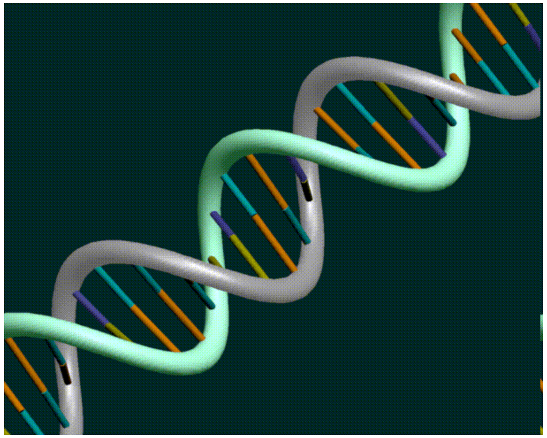
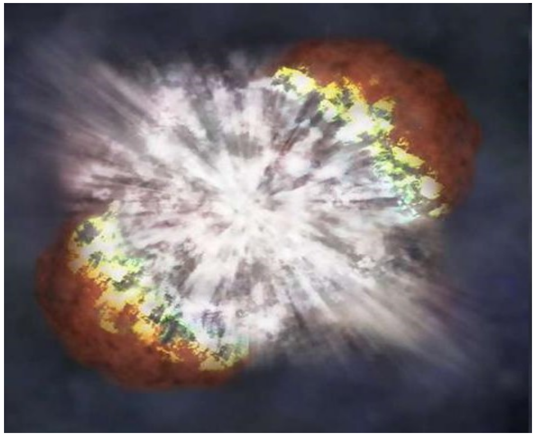
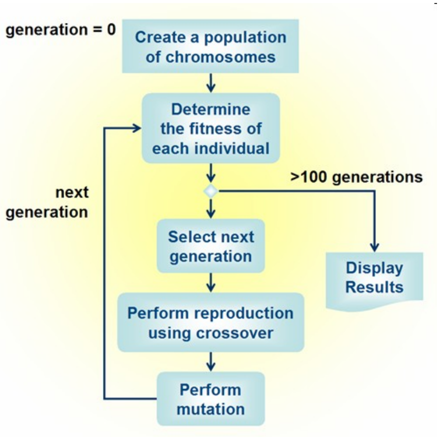
A model independent null test on the cosmological constant







Savvas Nesseris

Niels Bohr International Academy – DISCOVERY Center,

Copenhagen, Denmark

S.N., A. Shafieloo, arxiv: 1004.0960

Main points of the talk

- Observational status quo
- The Om statistic (null test)
- The Genetic Algorithms (GAs)
- Results

The Standard Cosmological model

Einstein equations:

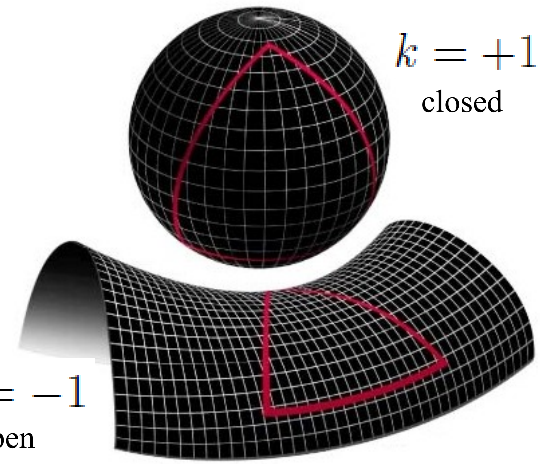
$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{G_{\mu\nu}} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Cosmological Constant

$$T^{\mu}_{\nu} = P g^{\mu}_{\nu} + (\rho + P) U^{\mu} U_{\nu}$$

Robertson-Walker metric:

$$ds^2 = c^2 dt^2 - \alpha(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + sin(\theta)^2 d\phi^2) \right)$$



$k = +1$

closed

$k = -1$

open

Friedmann equations:

$$H^2(\alpha) = \left(\frac{\dot{\alpha}}{\alpha}\right)^2 = \frac{8\pi G}{3}\rho(\alpha) - \frac{k}{\alpha^2}$$

$$\frac{\ddot{\alpha}}{\alpha} = -\frac{4\pi G}{3}(\rho(\alpha) + P(\alpha))$$

Continuity:

(from Bianchi identities)

$$\nabla_{\nu} T^{\mu\nu} = 0 \Longrightarrow \dot{\rho} + 3H(\rho + P) = 0$$

The Standard Cosmological model

Hubble (1929): Universe is expanding

From redshift of distant galaxies

Riess et al. (1998): … and actually is accelerating

from supernovae type Ia

2nd Friedmann equation: $\frac{\ddot{\alpha}}{\alpha} = -\frac{4\pi G}{3}(\rho(\alpha) + 3P(\alpha))$ $\Rightarrow$ $P < -\frac{\rho}{3}$

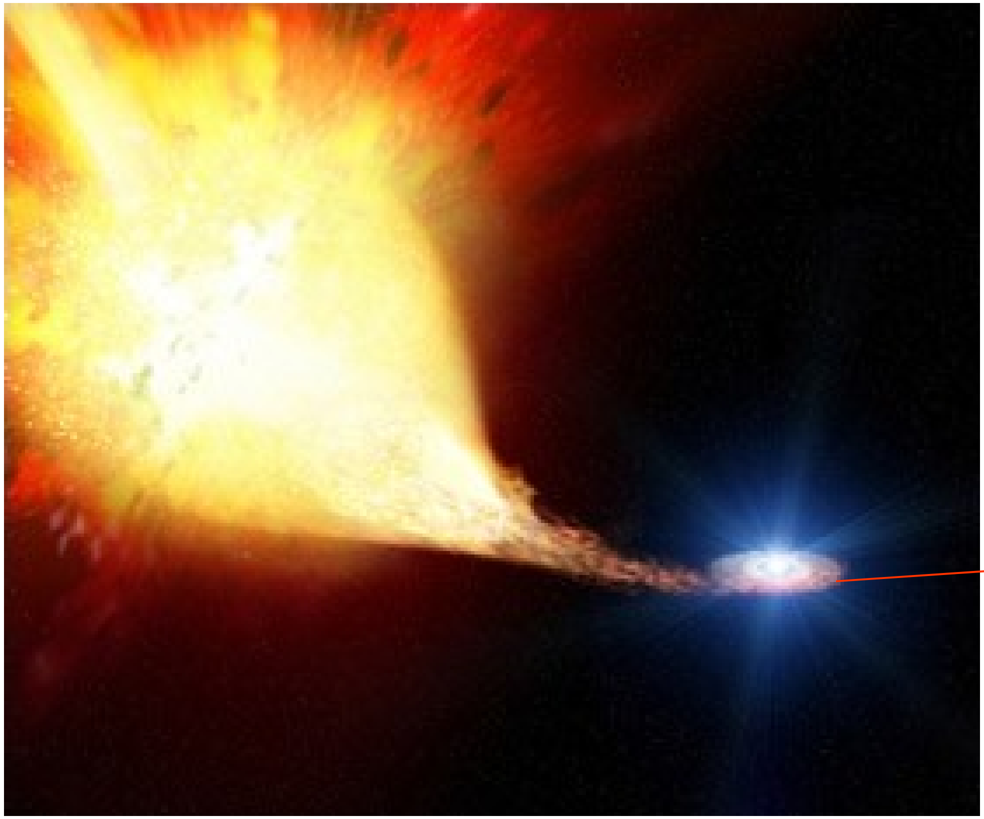
Equation of state $P = w\,\rho$

- $w = 0$ Non-relativistic matter $P << \rho$
- $w = \frac{1}{3}$ Radiation $P = \frac{1}{3}\rho$

$$P < -\frac{\rho}{3} \Rightarrow w < -\frac{1}{3}$$

Known kinds of matter cannot explain the accelerated expansion of the universe!

Type Ia supernovae



$$\boxed{M \to M_{Chand} \simeq 1.38 M_{\odot}}$$

Type Ia supernovae

The amount of light from the explosion that reaches an observer is the apparent luminosity l:

d_L

$$l = \frac{L}{4\pi\, d_L^2}$$

A measure of the brightness of the SnIa is the apparent magnitude m: $m = -2.5\log_{10}\left[\, l(d_L)\right]$

and is related to the lum. distance through:

$$m = M + 5log_{10}\frac{d_L}{Mpc} + 25$$

absolute magnitude $\longrightarrow$ $M = m(d_L = 10\,pc)$

Type Ia supernovae

- SnIa data are given in terms of the distance modulus:

$$\mu_{obs}(z_i) \equiv m_{obs}(z_i) - M$$

- DE is described by w(z)

$$\boxed{w(z) \equiv \frac{P}{\rho}}$$

$$w(z) = -1 + \frac{1}{3}(1+z)\frac{d\ln(\delta H(z)^2)}{d\ln z}$$

$$\delta H(z)^2 = H(z)^2/H_0^2 - \Omega_{0\mathrm{m}}(1+z)^3$$

- Theoretical prediction:

$$D_L(z) = (1+z)\int_0^z dz' \frac{H_0}{H(z';\Omega_{0\mathrm{m}}, w_0, w_1)}$$

$$\mu_{th}(z_i) \equiv m_{th}(z_i) - M = 5log_{10}(D_L(z)) + \mu_0$$

$$\mu_0 = 42.38 - 5log_{10}h$$

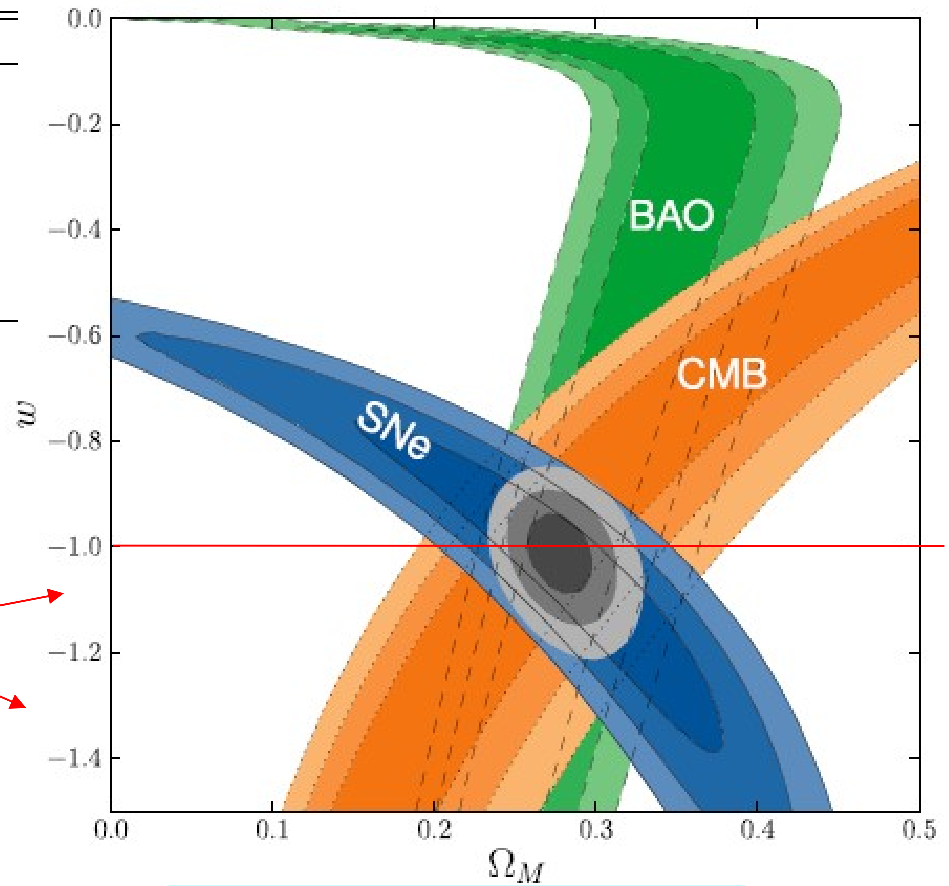
- Minimize to find the best fit parameters:

$$\chi^2_{SnIa}(\Omega_{0\mathrm{m}}, w_0, w_1) = \sum_{i=1}^{N} \frac{(\mu_{obs}(z_i) - \mu_{th}(z_i))^2}{\sigma^2_{\mu\ i}}$$

The status quo

Currently, the supernovae Ia data seem to allow for w<-1 !

Fit	Ω_M	w
SNe	$0.270^{+0.021}_{-0.021}$	-1 (fixed)
SNe+BAO+H_0	$0.309^{+0.032}_{-0.032}$	$-1.114^{+0.098}_{-0.112}$
SNe+CMB	$0.268^{+0.019}_{-0.017}$	$-0.997^{+0.050}_{-0.055}$
SNe+BAO+CMB	$0.277^{+0.014}_{-0.014}$	$-1.009^{+0.050}_{-0.054}$
SNe+BAO+CMB	$0.278^{+0.014}_{-0.014}$	-1 (fixed)
SNe+BAO+CMB	$0.281^{+0.016}_{-0.015}$	$-1.026^{+0.055}_{-0.059}$



Phantom behavior

Union2 dataset, arXiv:1004.1711

The current status

However, there are a few problems:

- Phantom fields (w<-1) violate the Dominant Energy Condition

  $\rho>0$ & $\rho + P \geq 0$

- Crossing w=-1 causes severe gravitational instabilities in the dark energy sector

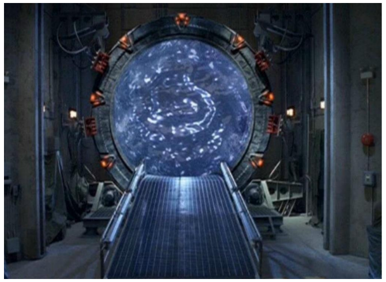
  A. Vikman 2005, astro-ph/0407107

For adiabatic perturbations:

$$c_s^2(a) = w(a) - a\frac{w'(a)}{3(1+w(a))}$$

- Stable wormholes $\longrightarrow$ time-travel?

  I.D. Novikov et al., 0911.4456

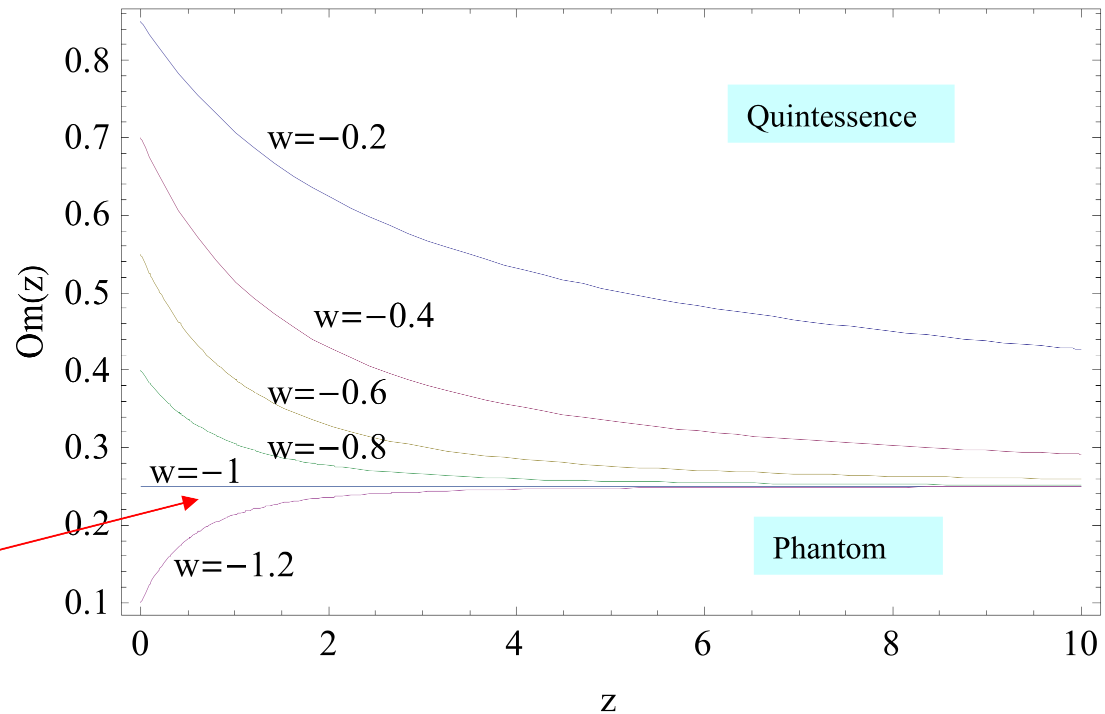


Test for the Cosmological Constant necessary!!!

The Om statistic

Definition: $$Om(x) \equiv \frac{h^2(x)-1}{x^3-1}, \qquad x = 1+z \qquad h(x) = H(x)/H_0$$

Example: $$h^2(x) = \Omega_{0m} x^3 + (1-\Omega_{0m}) x^\alpha \qquad \alpha = 3(1+w)$$



Constant only for w=-1!

V. Sahni, A. Shafieloo and A. A. Starobinsky, Phys. Rev. D **78**, 103502 (2008) [arXiv:0807.3548 [astro-ph]].

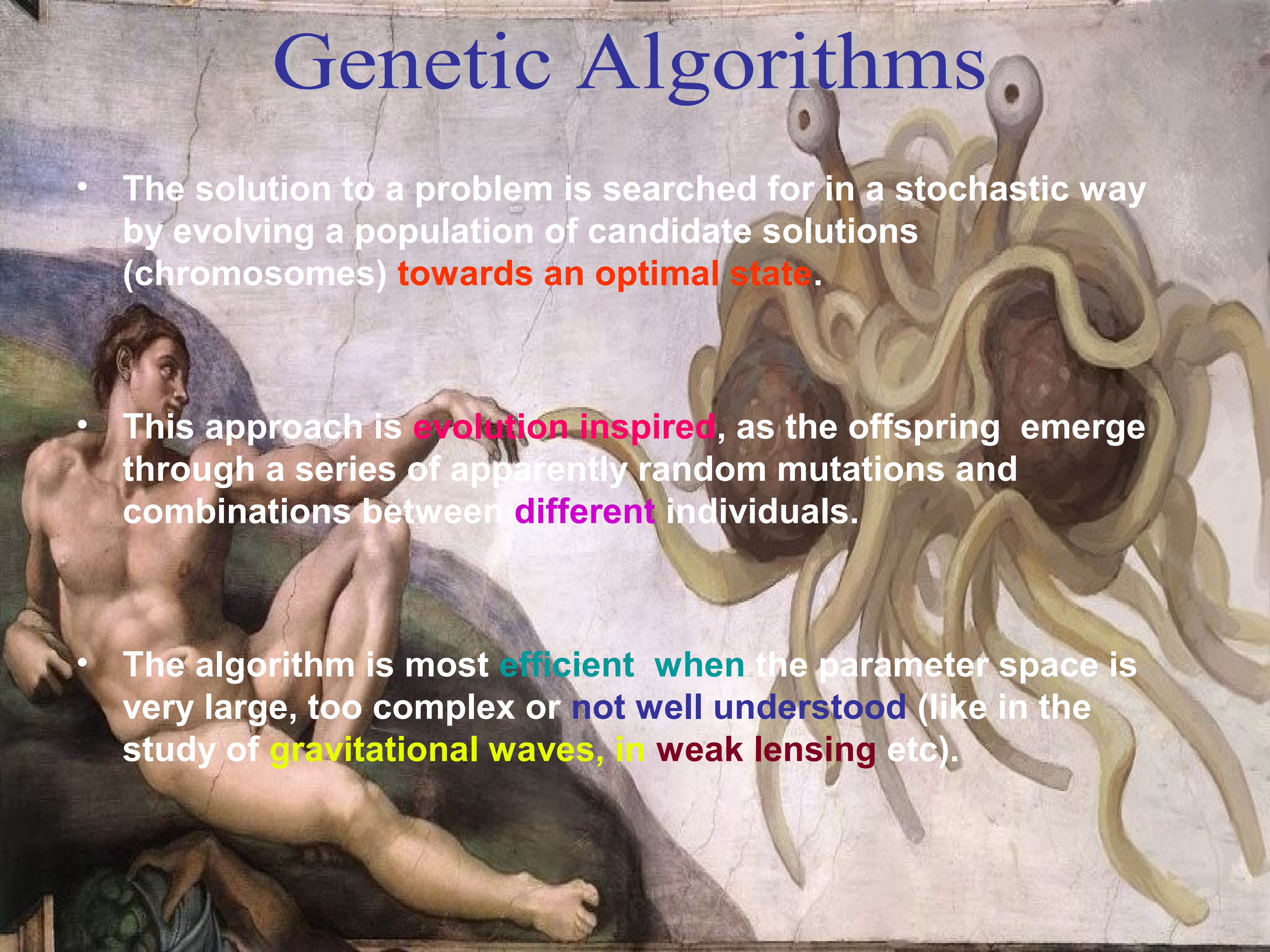
The Om statistic

$$Om(x) \equiv \frac{h^2(x) - 1}{x^3 - 1}, \qquad x = 1+z \qquad h(x) = H(x)/H_0$$

Note: Using a model for h(x) introduces bias into the results

Strategy:

- Fit the data with model independent method (Genetic Algorithms)
- Reconstruct Om(z) and test for deviations from w=-1



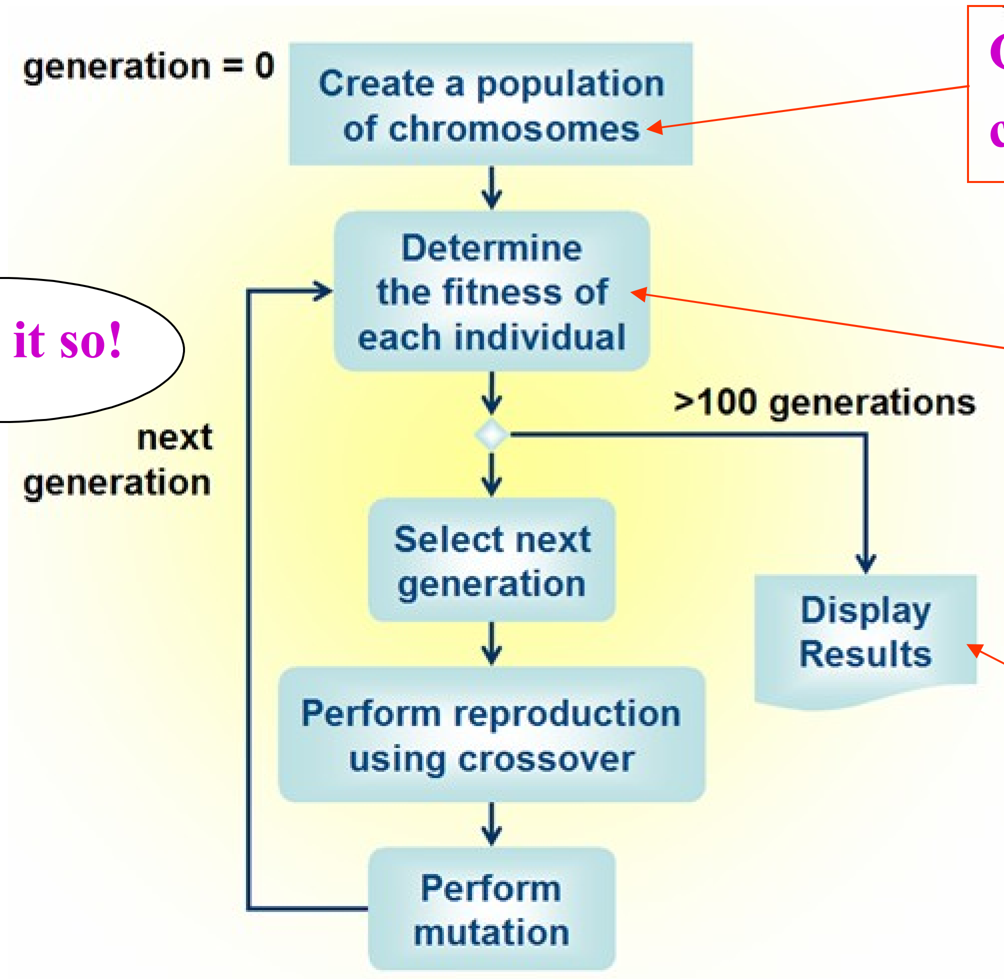
Genetic Algorithms

- The solution to a problem is searched for in a stochastic way by evolving a population of candidate solutions (chromosomes) towards an optimal state.

- This approach is evolution inspired, as the offspring emerge through a series of apparently random mutations and combinations between different individuals.

- The algorithm is most efficient when the parameter space is very large, too complex or not well understood (like in the study of gravitational waves, in weak lensing etc).

Genetic Algorithms: Outline



Group of random candidate solutions.

Measure of how well it fits the data

Really cool figures of the paper!

Genetic Algorithms: Selection

In GAs there are two possible methods:

1) Roulette wheel selection **(the selection probability is proportional to the fitness)**

- widely used method, but…
- a suboptimal solution may dominate the mating pool
- premature convergence…

2) Tournament selection
(find best individual)

- easy to implement
- in line with natural selection
- premature convergence avoided

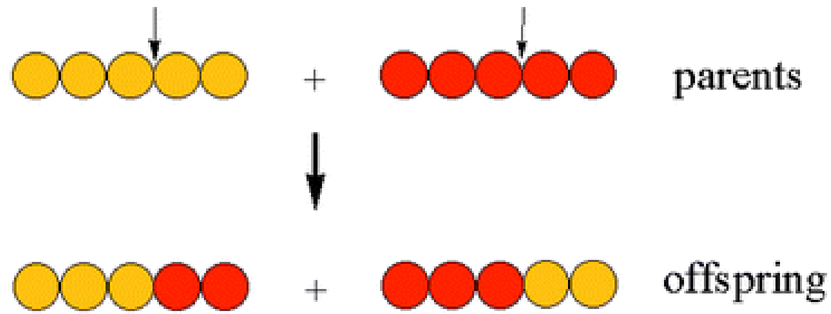
Genetic Algorithms: Reproduction

Reproduction can be done in two ways:

1) Crossover:

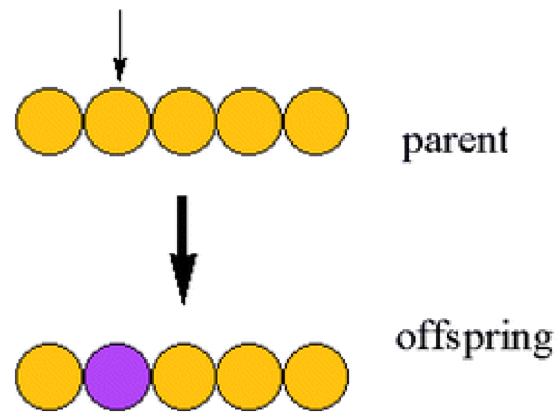
$$\mu_{GA,1}(z) = ln(z) \qquad \mu_{GA,2}(z) = -1 + z + z^2$$

$$\begin{aligned}\mu_{GA,1}(z) \oplus \mu_{GA,2}(z) &\rightarrow (\bar{\mu}_{GA,1}(z), \bar{\mu}_{GA,2}(z), \bar{\mu}_{GA,3}(z)) \\ &= (ln(z^2), -1 + ln(z^2), -1 + ln(z))\end{aligned}$$



2) Mutation:

$$\bar{\mu}_{GA,3}(z) = -1 + ln(z) \rightarrow -1 + ln(z^3)$$



Genetic Algorithms: Example

Genetic Algorithm

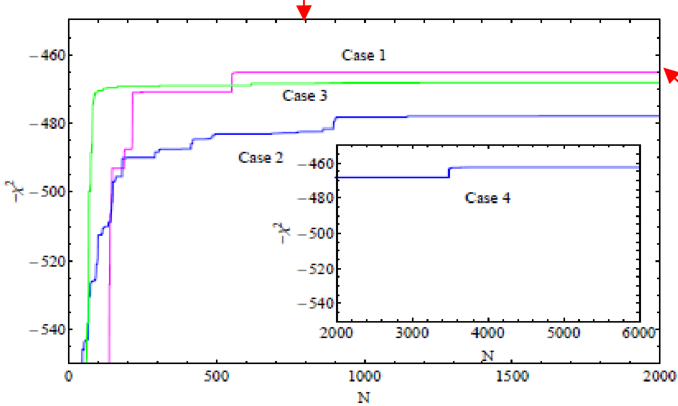
$x,\ x^2$ etc $\quad ln(x)$

$sin(x),\ cos(x) \quad e^x$

$$\mu_{GA,1}(z) = ln(z) \quad \mu_{GA,2}(z) = -1 + z + z^2$$

$$\mu_{GA,3}(z) = sin(z) \quad \ldots$$

Loop over generations, perform crossover + mutation, fitness is measured by χ^2



Grammar / Base pairs

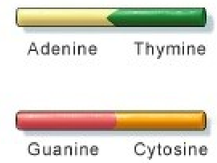
Chromosomes

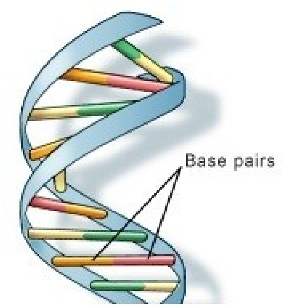
Evolution

Result

Best-fit

Nature





Genetic Algorithms: Strategy

Strategy:

- Apply the GA to the original data and obtain a solution $\mu_{GA}(z)$
- Generate a number of synthetic datasets by drawing points with replacement (bootstrap Monte-Carlo)
- Run the GA on the synthetic datasets.
- Obtain values for the Om statistic for each dataset
- The 95% error can be found by taking the 2.5 and 97.5 percentiles of the bootstrap distribution

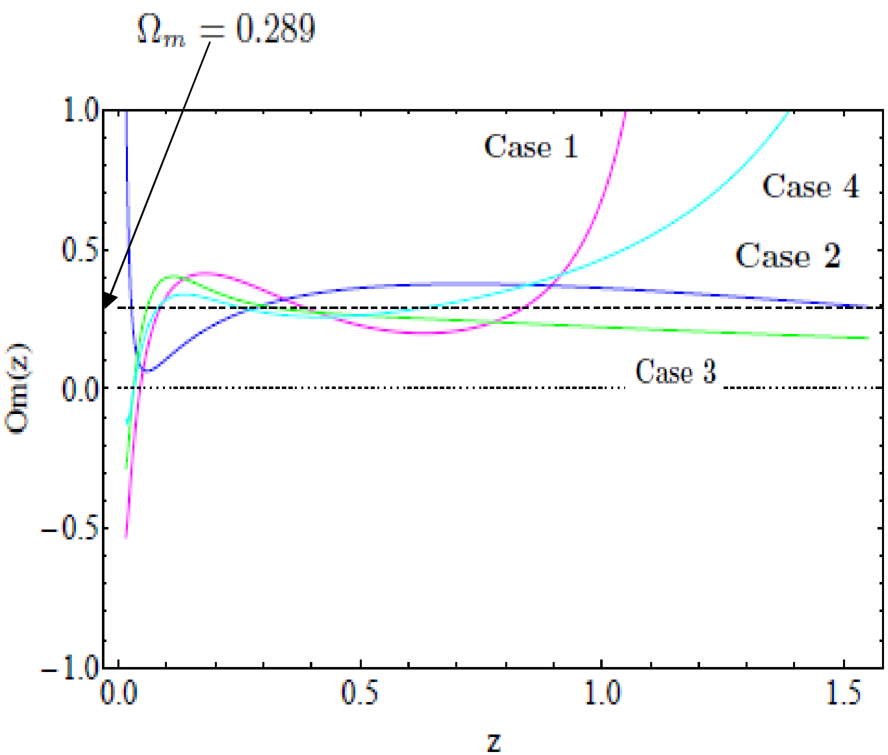
Results

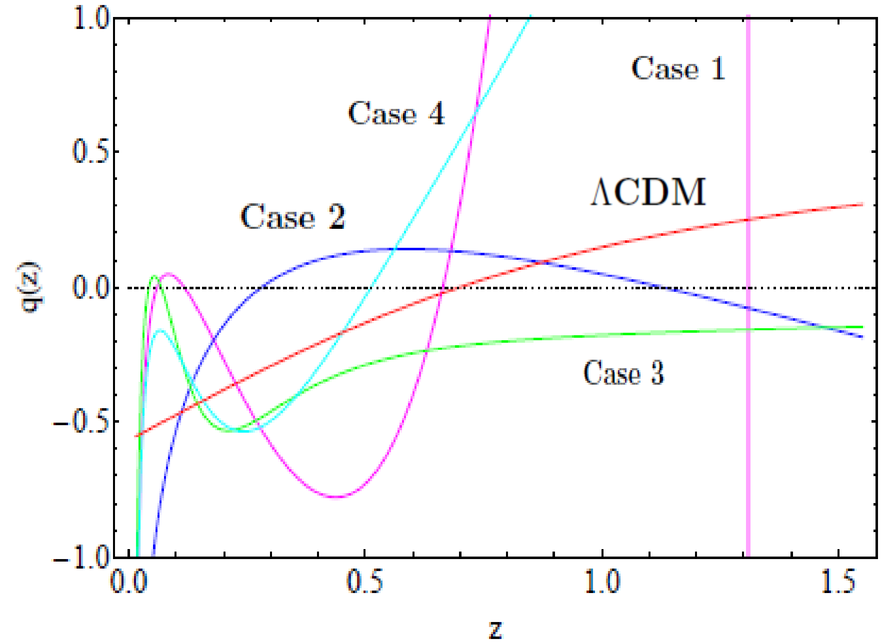
S.N., A. Shafieloo, arxiv: 1004.0960

$x,\ x^2$ etc $\quad ln(x)$

$sin(x),\ cos(x) \quad e^x$

Case 1	Basic grammar	$\chi^2_{min} = 465.19$
Case 2	Basic grammar + Legendre polyn.	$\chi^2_{min} = 477.87$
Case 3	Only polyn. in grammar (3000 gen.)	$\chi^2_{min} = 468.19$
Case 4	Only polyn. in grammar (6000 gen.)	$\chi^2_{min} = 462.56$
ΛCDM	$H(z)^2 = H_0^2(\Omega_m(1+z)^3 + 1 - \Omega_m)$	$\chi^2_{min} = 465.51$



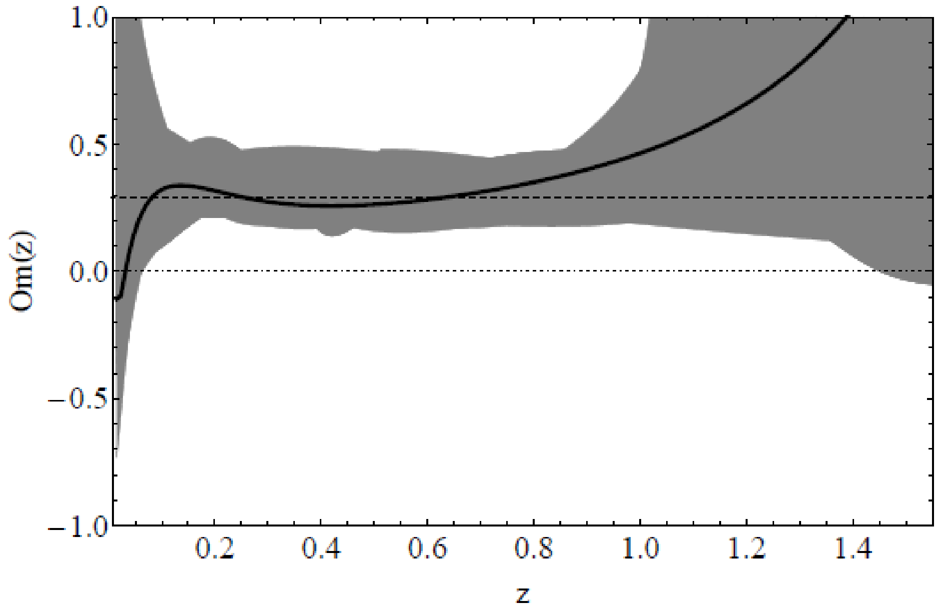


Results

The Om statistic

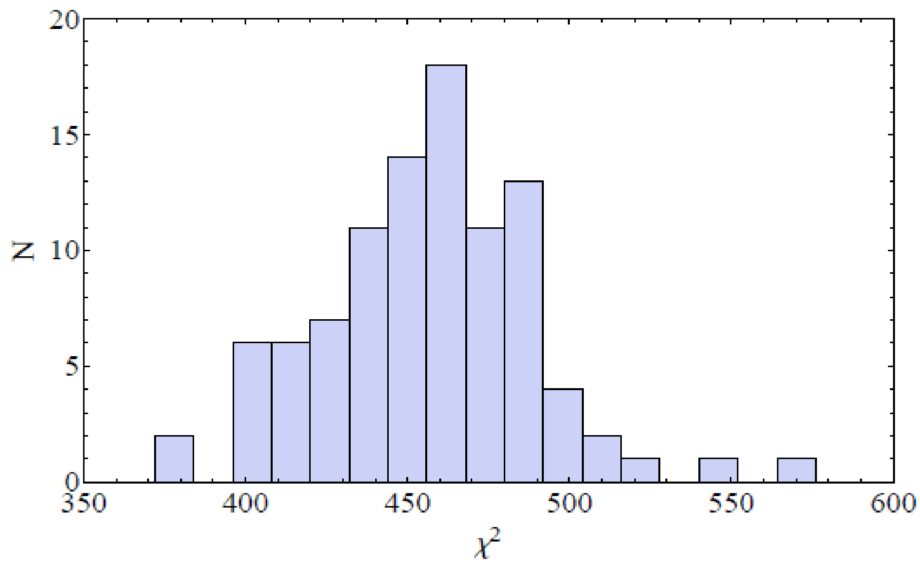
+

2σ error region:



Histogram of the

Bootstrap distribution:



Conclusions - Outlook

- Presented a null test of the Cosmological Constant
- Used the Genetic Algorithms as a model independent reconstruction method for H(z)
- Results in agreement with ΛCDM at the 2σ level

Open issues:

Use the newer SnIa sets (smaller errors – possible to discriminate?)

Compare all model independent methods (in progress SN, Y. Wang)