BULK DECAY OF (4+N) DIMENSIONAL BLACK HOLES: TENSOR TYPE GRAVITONS

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MOTIVATION

- Possibility of BH creation at LHC due to the existence of Large Extra Dimensions (ADD, RS models).

- Microscopic BH are higher-dimensional objects, so their decay pattern depends on the number and the nature (flat, warped) of the extra dimensions.

- The existence of both “bulk” and “brane” channel is a feature of Hawking radiation. The knowledge of the balance between them would be a guide to evaluate relevant detector signals (if any!).

Why Simply Rotating BH?

- Most interesting case as colliding particles would form BH with angular momentum on our brane only.
- Its line-element is a simplified Myers-Perry solution:

$$ds^2 = -\left(1 - \frac{\mu}{\Sigma r^{n-1}}\right)dt^2 - \frac{2\alpha\mu sin^2\theta}{\Sigma r^{n-1}}dtd\varphi + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$
$$+\left(r^2 + \alpha^2 + \frac{\alpha^2\mu sin^2\theta}{\Sigma r^{n-1}}\right)sin^2\theta d\varphi^2 + r^2cos^2\theta d\Omega_n^2$$

where $\Delta = r^2 + \alpha^2 - \frac{\mu}{r^{n-1}}$, $\Sigma = r^2 + \alpha^2 cos^2\theta$, $\mu = M_{BH}\frac{16\pi G_D}{(n+2)A_{n+2}}$, $\alpha = \frac{(n+2)J}{2M_{BH}}$,

n the number of extra dimensions,

G_D the (4+n) Newton's constant and An+2 the area of a (4+n) unit sphere

- There is a known eq. for tensor-type gravitational perturbations in this space-time background, provided that $n \geq 3$

Tensor-type Gravitational Perturbations (G_{ij})

For the aforementioned line-element [Kodama(2008)]

$\delta G_{ij} = 2S^2(x)\sum_{l,a} H_T^{(l,a)}(x) T_{ij}^{(l,a)}(y)$ with the harmonic tensor T_{ij} satisfying the eigenvalue eq.

$[\hat{\Delta} + l(l+n-1) - 2] T_{ij}^{(l,a)} = 0$ (Δ: Laplace-Beltrami operator)

Einstein's vacuum eq. with the use of the rather simple ansatz

$$H_T(x) = e^{-i\omega t} e^{im\varphi} R(r) Q(\theta)$$

give

$$\frac{1}{r^n}\partial_r(r^n \Delta \partial_r R) + \left(\frac{K^2}{\Delta} - \frac{l(l+n-1)}{r^2} - \Lambda_{jlm}\right) R = 0$$

$$\frac{1}{\sin\theta \cos^n\theta}\partial_\theta(\sin\theta \cos^n\theta Q) + \left(\omega^2 \alpha^2 \cos^2\theta - \frac{m^2}{\sin^2\theta} - \frac{l(l+n-1)}{r^2} + E_{jlm}\right) Q = 0$$

where $K = (r^2 + \alpha^2)\omega - am$, $\Lambda_{jlm} = E_{jlm} + a^2\omega^2 - 2am\omega$ and E_{jlm} the eigenvalue of the angular function Q(θ)

They satisfy the same field eq. with massless scalars (!), only with $l \geq 2$

Solving the radial eq. near the horizon

- For $r \cong r_H$ ($r_H^{n+1} = \frac{\mu}{1+\alpha_*^2}$, where $\alpha_* \equiv \frac{\alpha}{r_H}$, which follows from $\Delta(r_H) = 0$) the radial eq. takes the form of a hypergeometric DE.

- Boundary condition: no outgoing modes exist near BH's horizon

- Solution: $$R_{NH}(f) = A f^{\alpha}(1-f)^{\beta} F(a,b,c;f)$$

where $f(r) = \frac{\Delta(r)}{r^2+\alpha^2}$, $\alpha = i\frac{K_*}{A_*}$, $\beta = \frac{1}{2}[(2-D_*) - \sqrt{(D_*-2)^2 - 4[\frac{K_*^{\,2} - [l(l+n-1)\alpha_*^2 + \Lambda_{jlm}](1+\alpha_*^2)}{A_*^{\,2}}]}]$

$$A_* = (n+1) + (n-1)\alpha_*^2$$

$$D_* \equiv 1 - \frac{4\alpha_*^2}{A_*^{\,2}}$$

$$K_* = (1+\alpha_*^2)\omega_* - \alpha_* m \qquad \omega_* \equiv \omega r_H$$

Solving the radial eq. far away

- For $r \gg r_H$ the radial eq. takes the form of a Bessel DE.
- Solution: $$R_{FF}(r) = \frac{B_1}{r^{\frac{n+1}{2}}} J_\nu(\omega r) + \frac{B_2}{r^{\frac{n+1}{2}}} Y_\nu(\omega r)$$

where J_ν : Bessel functions of the first kind
and B_ν : Bessel functions of the second kind

parameter ν stands for $$\nu = \sqrt{E_{jlm} + a^2\omega^2 + (\frac{n+1}{2})^2}$$

Matching the two solutions

- R_{NH} needs to be stretched towards larger values of r:

$$\widetilde{R_{NH}} \cong A_1 r^{-(n+1)\beta} + A_2 r^{(n+1)(\beta+D_*-2)}$$

- R_{FF} needs to be shifted towards smaller values of r:

$$\widetilde{R_{FF}}(r) \cong \frac{B_1(\frac{\omega r}{2})^{\nu}}{r^{\frac{n+1}{2}}\Gamma(\nu+1)} - \frac{B_2\Gamma(\nu)}{\pi r^{\frac{n+1}{2}}(\frac{\omega r}{2})^{\nu}}$$

- Matching $\widetilde{R_{NH}}$ and $\widetilde{R_{FF}}$ is possible only for $\alpha_* < 1$ and $\omega_* < 1$. So we get

$$\frac{B_1}{B_2} = \frac{(\frac{2}{\omega r_H})^{2j+n+1}\Gamma^2(\nu)\Gamma(\alpha+\beta+D_*-1)\Gamma(\alpha+\beta)\Gamma(2-2\beta-D_*)}{\pi(1+\alpha_*^2)^{\frac{2j+n+1}{n+1}}\Gamma(2\beta+D_*-2)\Gamma(2+\alpha-\beta-D_*)\Gamma(1+\alpha-\beta)}$$

Defining Absorption Probability

- For $r \to \infty$ one expects only outgoing and incoming spherical waves to exist (gravitational potential $\to 0$ there)

- The far-field solution becomes:

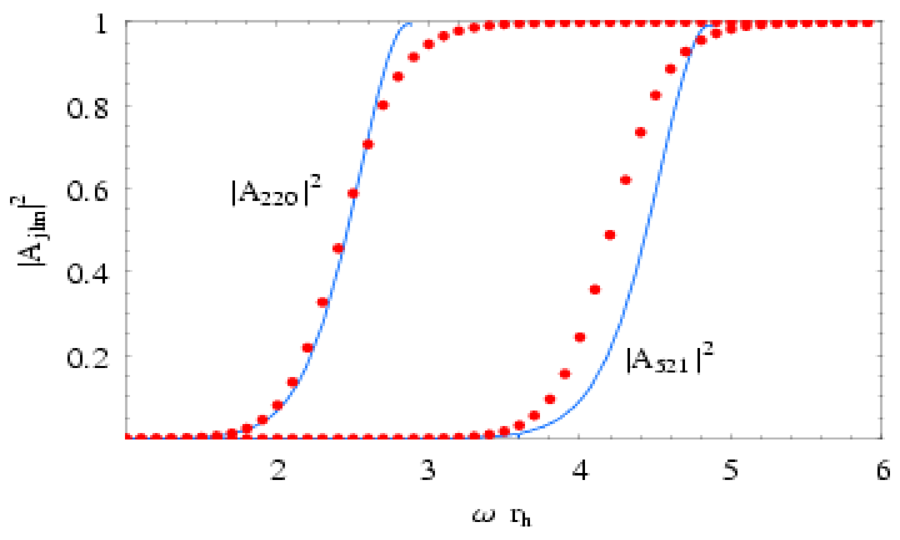
$$\widetilde{R_{FF}}(r) \cong \frac{1}{r^{\frac{n+2}{2}}\sqrt{2\pi\omega}}\left[(B_1 + iB_2)e^{-i\left(\omega r - \frac{\pi}{2}\nu - \frac{\pi}{4}\right)} + (B_1 - iB_2)e^{i\left(\omega r - \frac{\pi}{2}\nu - \frac{\pi}{4}\right)}\right]$$

- Absorption probability is determined as:

$$|A_{jlm}|^2 \equiv 1 - \left|\frac{B_1 - iB_2}{B_1 + iB_2}\right| = \frac{2i(B^* - B)}{BB^* + i(B^* - B) + 1}$$

, where $B \equiv \frac{B_1}{B_2}$

Comparing Analytic – Numerical Results



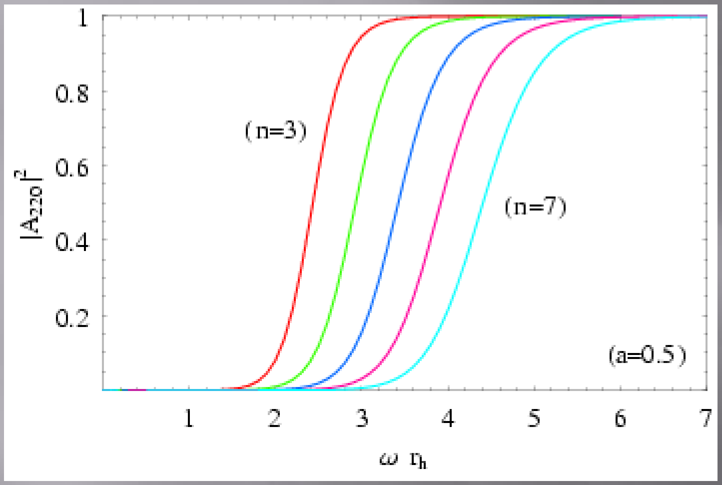
j=l=2, m=0

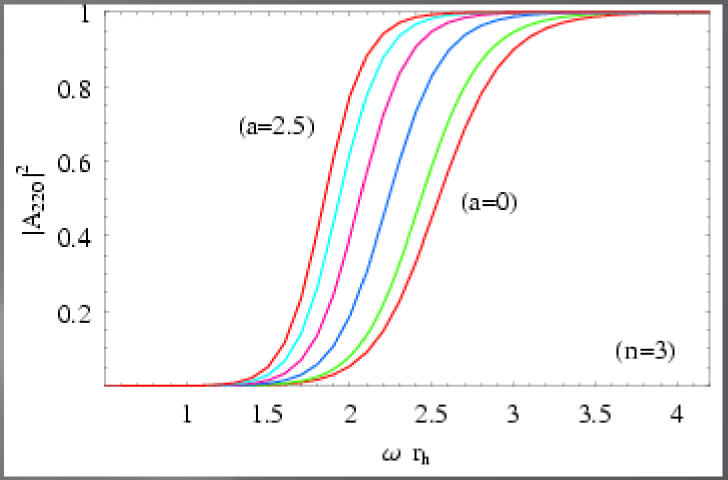
j=5, l=2, m=1

a = 0.5

n = 3

Absorption Probability





j=2, l=2, m=0

n = 3, 4, 5, 6, 7

α = 0.5

Its value **decreases** with the increase of n

α = 0, 0.5, 1, 1.5, 2, 2.5

n =3

Its value **increases** with the increase of α

Energy Emission Rate

$$\frac{d^2E}{d\omega dt} = \frac{1}{2\pi}\sum_{j,l,m}\frac{\omega}{e^{\tilde{\omega}/T_H}-1}N^l_{ST}(S^n)\left|A_{jlm}\right|^2$$

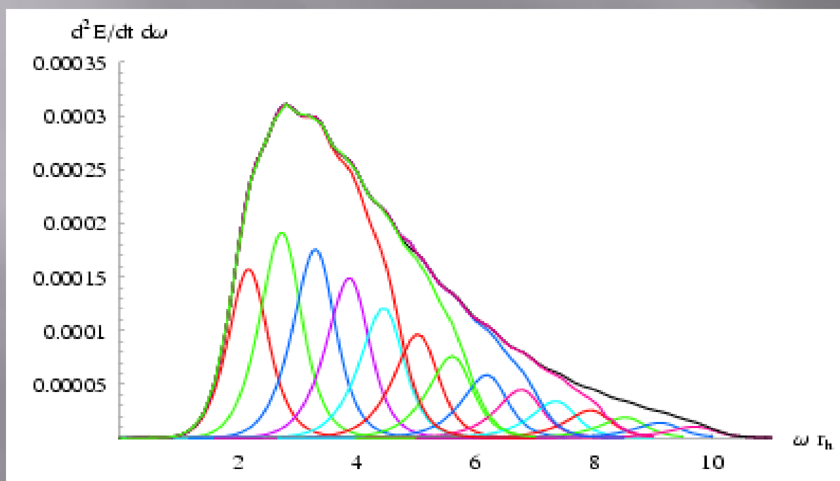
where

$$T_H = \frac{(n+1)+(n-1)\alpha_*^2}{4\pi(1+\alpha_*^2)r_H}$$

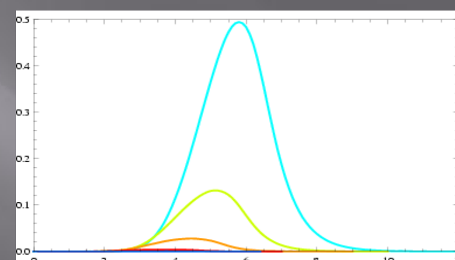
$$\tilde{\omega} \equiv \omega - \frac{\alpha m}{\alpha^2 + {r_H}^2}$$

$$N^l_{ST}(S^n) = \frac{(n+1)(n-2)(n+l)(l-1)(n+2l-1)(n+l-3)!}{2(l+1)!(n-1)!}$$

[Rubin,Ordonez(1984), Kodama(2009)]



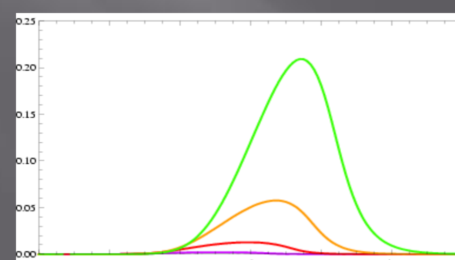
n= 3, α_*=1, (j_{max} = 6, 8, 10, 12, 15)



α_*=1

n=3, 4, 5, 6, 7

(top to bottom)



n=3

α_*=0, 0.5, 1, 1.2

- The angular momentum emission rate exhibits the same behavior with respect to n and α_* parameters

$$\frac{d^2J}{d\omega dt} = \frac{1}{2\pi}\sum_{j,l,m}\frac{m}{e^{\tilde{\omega}/T_H}-1}N^l_{ST}(S^n)\left|A_{jlm}\right|^2$$

Comparison with scalar emission

n	Scalar field	Tensor - gravitons	ratio
3	0.1646	0.0013	0.8%
4	0.3808	0.0222	5.8%
5	0.7709	0.1853	24%

- As n takes larger values, graviton modes play an increasingly important role as energy carriers.

Conclusions

- We have shown that gravitons emitted by a higher-dimensional BH in the bulk carry away a significant part of the BH energy.
- In order to credibly determine the balance between the "brane" and "bulk" emission channels we have to consider the graviton contribution to the process as well.
- To complete the analysis similar calculations have to be made for vector and scalar gravitational modes (a future work maybe?).