Non-Closed Universe in Normalized General Relativity

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General Relativity

$$I_{EH} = \int d^4x\sqrt{-g}\left(-R - 2\Lambda + L_M\right)$$

$$\boxed{R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \Lambda g_{\mu\nu} - \frac{1}{2}T_{\mu\nu}}$$

Can we determine Λ?

Unimodular Gravity

$$\boxed{g = \det\left(g_{\mu\nu}\right) = 1}$$

tracefree equations

$$R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} = -\frac{1}{2}\left(T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu}\right)$$

Bianchi identities

$$-R_{,\mu} + \frac{1}{2} T_{,\mu} = 0 \Leftrightarrow -R + \frac{1}{2} T = 4\Lambda$$

Λ is a constant of integration

Normalized General Relativity

$$I_{NGR} = \frac{\int \sqrt{-g}\left(-R + L_M\right)}{\int \sqrt{-g}}$$

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Equations of motion

$$\delta I_{NGR} = \frac{1}{\int \sqrt{-g}} \left(\delta I_{EH} - \frac{I_{EH}}{\int \sqrt{-g}} \delta \int \sqrt{-g} \right)$$

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$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \Lambda g_{\mu\nu} - \frac{1}{2} T_{\mu\nu}$$

$$\Lambda = \frac{I_{EH}^{(classical)}}{2}$$

Self consistency condition

Machian Principle

$$\Lambda = \frac{I_{EH}^{(classical)}}{2}$$

”Mass there governs inertia here”

”Local physical laws are determined by the large-scale structure of the Universe”

Simple example

Empty spacetime $T_{\mu\nu} = 0$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \Lambda g_{\mu\nu}$$

Taking trace

$$-R = 4\Lambda$$

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Self consistency equation

$$\Lambda = \frac{I_{NGR}^{classical}}{2}$$

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Self consistency equation + equations of motion

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Self consistency equation + equations of motion

$$\Lambda = \frac{I_{NGR}^{classical}}{2} = \frac{\int \sqrt{-g}\left(-R\right)}{2\int \sqrt{-g}} = 2\Lambda$$

Simple example

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$$-R = 4\Lambda$$

Self consistency equation + equations of motion

$$\Lambda = \frac{I_{NGR}^{classical}}{2} = \frac{\int \sqrt{-g}\,(-R)}{2\int \sqrt{-g}} = 2\Lambda \quad \boxed{\Rightarrow \Lambda = 0}$$

Simple example

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Taking trace

$$-R = 4\Lambda$$

Self consistency equation + equations of motion

$$R_{\mu\nu} = 0$$

Simple example

Starting from $\dfrac{\int f(R)\sqrt{-g}\,d^4x}{\int \sqrt{-g}\,d^4x}$

we are led towards $\left\langle R\dfrac{df}{dR}\right\rangle = 0$

FLRW model

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

$$T_{\mu\nu} = diag\{-\rho, p, p, p\}$$

$k = -1$ open

$k = 0$ flat

$k = 1$ closed

$$\boxed{\frac{\dot{a}^2 + k}{a^2} = \frac{1}{3}(\rho + \Lambda)}$$

Normalized FLRW model

Plugging in the FLRW line element one is led towards the reduced action

$$I_{NGR} = \frac{\int \left(6\frac{a\ddot{a} + \dot{a}^2 + k}{a^2} - 2\rho\left(a\right) \right) a^3 dt}{\int a^3 dt}$$

Its first variation is:

$$2\ddot{a}a + \dot{a}^2 + k = (\Lambda + \rho(a))a + \frac{a^3 \rho'(a)}{3}$$

Self Consistency $\Lambda = \frac{I_{NGR}}{2}$

Normalized FLRW model

Eliminating $\dot{a}$ and $\ddot{a}$ we are led towards

$$\Lambda = -\left\langle \rho(a) + \frac{1}{2}\rho'(a)a \right\rangle$$

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Employing the local conservation law

$$\rho'(a) + \frac{3}{a}\left(\rho(a) + p(a)\right) = 0$$

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The self consistency may be written as

$$\boxed{\Lambda = \frac{1}{2}\langle \rho + 3p \rangle \iff \left\langle \frac{\ddot{a}}{a} \right\rangle = 0}$$

Normalized FLRW model

Starting from the equation of state $p = w\rho$

$$\rho\left(a\right) = c^2 a^{-3(1+w)}$$

$\rho + 3p > 0$ implies $-3(1+w) < -2$

To fix Λ we substitute the solution back into the self consistency

$$\Lambda = \frac{c^2}{2}(1+3w)\left\langle a(t;\Lambda)^{-3(1+w)} \right\rangle$$

In case spacetime volume increases without bound $\Lambda \to 0^+$

Normalized FLRW model

$$\frac{\dot{a}^2 + k}{a^2} = \frac{1}{3}(\rho + \Lambda)$$

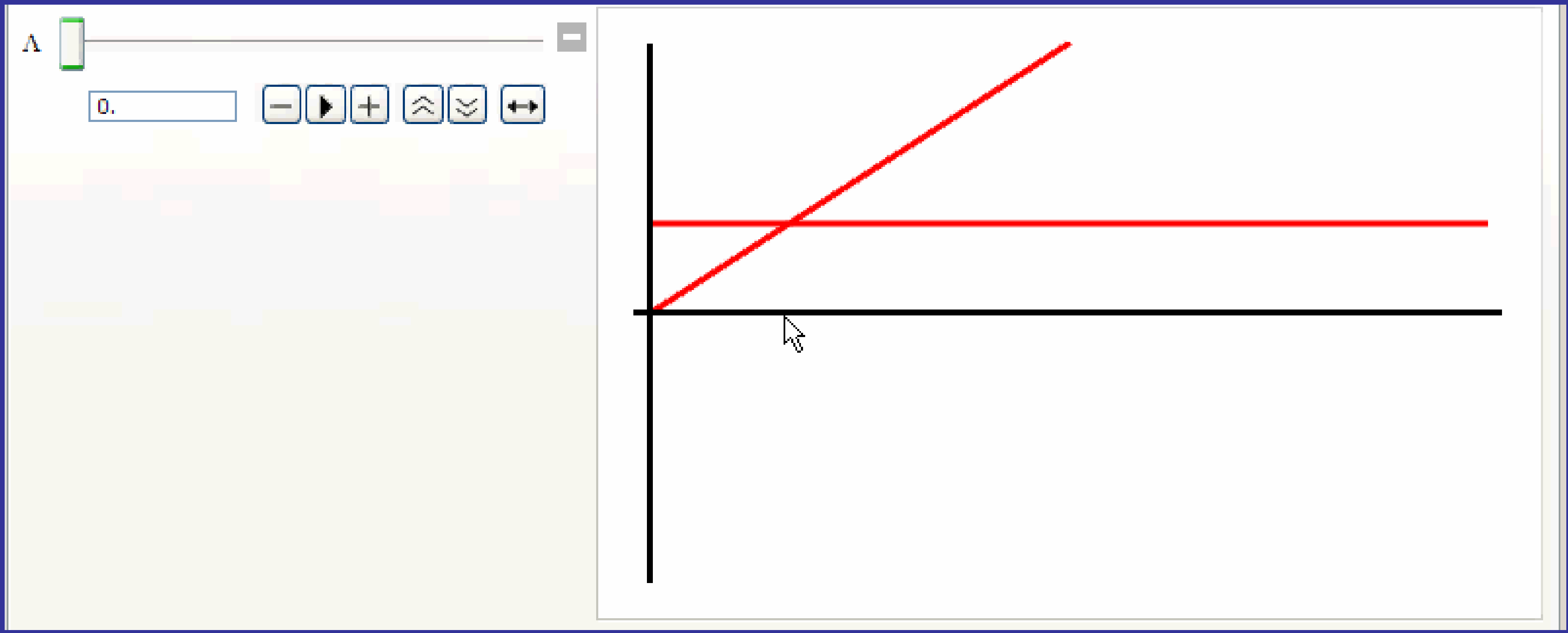
Use new variable $x = a^2$ and consider radiation $\rho = \frac{c^2}{a^4}$

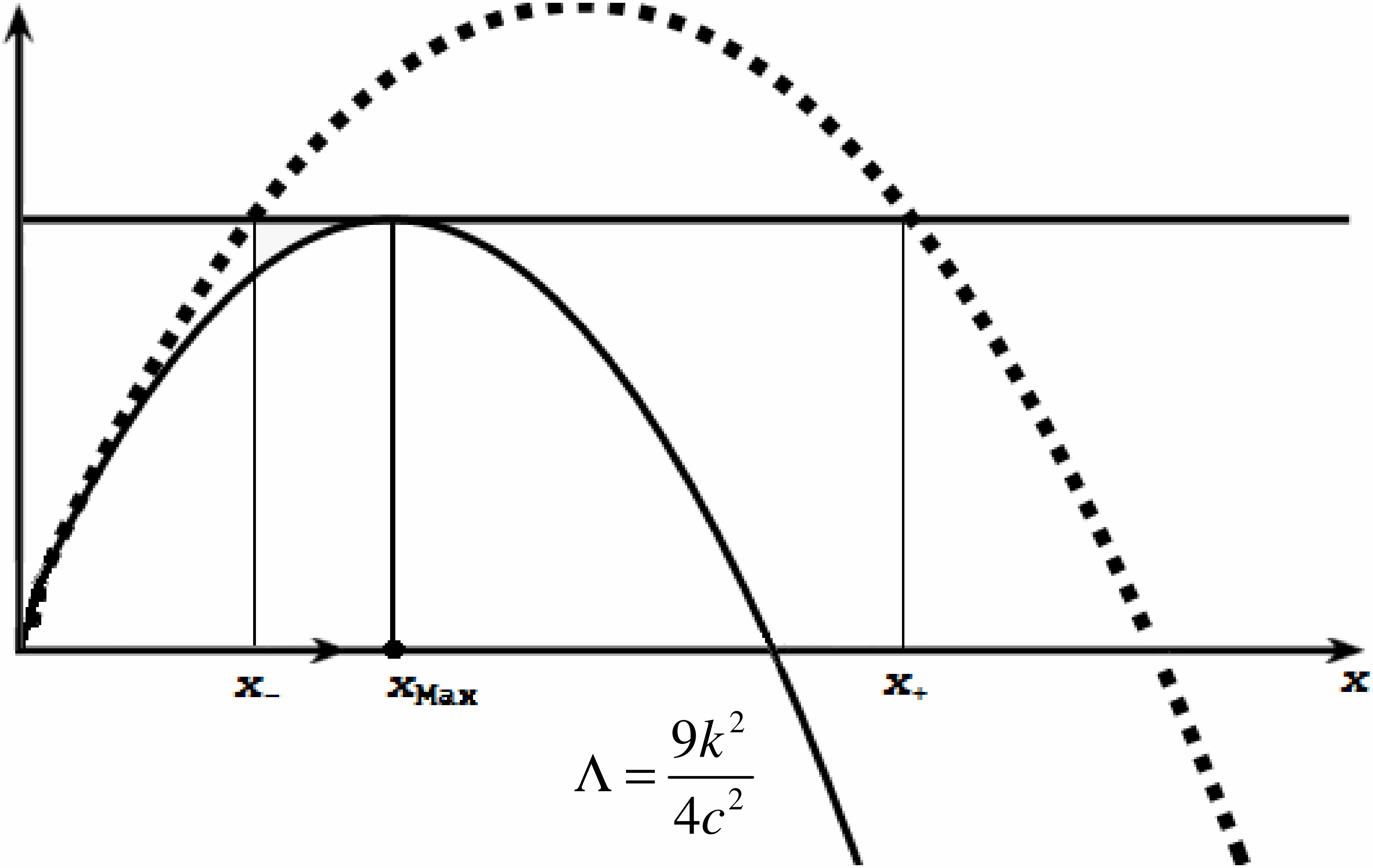
$$\boxed{\frac{1}{4}\dot{x}^2 + \left(-\frac{\Lambda}{3}x^2 + kx\right) = c^2}$$

Normalized FLRW model

What values of Λ are allowed?

$$\frac{1}{4}\dot{x}^2 + \left(-\frac{\Lambda}{3}x^2 + kx\right) = c^2$$





$$\Lambda = \frac{9k^2}{4c^2}$$

NGR Features

- General covariant

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- General covariant
- No extra degrees of freedom
- Underlying Symmetry principle

Non-local from local?
1D example

$$S = \int_{t_1}^{t_2} \left(\frac{m}{2} \left(\dot{r}^2 + r^2 \dot{\varphi}^2 \right) - V\left(r \right) \right) dt$$

Non-local from local?
1D example

$$S = \int_{t_1}^{t_2} \left(\frac{m}{2}\left(\dot{r}^2 + r^2\dot{\varphi}^2\right) - V(r) \right) dt$$

Integrate Euler-Lagrange for φ between t_1 and t_2 to obtain

$$mr^2\dot{\varphi} = c \Rightarrow \dot{\varphi} = \frac{c}{mr^2} \Rightarrow \varphi_2 - \varphi_1 = \frac{c}{m}\int_{t_1}^{t_2} \frac{dt}{r^2} \Rightarrow \frac{c}{m} = \frac{\varphi_2 - \varphi_1}{\int_{t_1}^{t_2} \frac{dt}{r^2}}$$

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$$S_{wrong} = \int_{t_1}^{t_2} \left(\frac{m}{2} \dot{r}^2 + \frac{c^2}{mr^2} - V(r) \right) dt$$

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$$mr^2\dot{\varphi}=c\Rightarrow\dot{\varphi}=\frac{c}{mr^2}\Rightarrow\varphi_2-\varphi_1=\frac{c}{m}\int_{t_1}^{t_2}\frac{dt}{r^2}\Rightarrow\frac{c}{m}=\frac{\varphi_2-\varphi_1}{\int_{t_1}^{t_2}\frac{dt}{r^2}}$$

$$S_{correct}=\int_{t_1}^{t_2}\left(\frac{m}{2}\dot{r}^2-V\left(r\right)\right)dt+\frac{\left(\varphi_2-\varphi_1\right)^2}{2m\int_{t_1}^{t_2}\frac{dt}{r^2}}$$

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