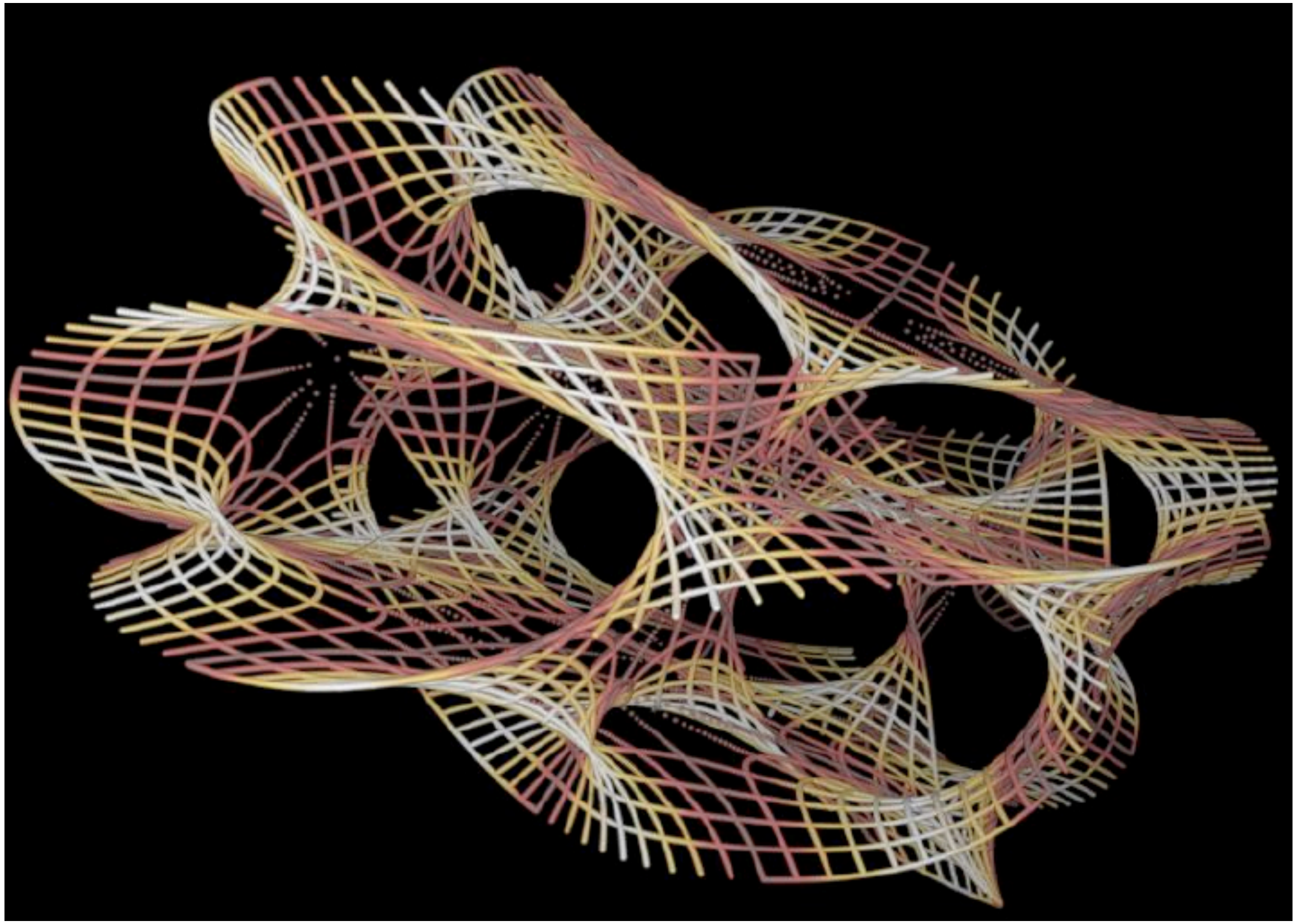
There is *no* evidence for any change in the inverse-square law of gravitation at the 'dark energy' scale: $\Omega_\Lambda^{-1/4} \sim (H_0 M_P)^{-1/2} \sim 0.1$ mm

$$V(r) = -G\frac{m_1 m_2}{r}[1 + \alpha \exp(-r/\lambda)]$$



Kapner *et al* (2007)

In string/M-theory, the sizes and shapes of the extra dimensions ('moduli') must be stabilised … e.g. by turning on background 'fluxes'

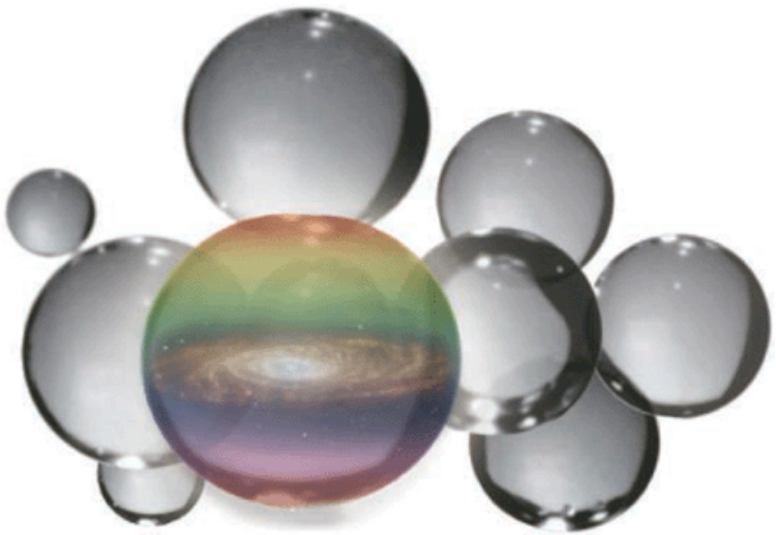


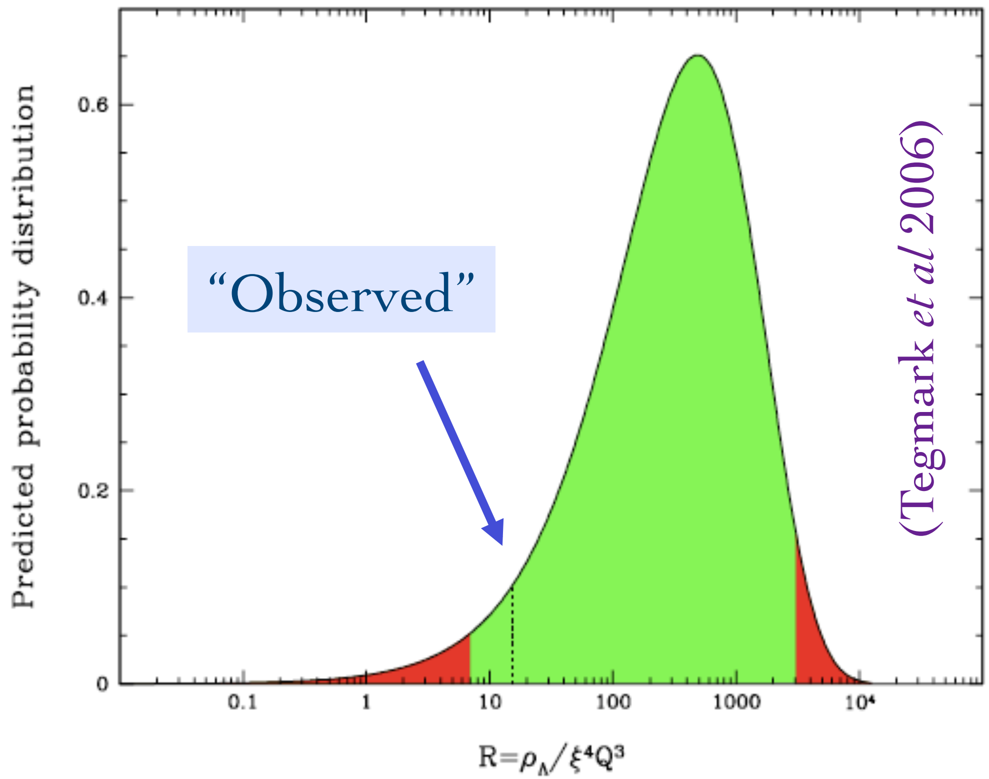
Given the variety of flux choices and the number of local minima in the flux potential, the total number of vacuua is *very* large - perhaps 10^{500}

The existence of the huge *landscape* of possible vacuua in string theory (with moduli stabilised through background fluxes) has remotivated **attempts at an 'anthropic' explanation for $\Omega_\Lambda \sim \Omega_m$**

Perhaps it is just "observer bias" … galaxies would not have formed if Λ had been much *higher* (Weinberg 1989, Efstathiou 1995, Martel, Shapiro, Weinberg 1998 …)

(Credit: K. Buckheit Krause/Science)





(Tegmark *et al* 2006)

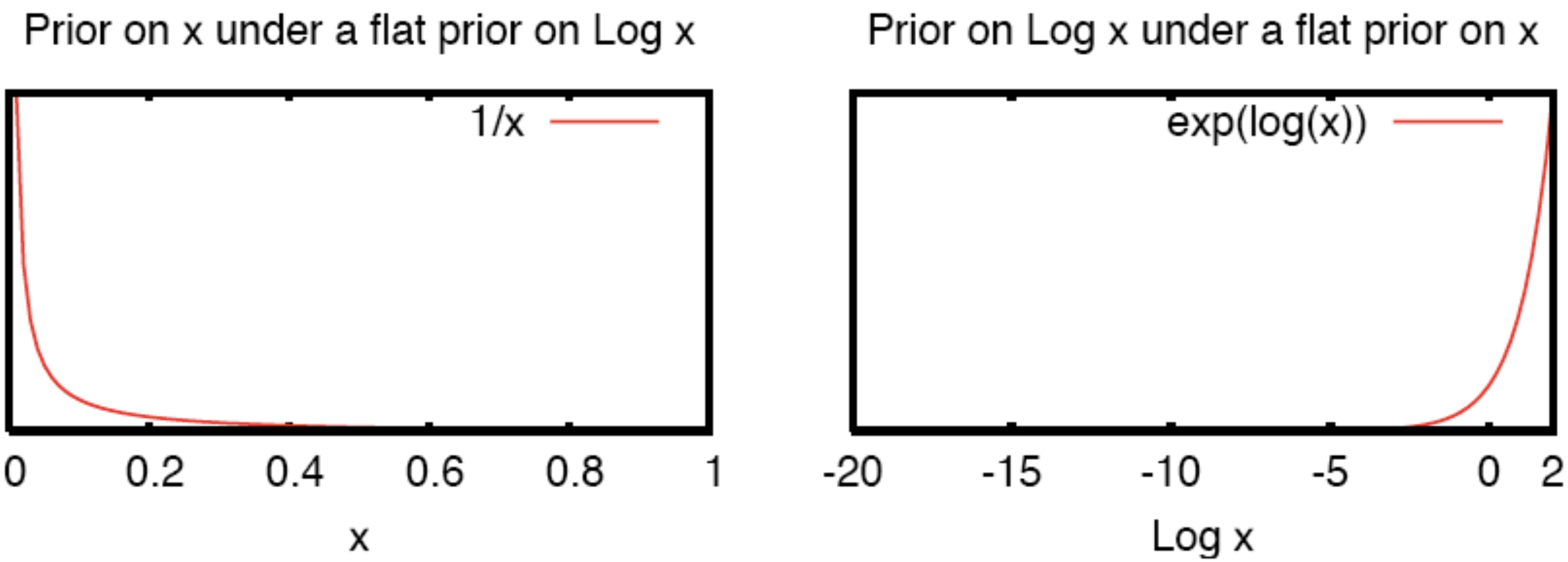
But the 'anthropic prediction' of Λ from considerations of galaxy formation is significantly *higher* than the observationally inferred value

Moreover this assumes the prior distribution to be *flat* in the range $0 \rightarrow 10^{-120}\ M_P^4$

Since we have *no* physical understanding of Λ, this may *not* be reasonable

If the relevant physical variable is e.g. *log* Ω_Λ, then $\boldsymbol{\Omega_\Lambda = 0}$ would be favoured!

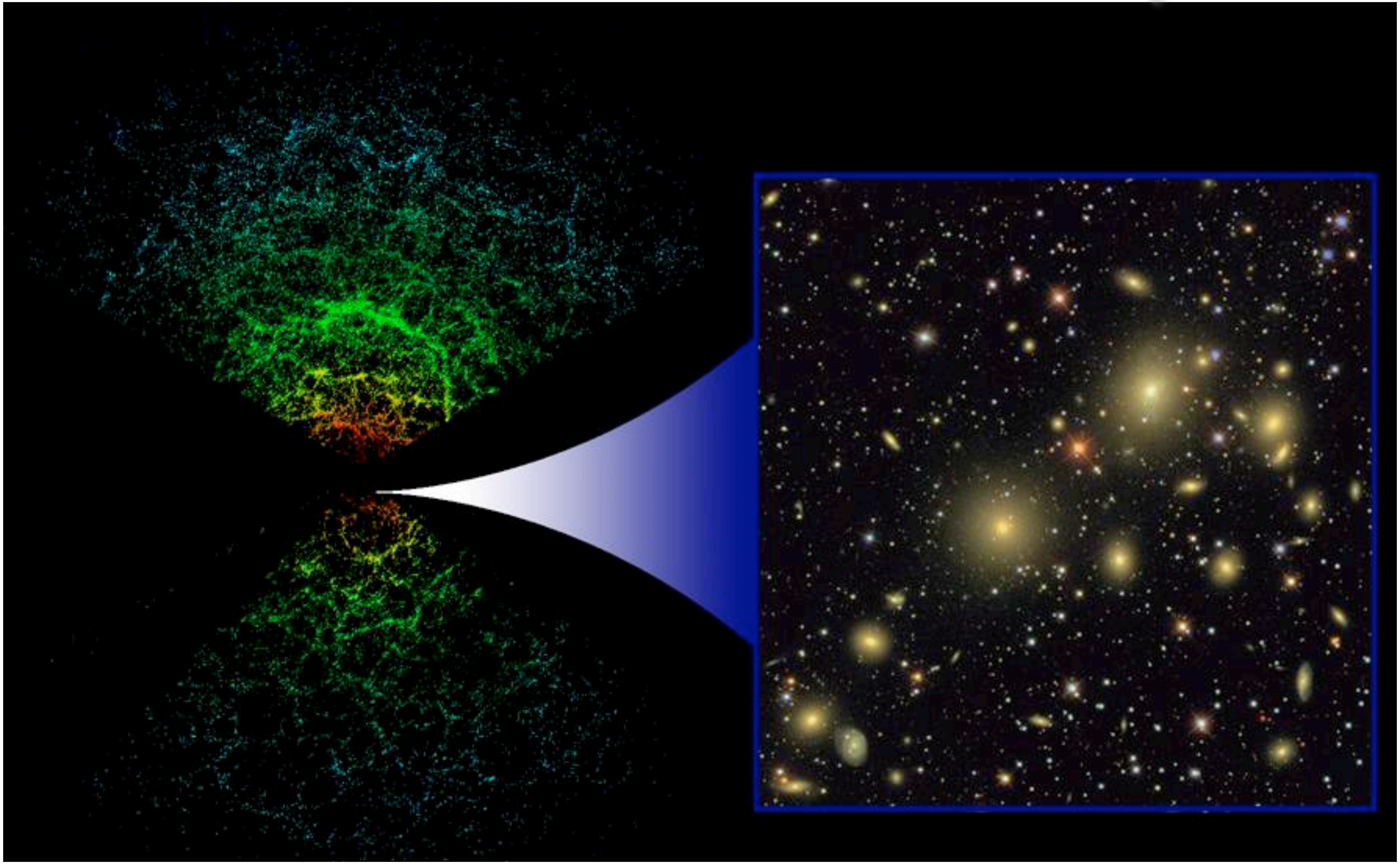
$$x \in [0, 1] \Longleftrightarrow \log x \in [-\infty, 0]$$



$$\text{Posterior} = \text{Prior} \times \text{Likelihood}$$

So it is far from clear that $\Lambda \sim H_0^2$ has an anthropic explanation

Galaxies are seen to trace out a cosmic 'web' of filamentary structure



Averaged on *large* scales the universe may well be homogeneous but how would it bias cosmological inferences if e.g. we are located in a void?

New H-band Galaxy Number Counts

Are we located in an underdense region in the galaxy distribution?

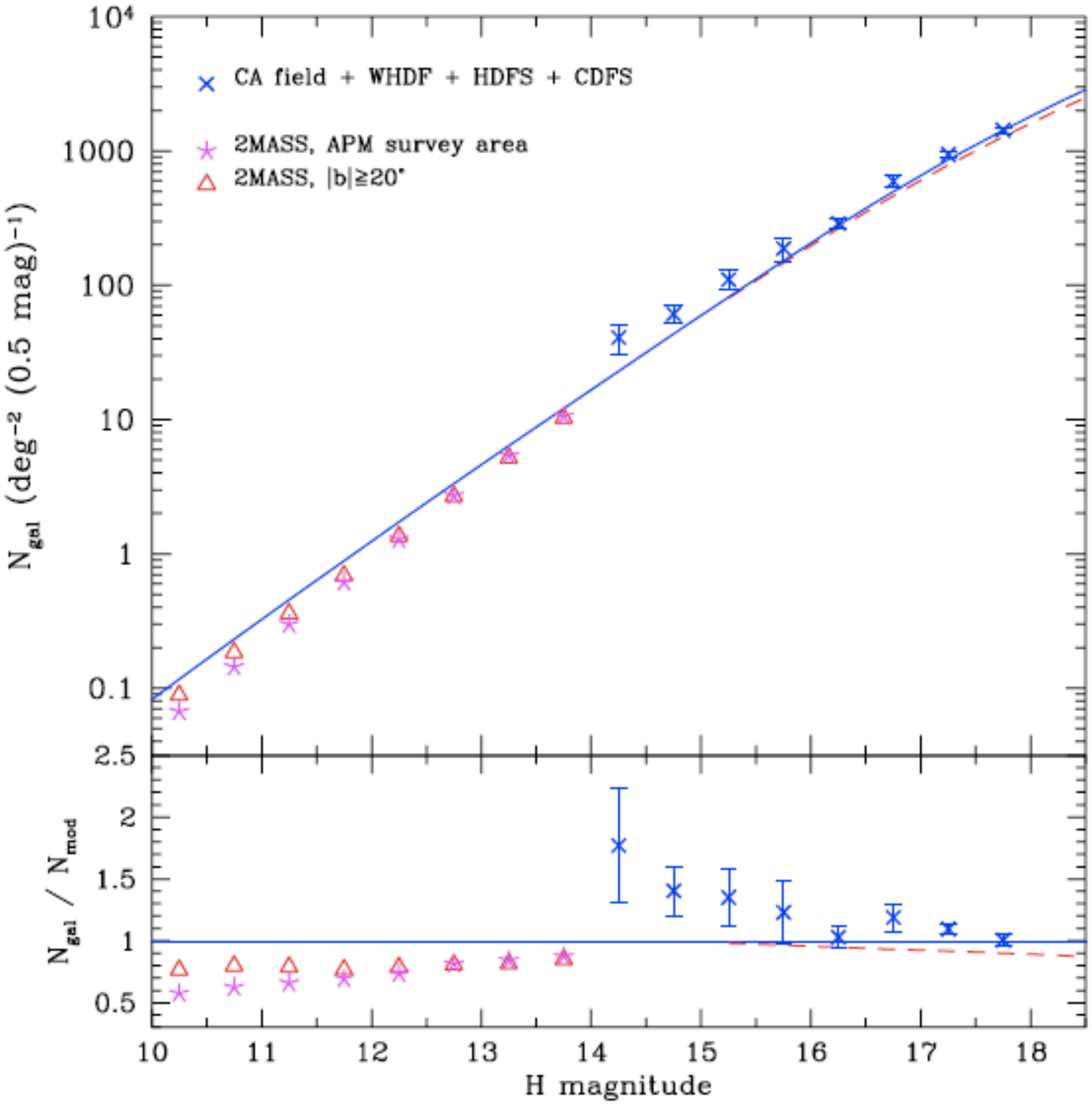
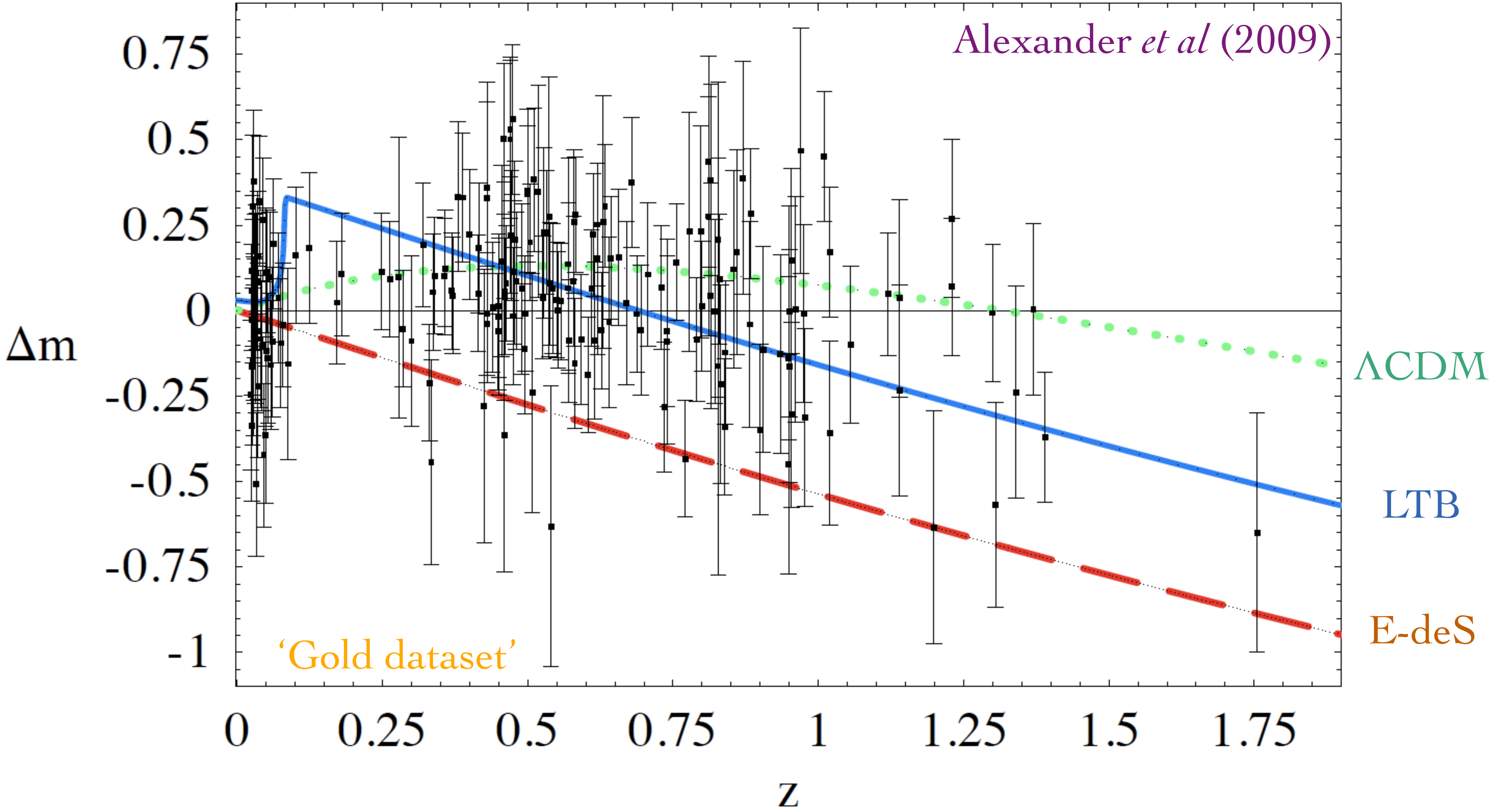


Figure 8. Here we show the faint H-band data from the two fields presented in this work (CA field and WHDF) and the two fields published by the LCIRS (HDFS and CDFS; Chen et al. 2002) applying a zeropoint to the LCIRS data consistent with the bright H-band 2MASS data (and hence the CA field and WHDF also), as shown in Fig. *7*. The errorbars at faint magnitudes indicate the field-to-field error, weighted in order to account for the different solid angles of each field. Bright H-band counts extracted from 2MASS for the APM survey area and for |b| >20° are shown as previously. In the lower panel, the counts are divided through by the pure luminosity evolution homogeneous prediction as before.

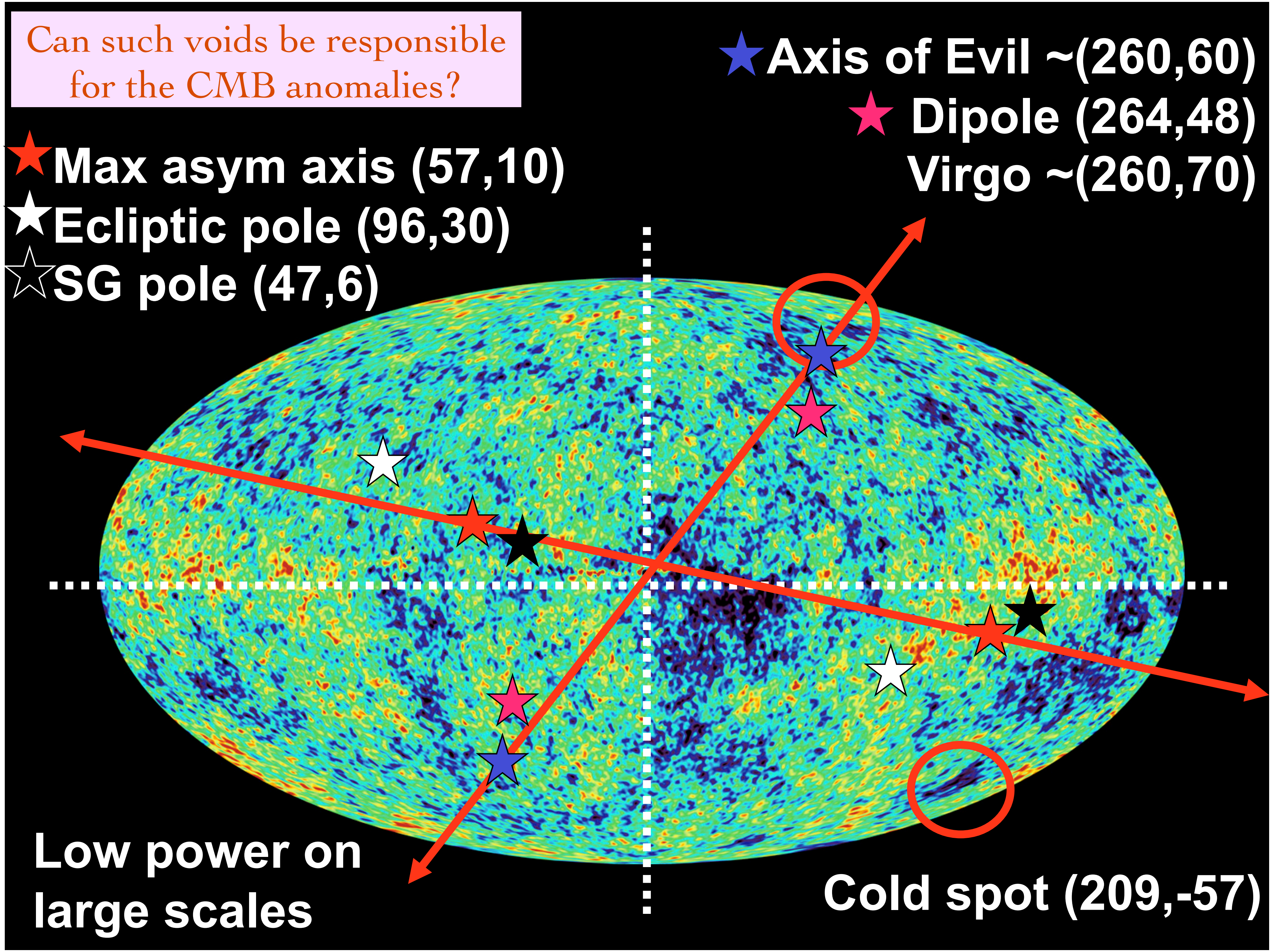
Frith, Metcalfe & Shanks (2006)

If so, the SN Ia Hubble diagram can be explained *without invoking acceleration*, since distant supernovae would be in a *slower* Hubble flow than the nearby ones within the local void (inhomogeneous **Lemaitré-Tolman-Bondi model**)

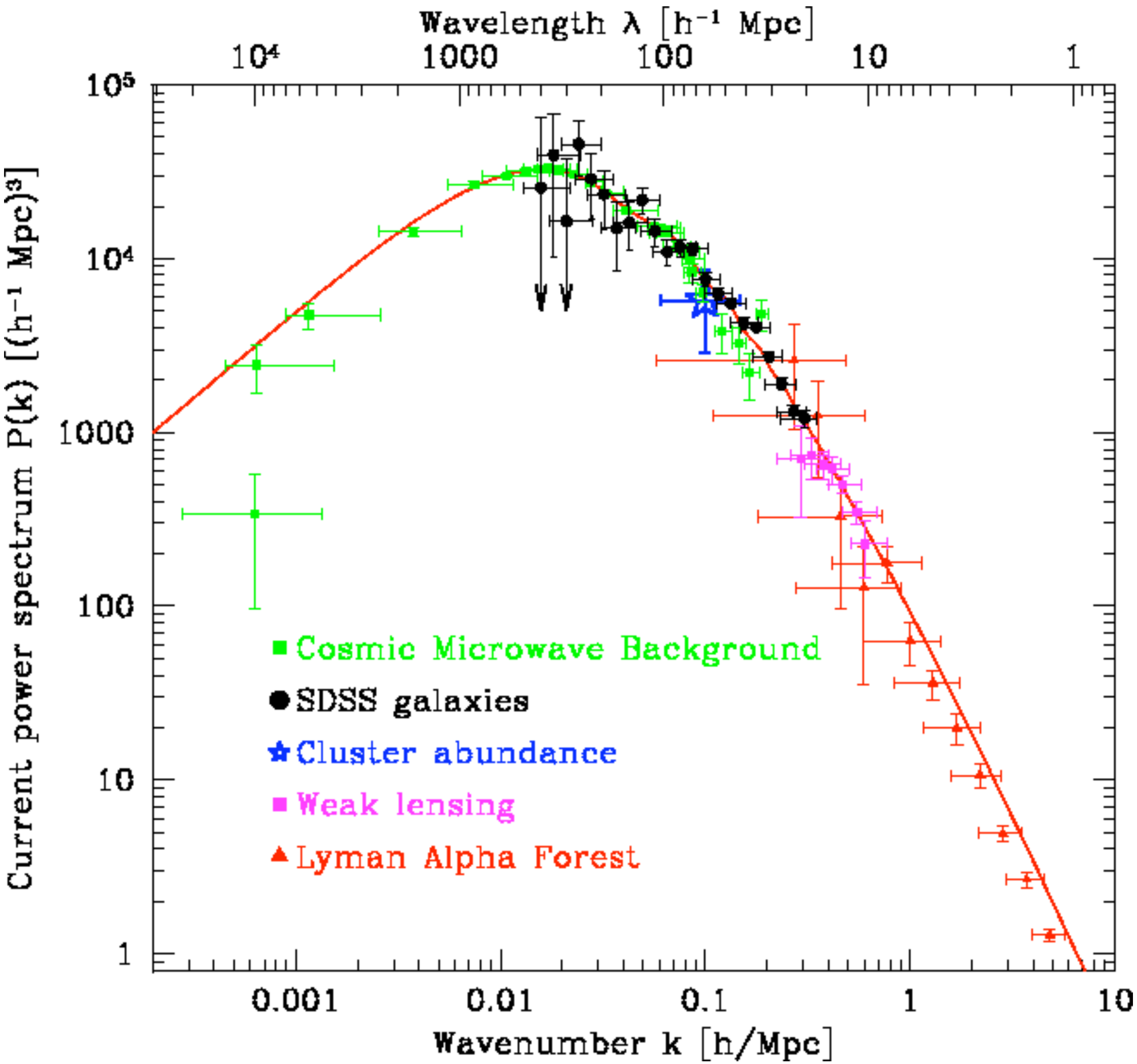


More interesting are the **Szekeres models** which do not even assume isotropy

Celerier, Bolejko, Krasinsky … (2009, 2010)


Can such voids be responsible for the CMB anomalies?
Axis of Evil ~(260,60)
Dipole (264,48)
Virgo ~(260,70)
Max asym axis (57,10)
Ecliptic pole (96,30)
SG pole (47,6)
Low power on large scales
Cold spot (209,-57)

The original argument for ΛCDM came from observations of large-scale structure …
assuming the primordial fluctuations are *adiabatic* and *~scale-invariant*
(as is apparently "expected in the simplest models of inflation")



Tegmark (2004)

The formation of large-scale structure is akin to a scattering experiment

The **Beam:** **inflationary density perturbations**

No 'standard model' – usually *assumed* to be adiabatic and ~scale-invariant

The **Target:** **dark matter (+ baryonic matter)**

Identity unknown - usually taken to be cold (sub-dominant 'hot' component?)

The **Detector:** **the universe**

Modelled by a 'simple' FRW cosmology with parameters h, Ω_{CDM}, Ω_{b}, Ω_{Λ}, Ω_{k} …

The **Signal:** **CMB anisotropy, galaxy clustering …**

measured over scales ranging from ~ 1 – 10000 Mpc ($\Rightarrow$ ~8 e-folds of inflation)

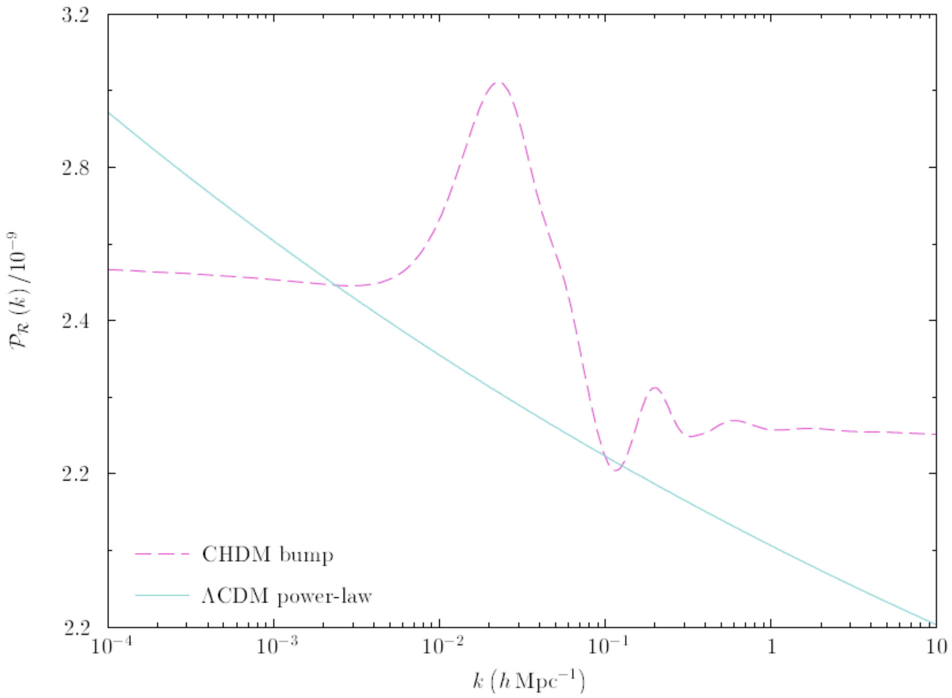
We cannot simultaneously determine the properties of *both* the **beam** *and* the **target** with an unknown **detector**

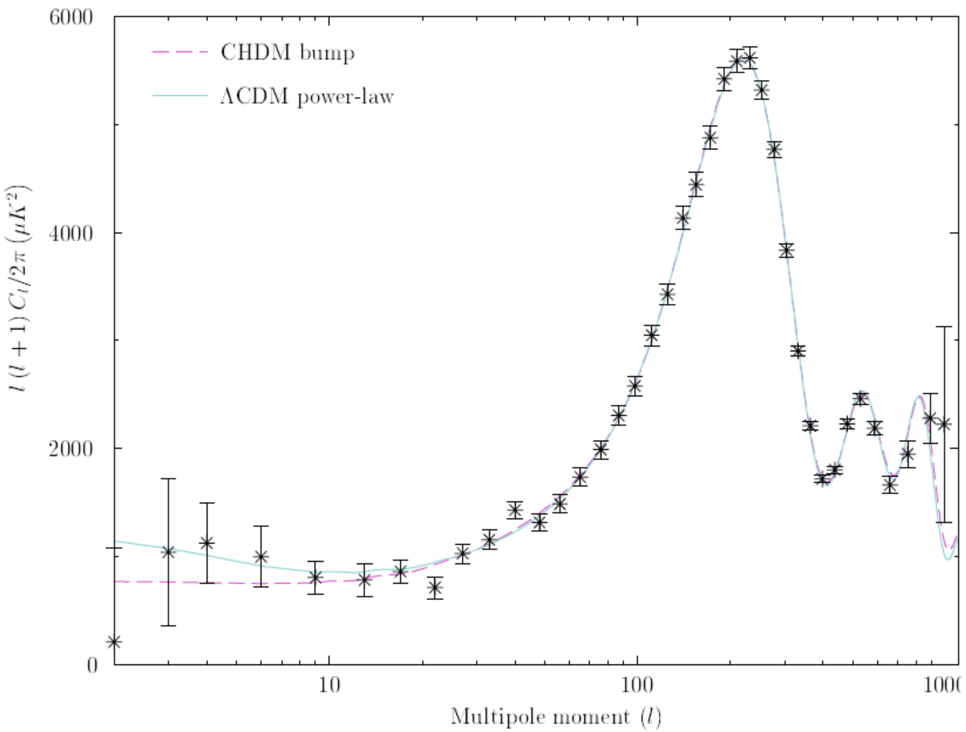
… hence need to adopt suitable 'priors' on *h*, Ω_{CDM}, etc in order to break inevitable parameter *degeneracies*

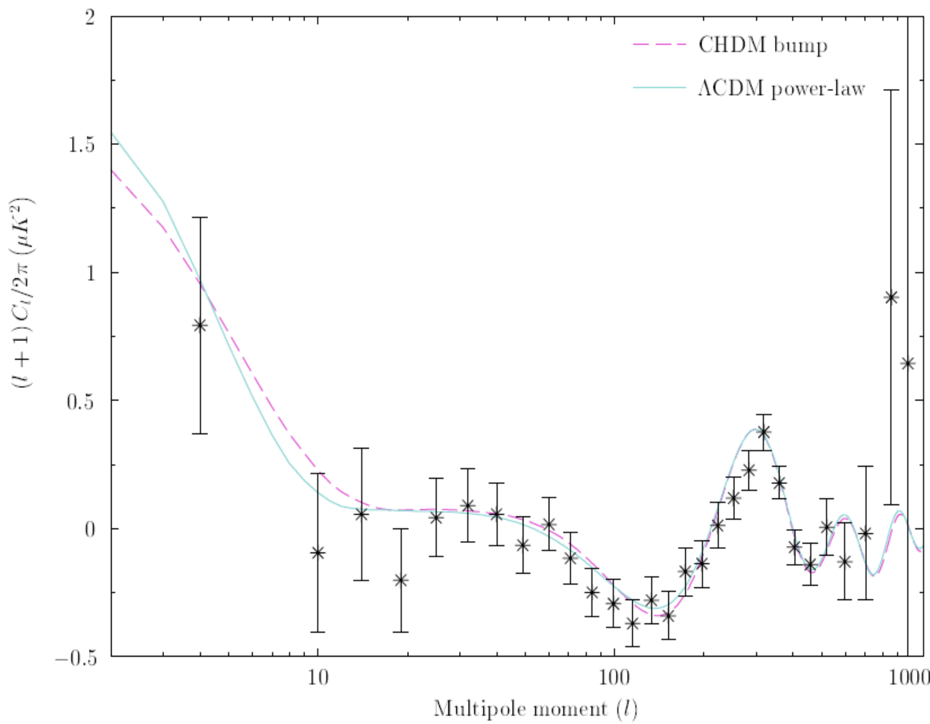
The WMAP 'precision' determination of cosmological parameters *assumes* a **scale-free primordial spectrum**

If there is a small 'bump' in the spectrum, the WMAP data can be fitted with *no dark energy* ($\Omega_m = 1$, $\Omega_\Lambda = 0$) if $h \sim 0.44$

(Hunt & Sarkar 2007, 2010)

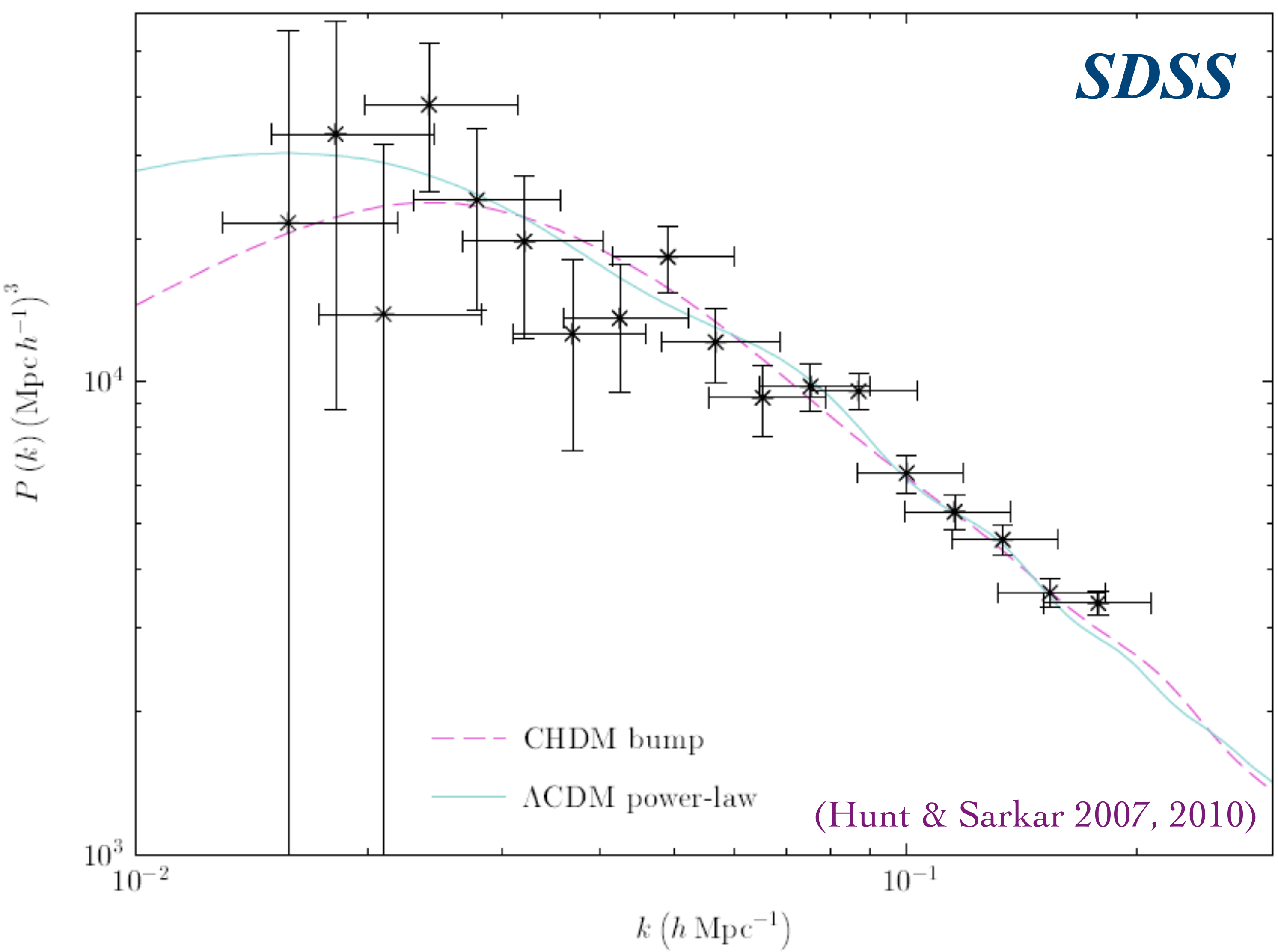






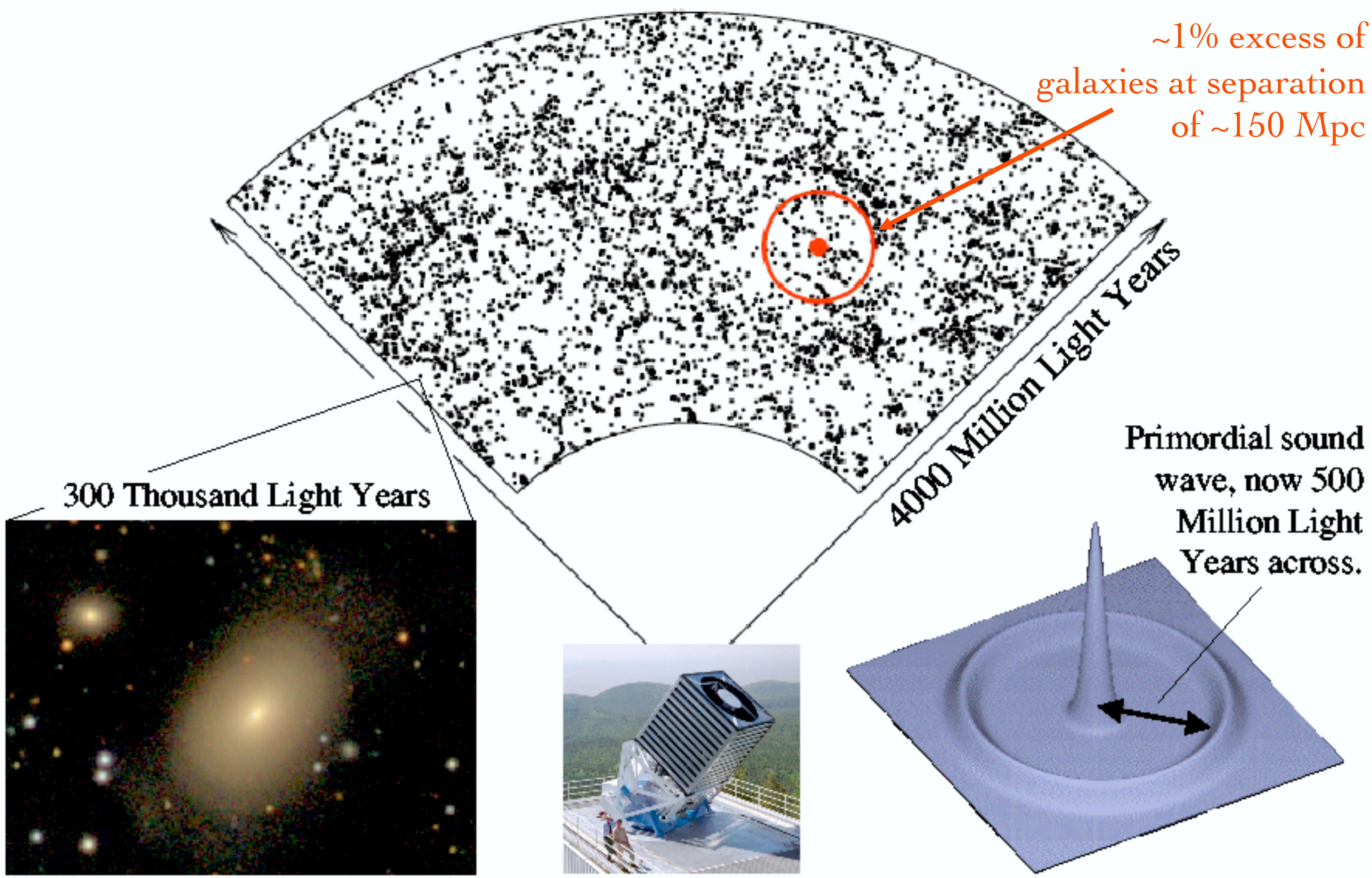
The small-scale power would be excessive unless damped by free-streaming

But adding 3 ν of mass 0.5 eV ($\Rightarrow \Omega_\nu \sim 0.1$) gives *good match* to large-scale structure



Fit gives $\Omega_b h^2 \approx 0.018 \rightarrow$ BBN √ ⇒ baryon fraction in clusters ~10% √

New Test: Baryon Acoustic Peak in the Large-Scale Correlation Function of *SDSS* Luminous Red Galaxies



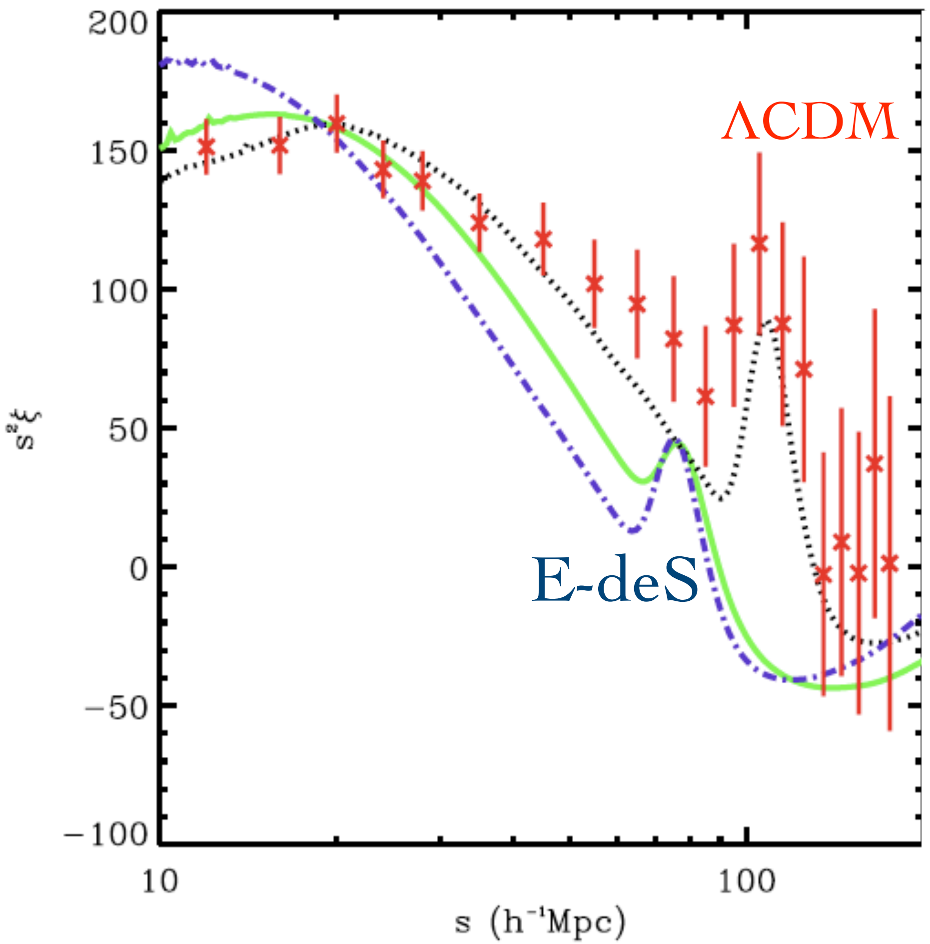
Eisenstein (2005)

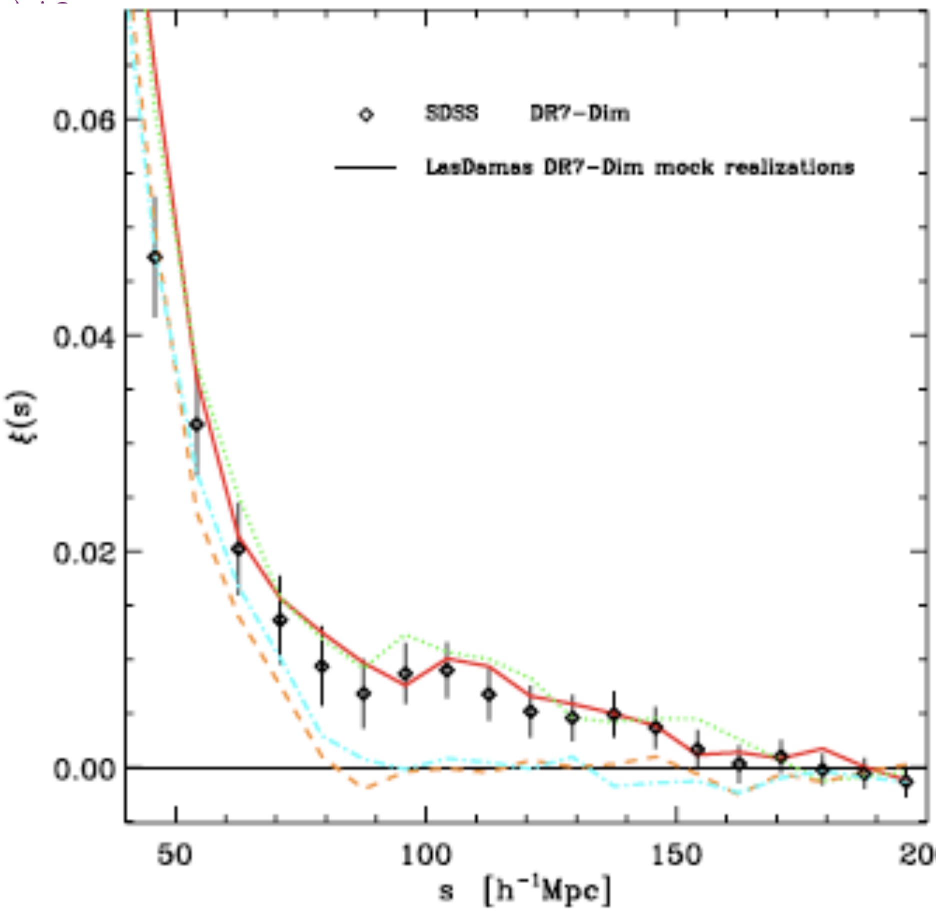
The E-deS model is *ruled out* by the 'baryon acoustic peak' observed at >4σ (DR3)

Eistenstein *et al* (2005)

… present at the ~same *physical scale*, but displaced in redshift space

Blanchard *et al* (2005)





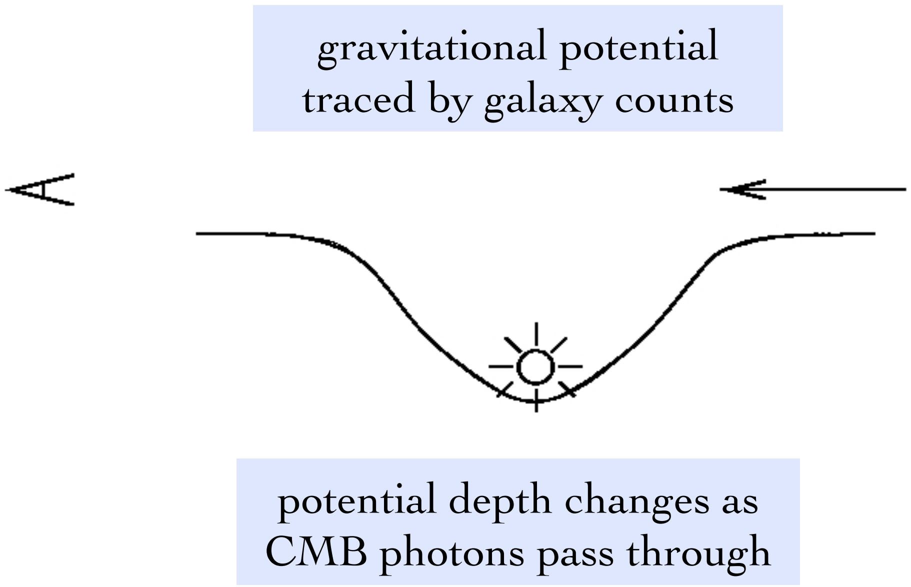
Kazin *et al* (2010)

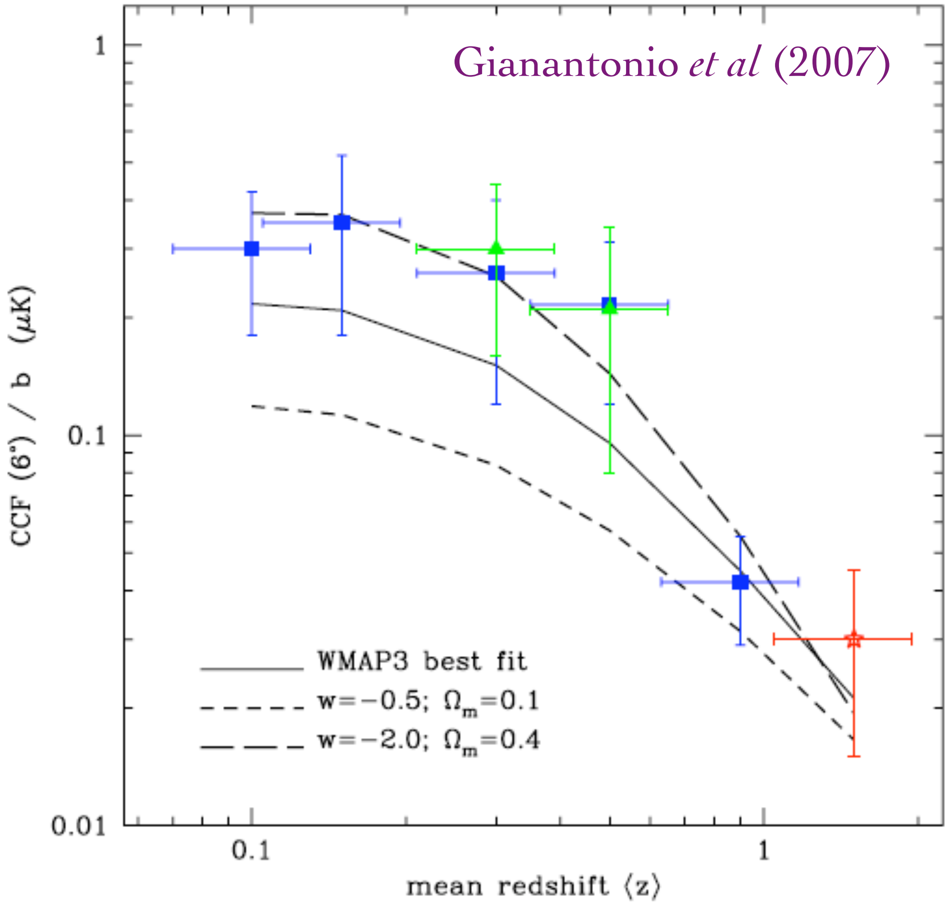
But why is *no* peak is evident when the statistics are trebled (DR7)?!

All the evidence for the 'standard model' is based on *geometrical* measurements

Is there direct *dynamical* evidence for Λ?

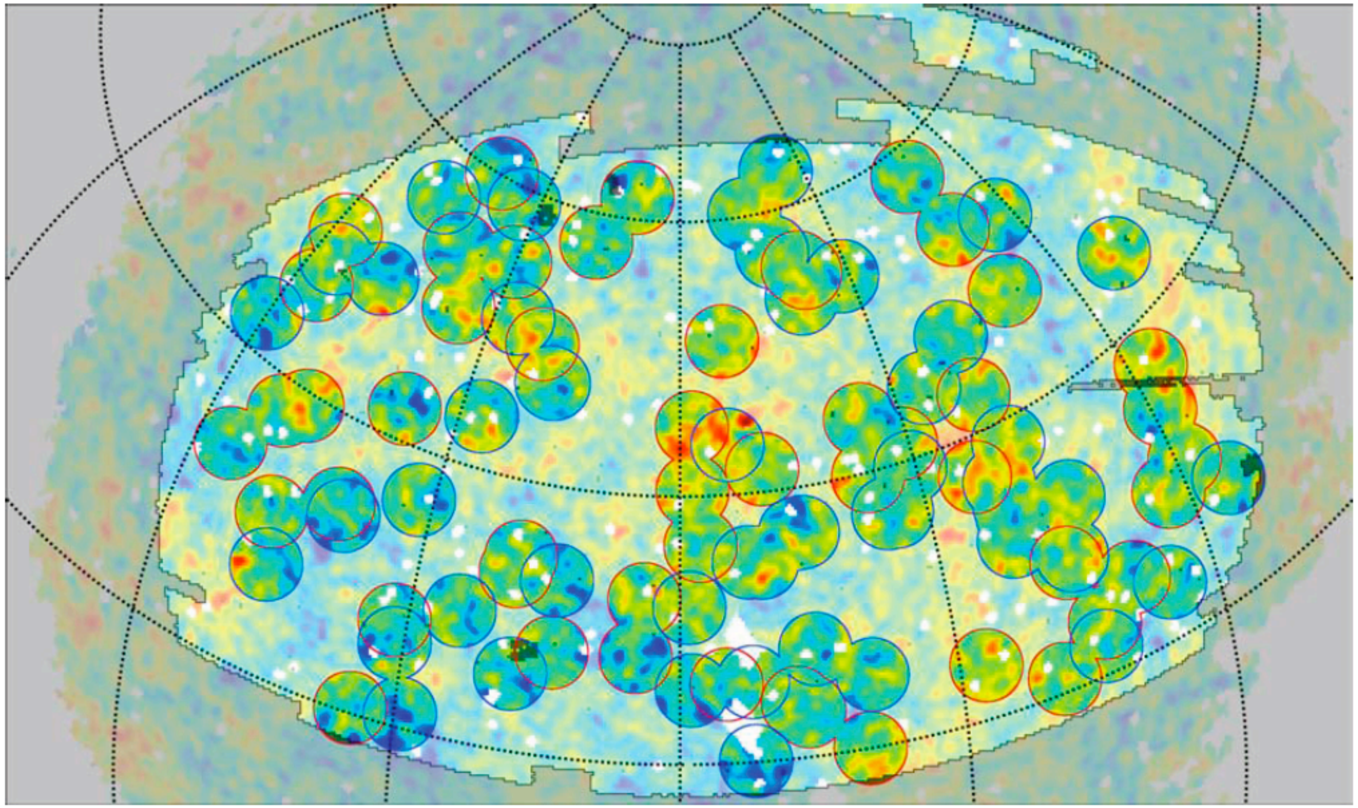
e.g. 'late integrated Sachs-Wolfe effect' – Turok & Crittenden (1996)





Present 'detections' are of *low* significance (< 3σ) … moreover the observed amplitude/*z*-dependence is *higher/steeper* than expected for Λ

It has been noted that there are *many* voids in the SDSS LRG sample



Granett *et al* (2009)

Figure 1: A map of the microwave sky over the SDSS area. The supervoids and superclusters used in our analysis are highlighted and outlined at a radius of 4°, blue for supervoids and red for superclusters. The compensated filter we use in our analysis approximately corrects for the large-angular-scale temperature variations that are visible across the map. The SDSS DR6 coverage footprint is outlined. Holes in the survey, *e.g.* due to bright stars, are displayed in black. Additionally, the WMAP Galactic foreground and point source mask is plotted (white holes). The disk of the Milky Way, which extends around the left and right border of the figure, is also masked. The map is in a Lambert azimuthal equal-area projection, centred at right ascension 180 and declination 35. The longitude and latitude lines are spaced at 30° intervals.

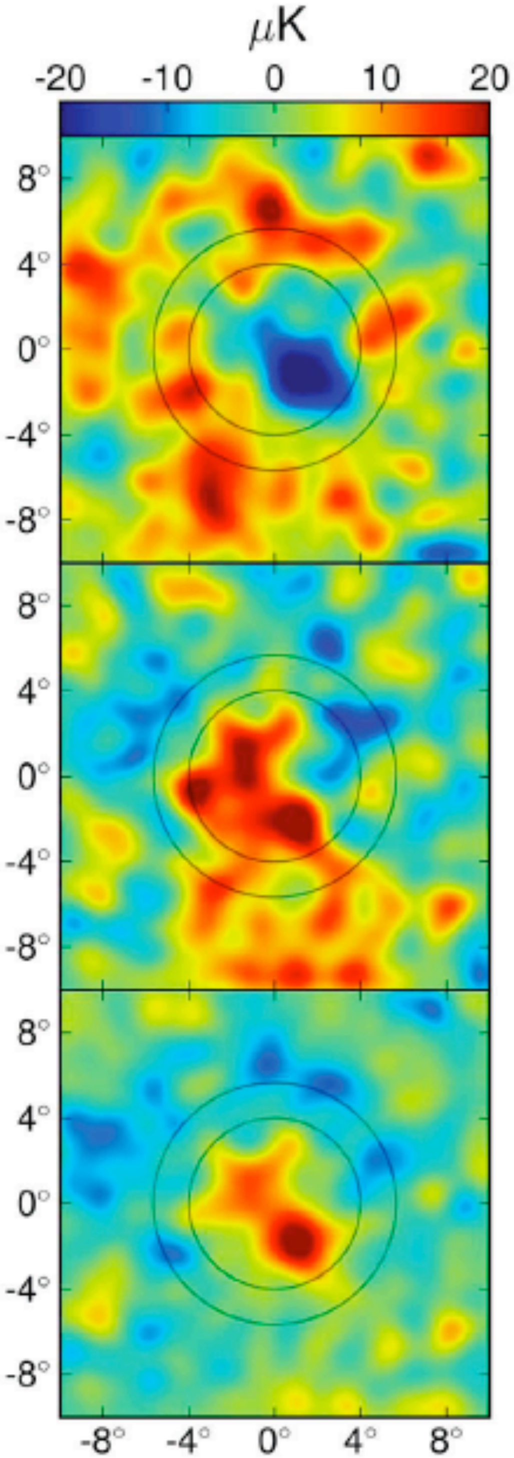


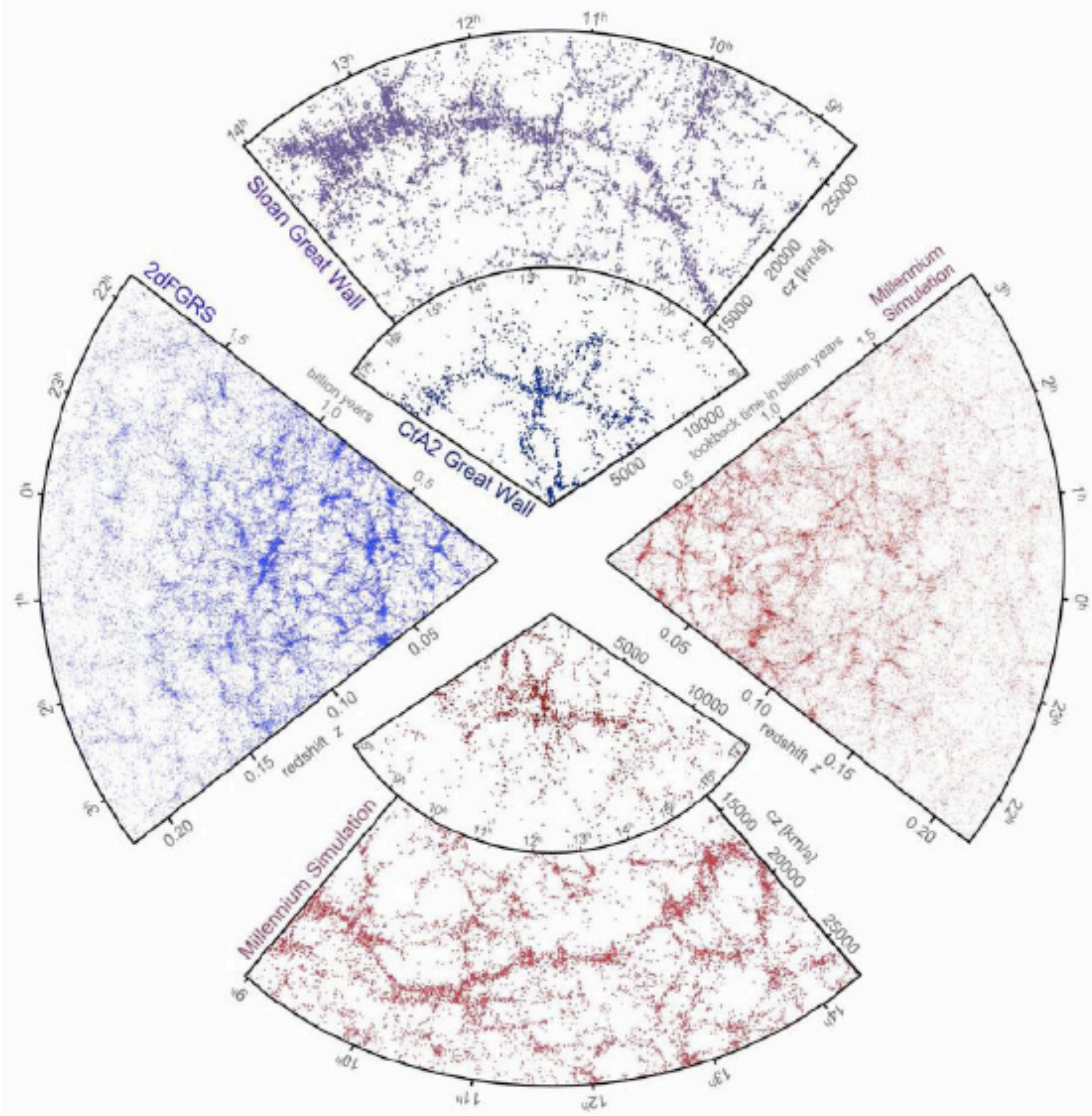
Figure 2: We stack regions on the CMB corresponding to supervoid and supercluster structures identified in the Sloan Digital Sky Survey. We averaged CMB cut-outs around 50 supervoids (top) and 50 superclusters (middle), and differenced these two samples (bottom). The individual cutouts from the CMB were aligned vertically in the image based on the measured orientations of the clusters and voids, but we do not scale or apply weights to the images. Although our statistical analysis uses the raw image, for this figure we smooth the images with a Gaussian kernel with width 0.5°. A hot spot and a cold spot are immediately recognizable in the cluster and void stacks, respectively, with a characteristic radius of 4°, corresponding to spatial scales of 100 h^{-1} Mpc. The inner circle (4° radius) and equal-area outer ring mark the extent of the compensated filter used in our analysis. The measured signal from these large structures is consistent with the ISW effect. There is a tantalizing hint of a hot ring around the cold spot. The observed morphology is consistent with the 'cosmic web'[30] picture in which voids are typically surrounded with 'walls' of higher density regions, while clusters fade gradually into the surrounding with filaments originating from them. Given the somewhat arbitrary rotations of each image in the stack, and the noise level, small-scale features should be interpreted cautiously.

Granett *et al* seek to detect the **'late ISW effect' due to dark energy** by *cross-correlating* SDSS red luminous galaxies with the WMAP-5 sky

However the observed temperature decrement of -11.3 ± 3.1 μK is ~10 times *more* than expected in the ΛCDM model

So the voids must be even *bigger* and *emptier* than indicated by the galaxy counts

Hunt & Sarkar (2010)



(Springel, Frenk, White 2007)

This is *not* consistent with simulations of structure formation in the ΛCDM model (which is said to agree with observations)

The *WMAP* and *SDSS* results constrain $P_{\mathrm{m}}(k) \Rightarrow$ use this to estimate $\delta_H = \delta H / H$.

The variance of δ_H on the scale R is Wang *et al.* 1997

$$\left\langle \delta_H^2 \right\rangle_R = \frac{\Omega_m^{1.2}}{2\pi^2 R^2} \int \mathrm{d}k\, P(k) \left[\frac{3}{k^2 R^2} \left(\sin kR - \int_0^{kR} \mathrm{d}x \frac{\sin x}{x} \right) \right]^2 .$$

Use MCMC to draw n samples $\boldsymbol{\theta}_i$ from $P(\boldsymbol{\theta}|\, \mathrm{data})$. Then estimate of distribution is

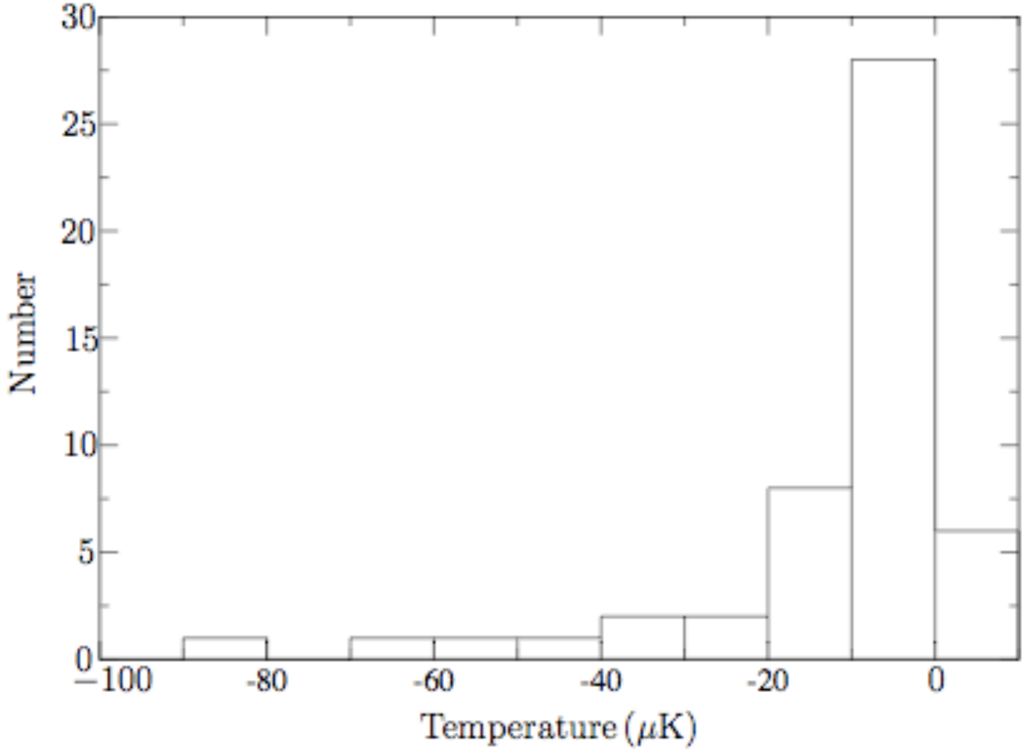
$$P(\boldsymbol{\theta}|\, \mathrm{data}) \simeq \frac{1}{n} \sum_{i=1}^{n} \delta(\boldsymbol{\theta} - \boldsymbol{\theta}_i) .$$

Hence

$$P(\delta_H|\, \mathrm{data}) = \int P(\delta_H|\, \boldsymbol{\theta})_R P(\boldsymbol{\theta}|\, \mathrm{data})\, \mathrm{d}\boldsymbol{\theta} \simeq \frac{1}{n} \sum_{i=1}^{n} P(\delta_H|\, \boldsymbol{\theta}_i)_R ,$$

where

$$P(\delta_H|\, \boldsymbol{\theta})_R = \frac{1}{\sqrt{2\pi \left\langle \delta_H^2 \right\rangle_R}} \exp\left(-\frac{\delta_H^2}{2 \left\langle \delta_H^2 \right\rangle_R} \right) .$$

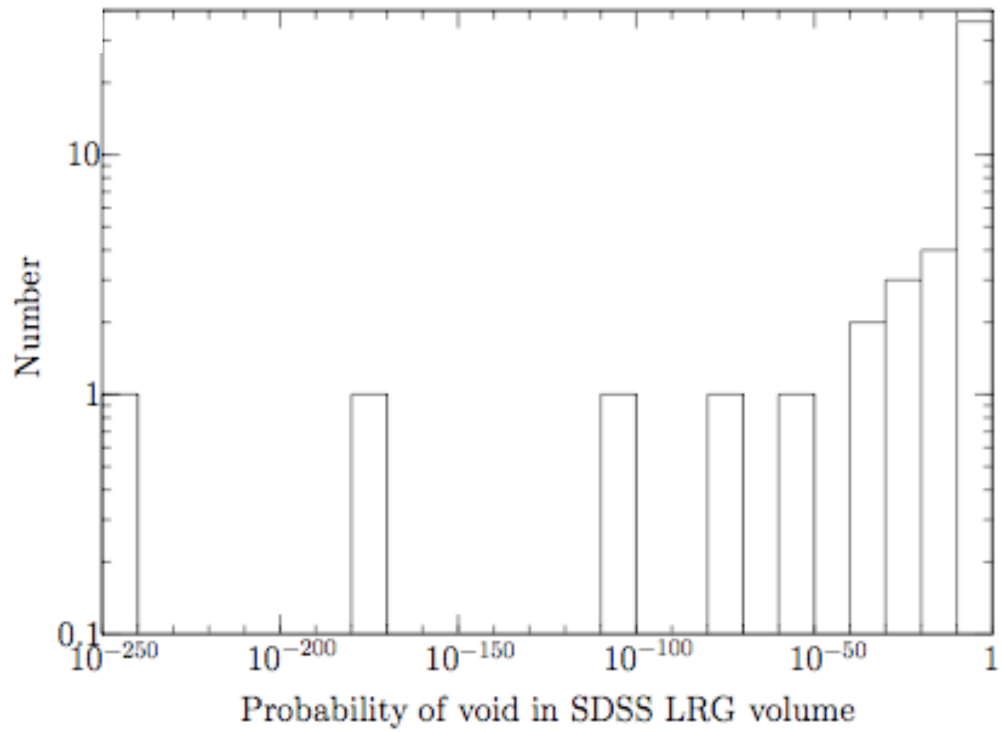


To yield the observed temperature decrements (*if* due to the ISW effect), the *observed* voids must have underdensities $\delta \sim$ - 0.7-0.9 and radii ~ 100-200 h^{-1} Mpc

$$\frac{\Delta T}{T} = -\frac{\Omega_{\rm m}}{2}\left(\frac{R}{c/H_0}\right)^3 \frac{(1+2z)^{1/2}}{(1+z)^2}\delta$$

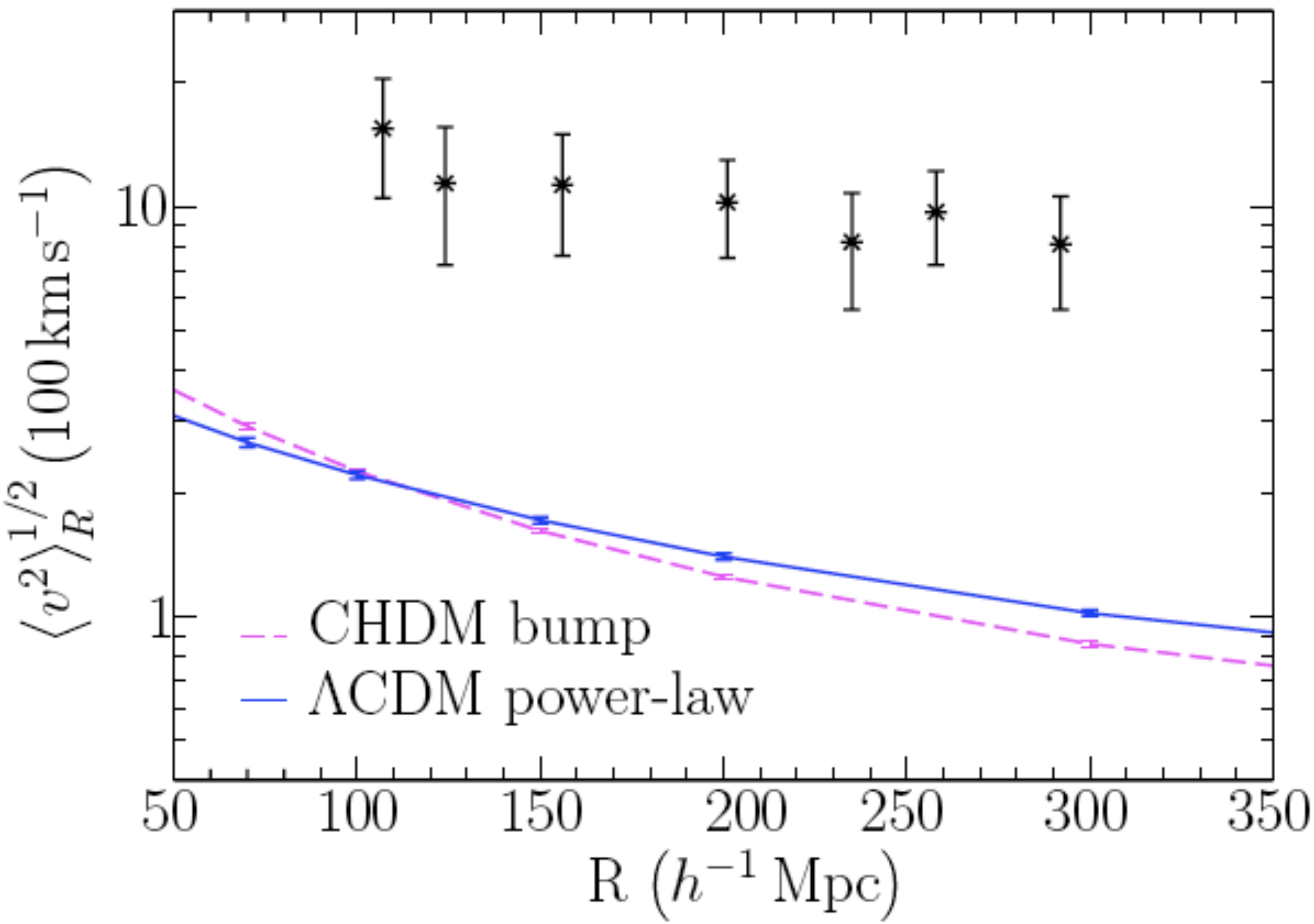
The probability of finding such huge underdense regions in the ΛCDM model (normalised to WMAP) is *vanishingly* small

Hunt & Sarkar (2010)



Unexpectedly large peculiar velocities have been detected recently

Kashlinsky *et al* (2009, 2010), Watkins *et al* (2009)



This too *cannot* be accounted for in the standard theory of structure formation (assuming gaussian adiabatic density fluctuations)

Conclusions

There has been a renaissance in cosmology but modern data is still interpreted in terms of an *idealised* model whose basic assumptions have not been rigorously tested

The standard FRW model naturally admits $\Lambda \sim H_0^2$ …
and this is being *interpreted* as dark energy: $\Omega_\Lambda \sim H_0^2 M_P^2$

More realistic models of our *inhomogeneous* universe may account for the SNIa Hubble diagram without acceleration

The CMB and LSS data can be equally well fitted if the primordial perturbations are *not* scale-free and $m_\nu \sim 0.5$ eV

Dark energy may just be an artifact of an oversimplified cosmological model