Quasi-equilibrium models of magnetized compact objects

CHARALAMPOS MARKAKIS
University of Wisconsin – Milwaukee
KOJI URYU
University of the Ryukyus – Okinawa
ERIC GOURGOULHON
Paris Observatory – Meudon

NEB 14, Ioannina
8 June 2010

Strong magnetic field in compact objects

- Comparing the energy of strong magnetic field (B-field) to that of gravitation in NS,

  $U \sim \frac{1}{2}\int d^3x B^2$ T : kinetic energy W : gravitational energy $T/|W| \sim 0.1$

  B-field may change the structure of NS for $U/|W| \sim 0.01$.

  Then, for NS $B \sim 4.4 \times 10^{16} \left(\frac{M[M_\odot]}{1.4M[M_\odot]}\right)\left(\frac{10[\mathrm{km}]}{R[\mathrm{km}]}\right)^2 [\mathrm{G}]$
- Magnetar observations: anomalous X-ray pulsars / soft γ-repeaters, may contain NS with $B \gg 10^{14} - 10^{15}\,\mathrm{G}$ at the NS surface
- Toroidal B-field of NS interior may be much stronger

STRONG MAGNETIC FIELD IN COMPACT OBJECTS

- How strong is the B-field of inspiraling BNS before merger?
  - The poloidal B-field (seen as the surface B-field) may have decayed but the toroidal field may remain strong (?) (Although, even for $B \sim 10^{14}$ G, $U/|W| \sim 10^{-6}$).
- Astrophysical motivation
  - B-field anisotropic
  - B-field may be amplified ~10 times by magnetic winding and MRI in a post-merger pre-collapse object.
  - Important candidate source of short γ-ray bursts.
  - Initial data, calculated from a self-consistent system of the Einstein-Maxwell and MHD-Euler equations assuming quasi-equilibrium, is desirable for merger simulations.

MODELING MAGNETIZED BINARY BLACK HOLES AND NEUTRON STARS

- A helically symmetric perfect fluid Einstein-Maxwell spacetime is used to model a magnetized neutron-star (or black-hole) binary.
- Generalized thermodynamic laws are derived for this system.
- Applying the ideal MHD theory by Bekenstein & Oron (PRD 2000), in which not only the magnetic flux (Alfven theorem) but also a magnetized circulation (generalized Kelvin theorem) are conserved, the first law satisfied by a sequence of equilibrium solutions becomes

  $\delta Q = 0$ or $\delta M = \Omega \delta J$ for an asymptotically flat system.
- A first integral of the MHD-Euler equation for a helically symmetric irrotational BNS is derived in the framework of an ideal MHD theory by Bekenstein & Oron.

$$S = \int \sqrt{-g} d^4x \left(-\epsilon + \frac{1}{16\pi} R - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} + A_\alpha J^\alpha\right)$$

GRMHD equations

- Perfect MHD condition

$$\boldsymbol{u} \cdot \mathbf{d}\boldsymbol{A} = 0 \quad \rightarrow \quad \boldsymbol{B} = \boldsymbol{u} \cdot *\mathbf{d}\boldsymbol{A}$$

- Maxwell equation

$$*\mathbf{d}(\mathbf{u} \wedge \mathbf{B}) = \mathbf{J}$$

- MHD-Euler equation

$$\boldsymbol{u} \cdot \mathbf{d}(h\boldsymbol{u}) = \mathbf{d}\boldsymbol{A} \cdot \boldsymbol{J} \,/\, \rho$$

h	specific enthalpy
$\boldsymbol{F} = \mathbf{d}\boldsymbol{A}$	Faraday tensor
ρ	rest mass density

$$S = \int \sqrt{-g} d^4x \left(-\epsilon + \frac{1}{16\pi} R - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} + F_{\alpha\beta} u^\alpha b^\beta\right)$$

GRMHD equations

- Perfect MHD condition

$$\boldsymbol{u} \cdot \mathbf{d}\boldsymbol{A} = 0 \quad \rightarrow \quad \boldsymbol{B} = \boldsymbol{u} \cdot *\mathbf{d}\boldsymbol{A}$$

- Bekenstein-Oron relation

$$\boldsymbol{u} \cdot \mathbf{d}\boldsymbol{a} = \boldsymbol{B} \quad \rightarrow \quad \boldsymbol{b} = \boldsymbol{u} \cdot *\mathbf{d}\boldsymbol{a}$$

- MHD-Euler equation

$$\boldsymbol{u} \cdot \mathbf{d}\boldsymbol{\pi} = 0 \qquad \boldsymbol{\pi} = h\boldsymbol{u} - \boldsymbol{b} \cdot \mathbf{d}\boldsymbol{A} \,/\, \rho$$

$$S = \int \sqrt{-g} d^4 x \left(-\epsilon + \frac{1}{16\pi} R - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} + F_{\alpha\beta} u^\alpha b^\beta\right)$$

GRMHD equations

- Perfect MHD condition

$$\boldsymbol{u} \cdot \mathbf{d}\boldsymbol{A} = 0 \quad \rightarrow \quad \boldsymbol{B} = \boldsymbol{u} \cdot *\mathbf{d}\boldsymbol{A}$$

- Bekenstein-Oron relation

$$\boldsymbol{u} \cdot \mathbf{d}\boldsymbol{a} = \boldsymbol{B} \quad \rightarrow \quad \boldsymbol{b} = \boldsymbol{u} \cdot *\mathbf{d}\boldsymbol{a}$$

- MHD-Euler equation

$$\boldsymbol{u} \cdot \mathbf{d}\boldsymbol{\pi} = 0 \qquad \boldsymbol{\pi} = h\boldsymbol{u} - \boldsymbol{b} \cdot \mathbf{d}\boldsymbol{A} \,/\, \rho$$

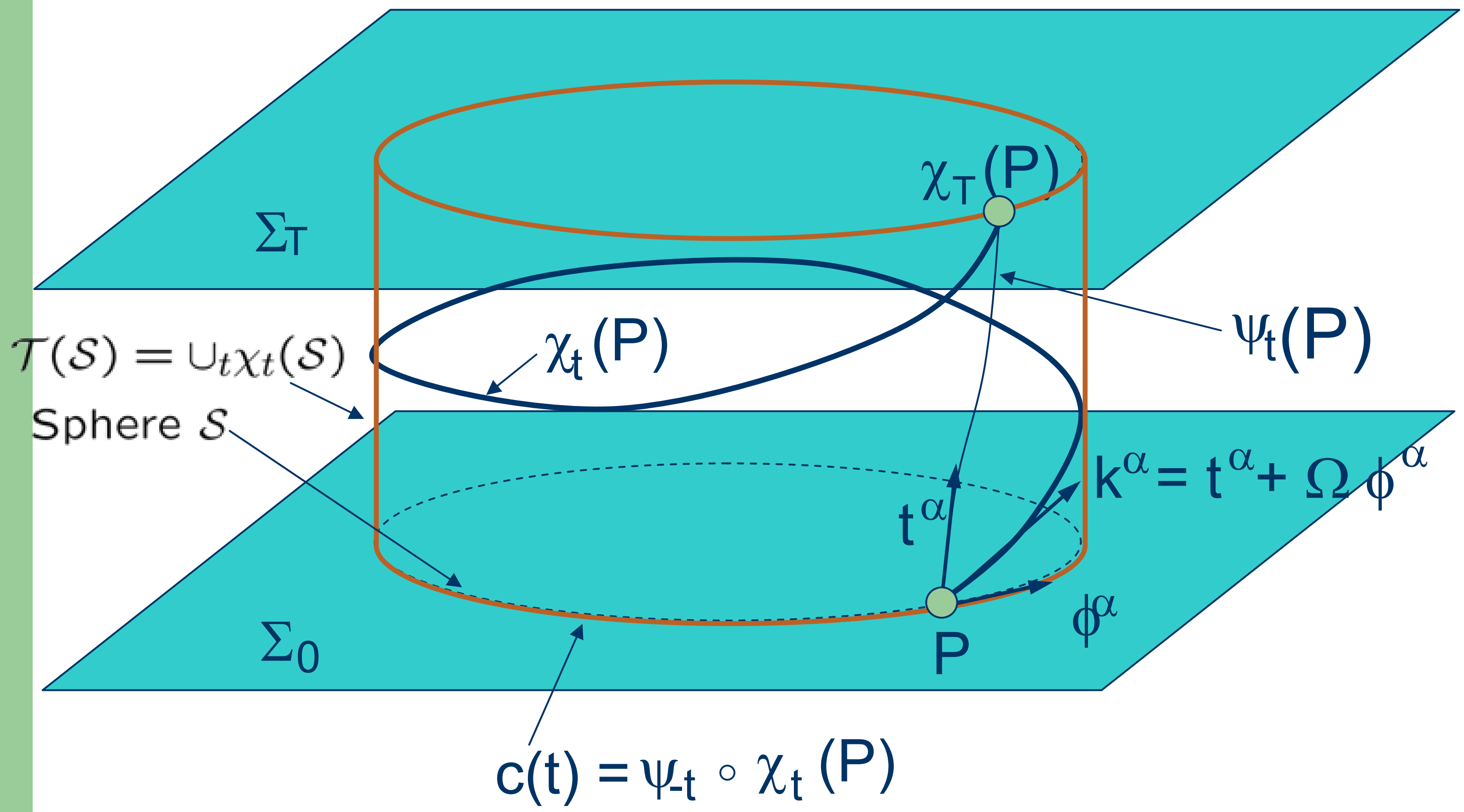
Alfven's theorem

$$\frac{d}{d\tau} \oint_c \boldsymbol{A} \cdot d\boldsymbol{l} = 0$$

Oron's theorem

$$\frac{d}{d\tau} \oint_c \boldsymbol{\pi} \cdot d\boldsymbol{l} = 0$$

Helical symmetry



First integral of the MHD-Euler equation

- Helical symmetry

$$\pounds_{\boldsymbol{k}} \boldsymbol{Q} = 0 \qquad \boldsymbol{Q} = (\boldsymbol{g}, \rho, h, \boldsymbol{u}, \boldsymbol{B}, \boldsymbol{A})$$

- Irrotational magneto-flow

$$\mathbf{d}\boldsymbol{\pi} = 0 \qquad \boldsymbol{\pi} = h\boldsymbol{u} - \boldsymbol{b} \cdot \mathbf{d}\boldsymbol{A} \,/\, \rho = \mathbf{d}\Psi$$

- Integrability condition

$$\pounds_{\boldsymbol{k}} \boldsymbol{b} \cdot \mathbf{d}\boldsymbol{A} \,/\, \rho = \mathbf{d}f$$

First integral

$$\boldsymbol{k} \cdot \boldsymbol{\pi} + f = \mathcal{E} = \text{constant}$$

(Similar result for corotational flow)

Initial data: method of solution

- Equations for the magneto-fluid

1. The first integral and $\boldsymbol{u} \cdot \boldsymbol{u} = -1$ are solved for u^t and h
2. Rest mass conservation is written as an elliptic equation for Ψ with Neumann boundary conditions
3. The Einstein equations are solved using e.g. the conformal thin-sandwich or the waveless formalism
4. The Maxwell + auxiliary equations are solved using helically symmetric ideal MHD inside star, and a waveless formalism outside.

- Must impose the ideal MHD condition during the iteration

Quasi-equilibrium sequences

- First law of thermodynamics (Bardeen-Carter-Hawking 1973, Carter 1973, Iyer-Wald 1995, Friedman-Uryu-Shibata 2002, et al), relating the change in the asymptotic Noether charge associated with helical symmetry to the changes in circulation, magnetic flux, baryon mass, entropy of the fluid and in the area and charge of black holes.
- First law extended to helically symmetric magnetofluids with no resistivity or viscosity (formally agrees with Carter 1973 but does not require stationarity and axisymmetry)
- The first law for helically symmetric binaries with magnetic field is useful for constructing a sequence of solutions in quasi-circular orbits modeling inspiral

  (Thermodynamics of magnetized binary compact objects,
  K. Uryu, E. Gourgoulhon, C. Markakis – submitted to Phys. Rev. D)