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A Conventional Form of Dark Energy

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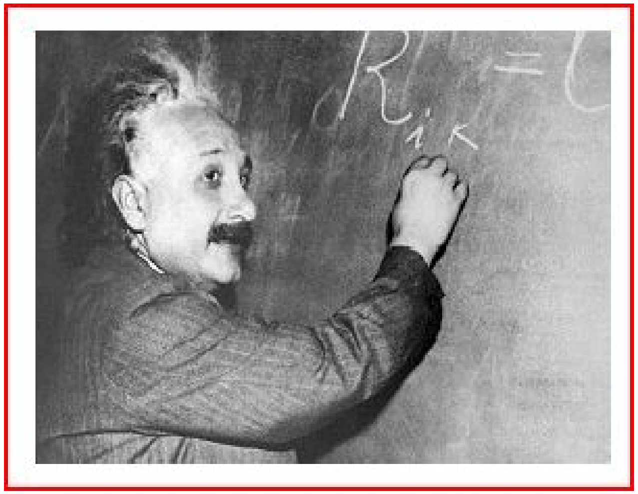
THEORY

Albert Einstein (1916)

General Theory of Relativity



$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{geometry} = -\frac{8\pi G}{c^4} \underbrace{T_{\mu\nu}}_{matter-energy}$$

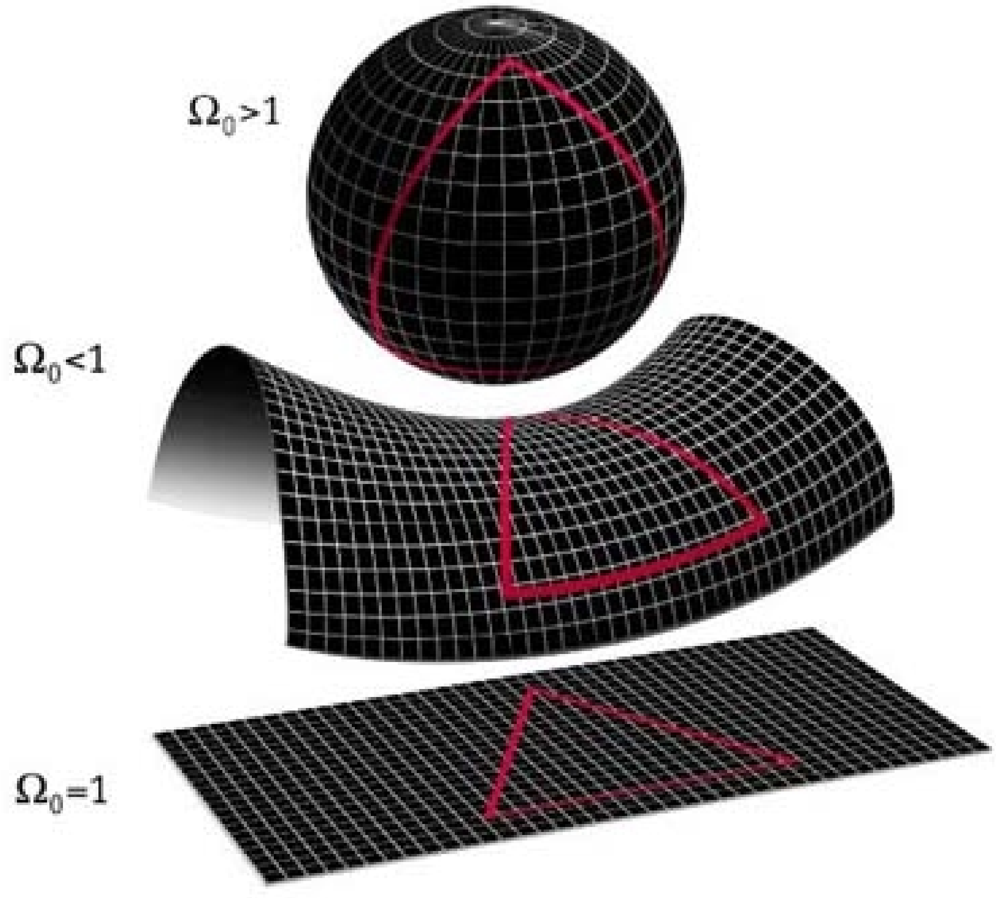
Geometry: **Robertson – Walker line-element** (1934)

$$ds^2 = c^2 dt^2 - S^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\vartheta^2 + \sin^2\vartheta \, d\phi^2 \right) \right]$$

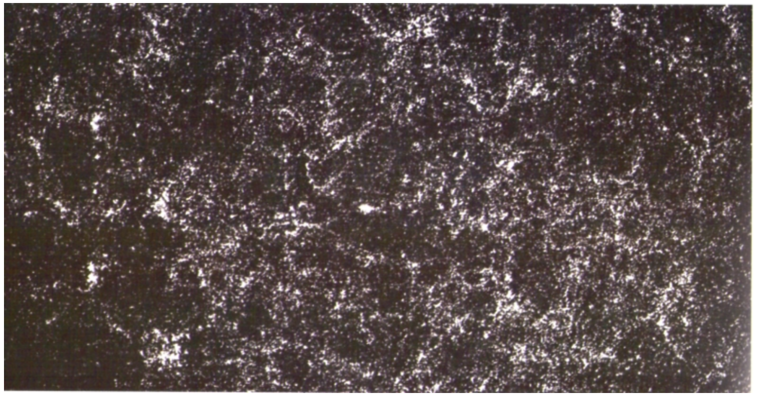
S(t): **Scale Factor :** Determines the evolution of the ***curvature radius*** of the spatial "slices", in time.

k : curvature parameter

$$k = \begin{cases} +1 & \text{closed spatial sections} \\ -1 & \text{open spatial sections} \\ 0 & \text{flat spatial sections} \end{cases}$$



Matter – energy content



Perfect Fluid

$$T^{\mu\nu} = (\varepsilon + p)u^{\mu}u^{\nu} - pg^{\mu\nu}$$

ε : total energy-density
p : pressure
u^{μ}: four–velocity (μ = 0, 1, 2, 3)

Conservation law:

$$T^{\mu\nu}_{\ ;\nu} = 0$$

$$\underbrace{T^{i\nu}_{\ ;\nu} = 0}_{spatial\ \text{part}} \text{ ή } \frac{du^i}{ds} + \Gamma^{i}_{\mu\nu} u^{\mu} u^{\nu} = \frac{1}{\varepsilon + p} h^{i\lambda} p_{,\lambda}$$

hydrodynamic flows of the volume elements

$$\underbrace{T^{0\nu}_{\ ;\nu} = 0}_{temporal\ \text{part}} \text{ ή } \dot{\varepsilon} + 3\frac{\dot{S}}{S}(\varepsilon + p) = 0$$

continuity equation

During the early 30´s:

- The Universe matter–content was thought to consist **ONLY** of what we were able **to "see"!**

$$\varepsilon = \rho c^2$$

ρ: the rest–mass density of the ordinary (baryonic) matter.

- The Universe matter-content appeared to be **"collisionless".**

$$p = \left(\frac{c_s}{c}\right)^2 \varepsilon \simeq 10^{-5} \varepsilon \Rightarrow p \simeq 0$$

Continuity equation: $\rho = \rho_0 \left(\frac{S_0}{S}\right)^3$ **Evolution of the rest–mass density**.

Albert Einstein (1917)

“The Universe is both static and closed!”

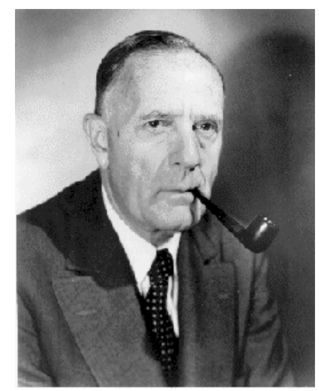
To avoid mathematical disaster, he decided to **“correct”** the field equations, introducing **the cosmological constant, Λ.**

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^{4}} T_{\mu\nu}$$

Λ – term: A **constant** energy – density.
In the Newtonian limit introduces **a repulsive gravitational force!**

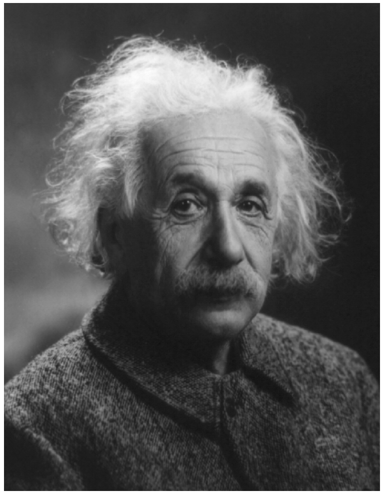
Edwin Hubble (1929) : *“The Universe expands!”*

Hubble parameter: $$H(t) = \frac{\dot{S}}{S} > 0$$



Hubble’ s law: **$\upsilon = H_0 r$** $H_0 = 70.5 \pm 1.3$ (km/sec)/Mpc (Komatsu et al. 2009)

Albert Einstein (1934)

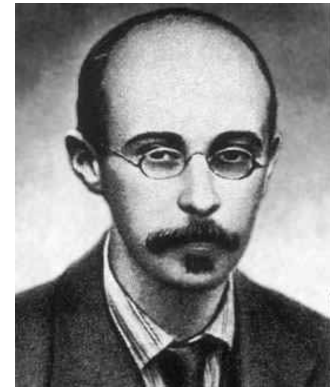


"…The introduction of Λ was the biggest **blunder** of my life!..."

Or (maybe) not?

The evolving Universe

Alexander Friedmann (1922)



$$\left(\frac{\dot{S}}{S}\right)^2 + k\frac{c^2}{S^2} = \frac{8\pi G}{3c^2}\varepsilon$$

$$\mathrm{H}^2 + k\frac{c^2}{S^2} = \frac{8\pi G}{3}\rho$$

For k = 0 (flat model) : $\rho_0 = \dfrac{3\mathrm{H}_0^{\,2}}{8\pi G} = \rho_c$ **critical rest–mass density**

$$\left(\frac{H}{H_0}\right)^2 + k\left(\frac{c/H_0}{S_0}\right)^2\left(\frac{S_0}{S}\right)^2 = \frac{\rho_0}{\rho_c}\left(\frac{S_0}{S}\right)^3$$

Introducing **the cosmological red shift** parameter : $1+z=\dfrac{S_0}{S}$

$$\left(\frac{H}{H_0}\right)^2 + k\left(\frac{\ell_{H_0}}{S_0}\right)^2 (1+z)^2 = \Omega_0 (1+z)^3$$

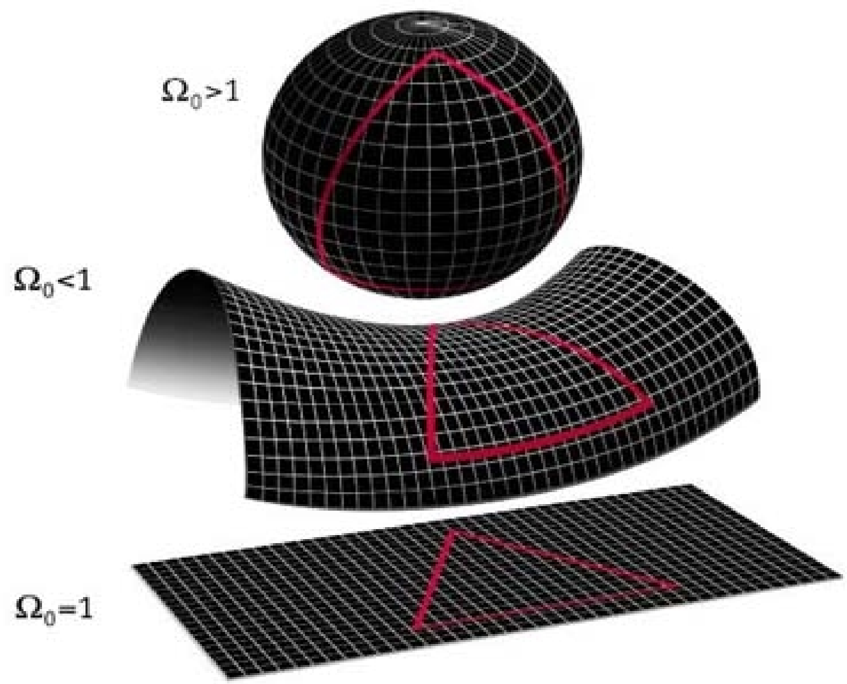
$\ell_{H_0} = c/H_0$ the "*horizon*" length

$\Omega_0 = \dfrac{\rho_0}{\rho_c}$ the **density parameter**

At the present epoch : $S = S_0$, $z = 0$, $H = H_0$

$$\Omega_0 = 1 + \underbrace{k\left(\frac{\ell_{H_0}}{S_0}\right)^2}_{\Omega_k}$$

- ✓ **Closed** model ($k = +1$) : $\Omega_0 > 1$
- ✓ **Open** model ($k = -1$) : $\Omega_0 < 1$
- ✓ **Flat** model ($k = 0$) : $\Omega_0 = 1$



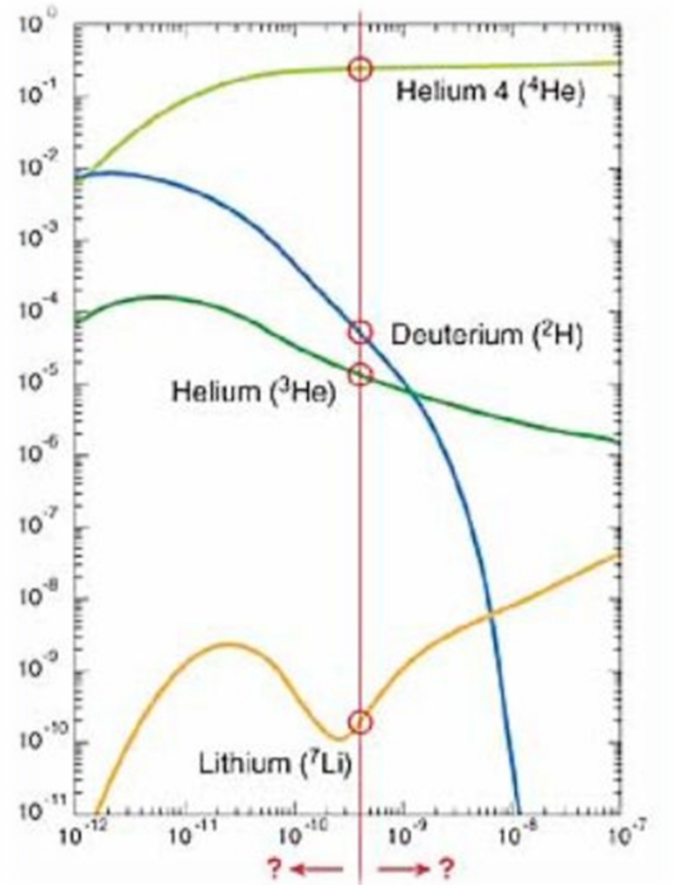
OBSERVATIONS

5 – year WMAP data (Komatsu et al. 2009) : $\rho_c \approx 1.04 \times 10^{-29}$ gr/cm^3

- **Ordinary (baryonic) matter:**

  Light chemicals' abundances suggest that (Olive et al. 2000)

  $\rho_{B_0} \approx 4\% \, \rho_c$ ή $\Omega_B \approx 0.04$



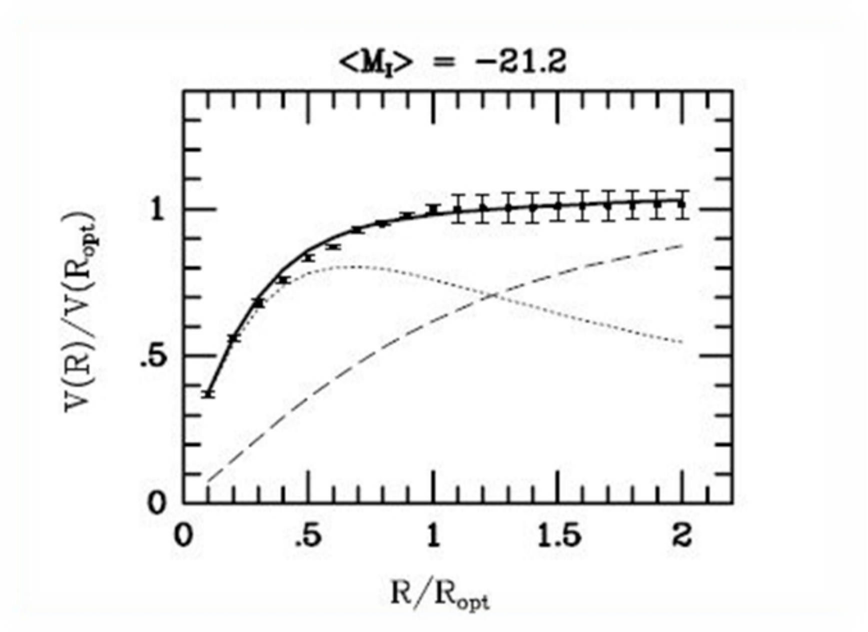
- **Dark Matter (DM):**

  F. Zwicky (1933) : Stability of large – scale structures!

  V. Rubin (1970): Confirmed in the case of our Galaxy!

Today :

- Galactic rotation curves.
- Weak gravitational lensing of distant galaxies.
- Weak modulation of strong lensing around massive elliptical galaxies.
- X-ray emission on the scale of galaxy clusters.



DM is assumed to consist of **non-relativistic WIMPs** (Cold Dark Matter – CDM), which are thought to be **collisionless(?)**

More than 80% (by mass) in the Universe consists of non–luminous (and non–baryonic) material !

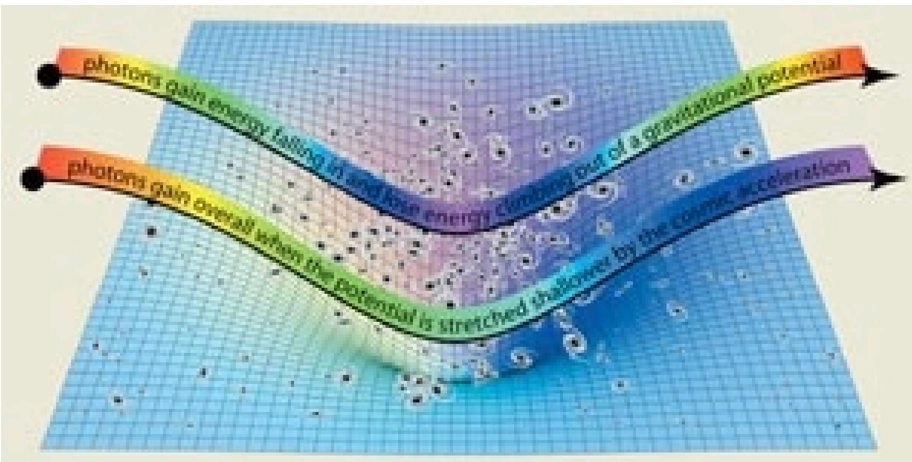
This amount contributes to the total rest–mass density of the Universe an additional 23% of ρ_c .

Hence: $\rho_{0_M} = \rho_{0_B} + \rho_{0_{DM}} = 27\ \%\ \rho_c$ ή $\Omega_M = \dfrac{\rho_{0_M}}{\rho_c} = 0.27 < 1$

Conclusion : We live in an **open** FRW model!

Probing the Cosmic Microwave Background (CMB)

Yakov B. Zel'dovich (1970)

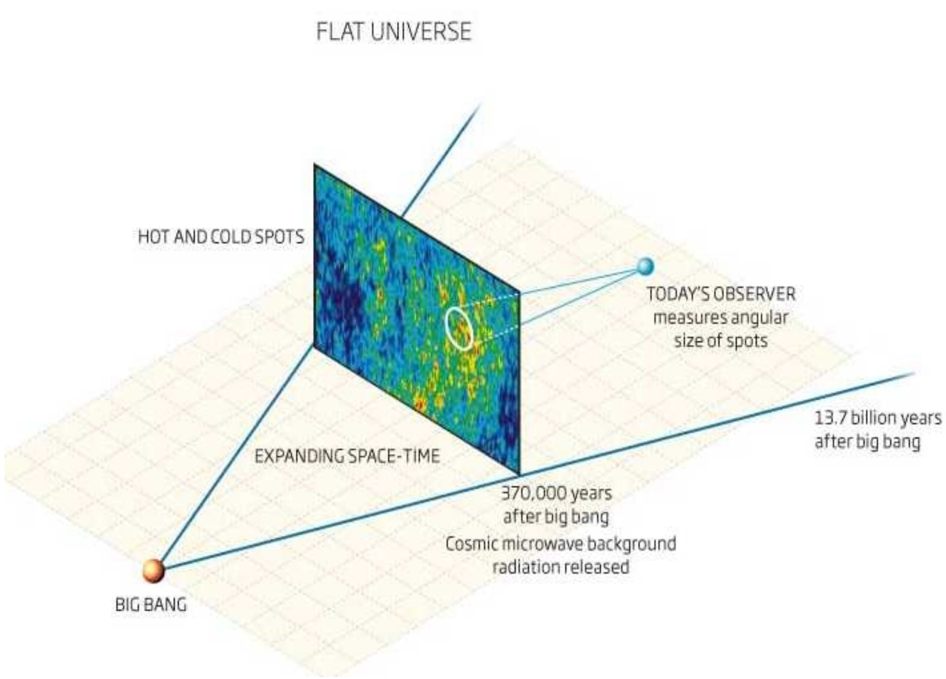


Energetics of the CMB photons as they fall in and climb out of the potential wells of the already evolved density perturbations, suggest that:

Deep potential well => considerable energy–loss => **Cold spot**

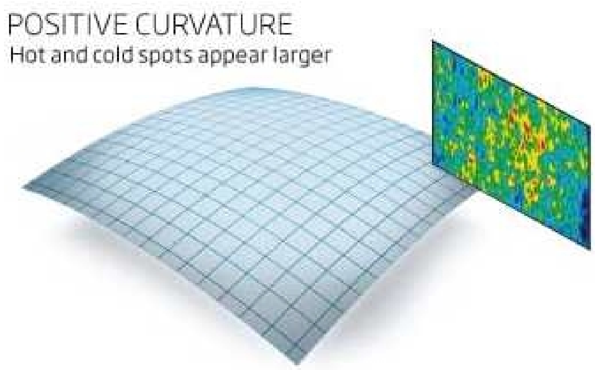
Shallow potential well => not too much energy–loss => **Hot spot**

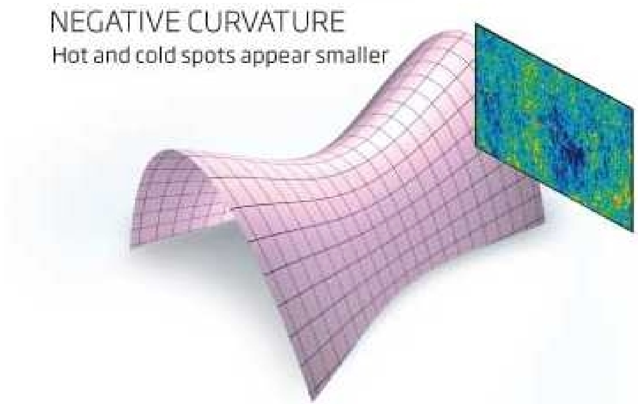
Angular size of the spots



k = 0 $\Delta\theta \approx 1^0$

k > 0 $\Delta\theta > 1^0$



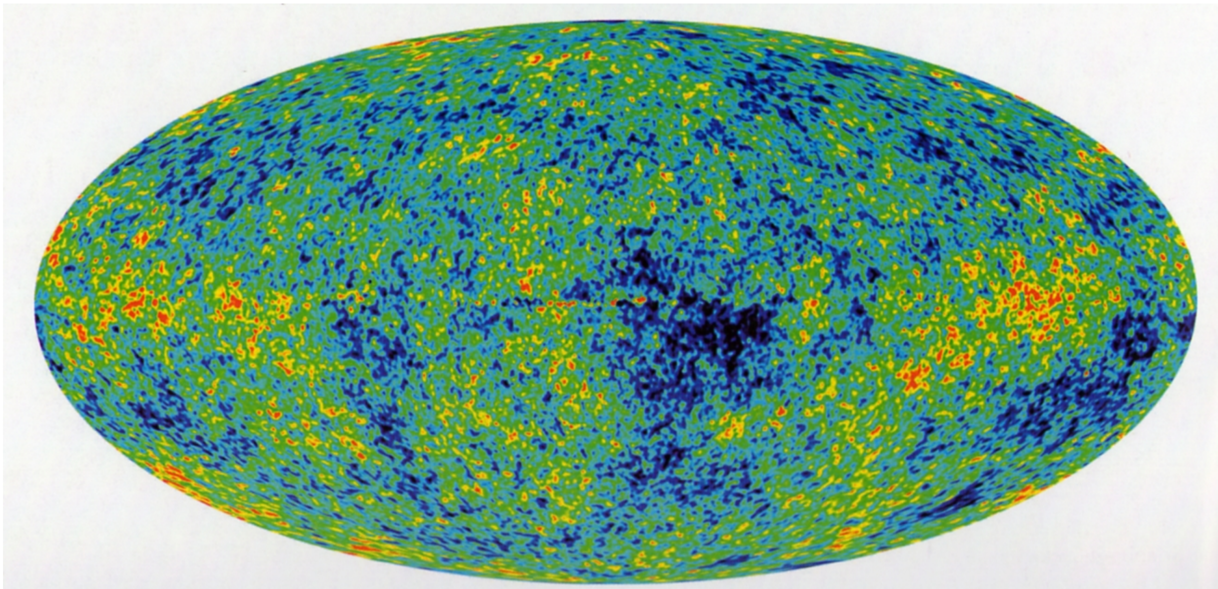


k < 0 $\Delta\theta < 1^0$

(1999) MAXIMA – BOOMERanG

Provided **strong evidence that $\Delta\theta \approx 1^0$**

(2003-08) Wilkinson Microwave Anisotropy Probe (**WMAP**)



Confirmed, beyond any doubt, that, **we do live** in a spatially–flat FRW model!

In other words, … today, $\Omega_0 = 1$.

Since $\Omega_M = 0.27 < 1$, we conclude that :

The Universe should contain a considerably larger amount of energy than the equivalent to the total rest–mass density of its matter content does!

Now, it gets even better!

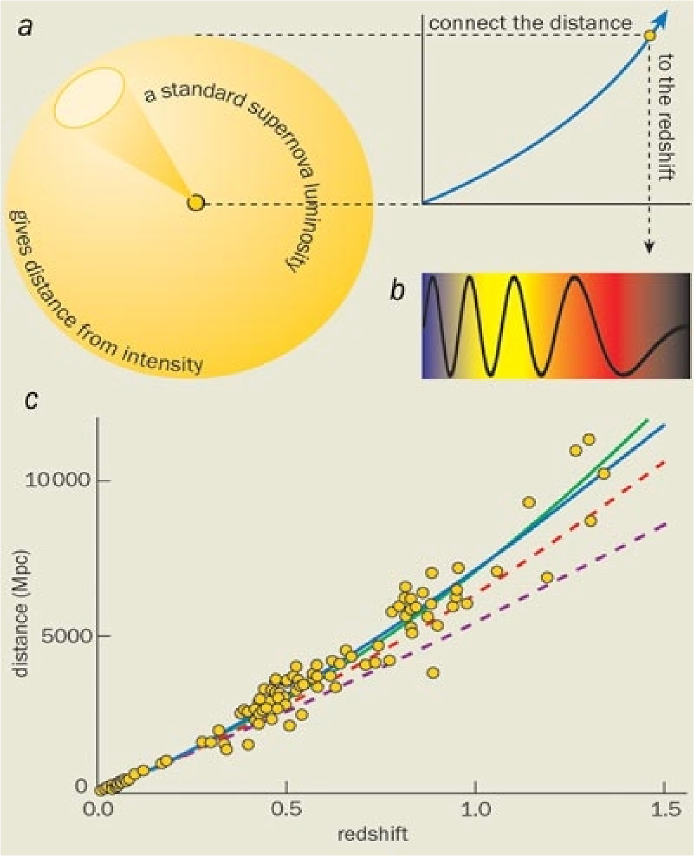
For many years it has been a common belief that, the Universe decelerates its expansion, due to its own gravity.

Deceleration parameter : $q = -\dfrac{\ddot{S}S}{\dot{S}^2} = \dfrac{dH/dz}{H(z)}(1+z) - 1 > 0$

In 1998 two scientific groups tried to determine the value of q, using SNe Ia as standard candles

"SN Cosmology Project"
S. Perlmutter et al. 1999

"SN search team"
A. Riess et al. 1998



The idea was to measure the red shift (**z**) and the apparent magnitude (**m**) of cosmologically–distant indicators (**standard candles**) whose absolute magnitude (**M**) is assumed to be known.

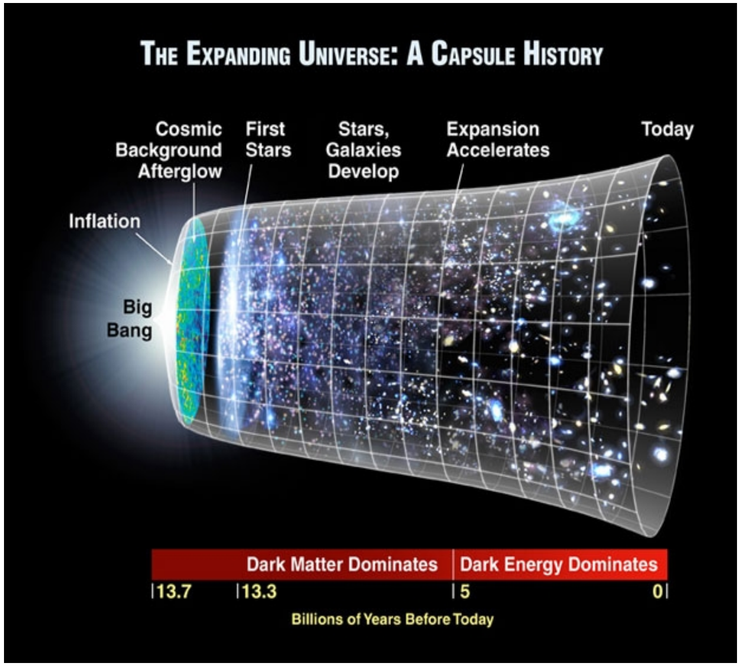
The corresponding results should have been arranged along the (theoretically predicted) curve of the:

Distance Modulus $$\mu(z) = m - M = 25 + 5\log_{10}\left(\frac{d_L(z)}{Mpc}\right),$$

where $d_L(z)$ is the luminosity distance in an <u>**open**</u> (as they used to assume) FRW model.

Surprisingly:

The SN Ia events, *at peak luminosity*, appear to be **dimmer**, i.e., they seem **to lie farther** than expected!



<u>**A possible explanation**</u>:

Recently, **the Universe accelerated its expansion**!

Moreover:

The best fit to the observational data was given by

$$d_L(z) = \frac{c}{H_0}(1+z)\int_0^z \frac{dz}{\sqrt{(1+z)^2(1+\Omega_M z) - z(2+z)\Omega_\Lambda}}$$

(Carroll et al. 1992)

Corresponding to a **spatially–flat FRW model** with

$$\Omega_M = 0.27 \quad \text{and} \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2} = 0.73$$

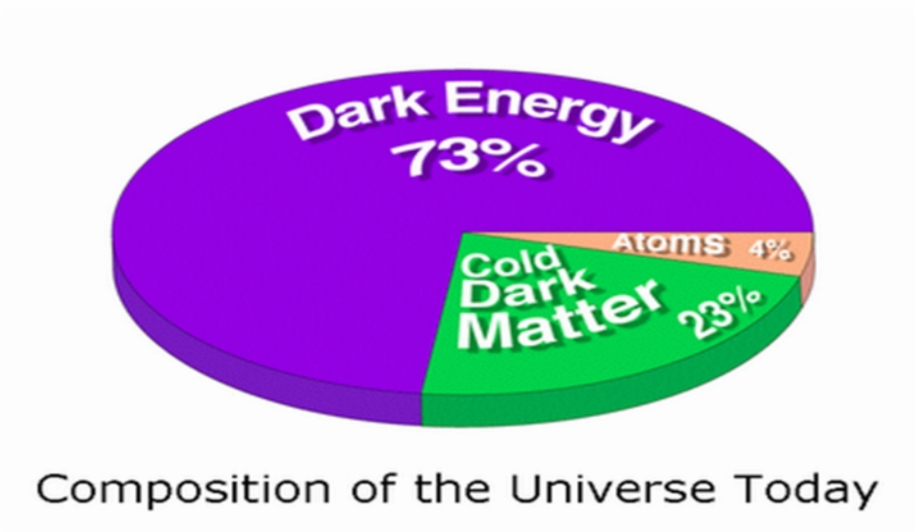
The cosmological constant strikes back!

The particle–physics' vacuum contributes an ***effective*** cosmological constant (repulsive in nature) and, therefore, it could justify for **both the spatial flatness and the accelerated expansion** of the Universe.

Unfortunately, **it is 10^{123} times larger than what is observed**!

But, if **not Λ**, then what?

Cosmologists came up with a name that reflects our ignorance on the nature of the most abundant Universe constituent: **The Dark Energy!**



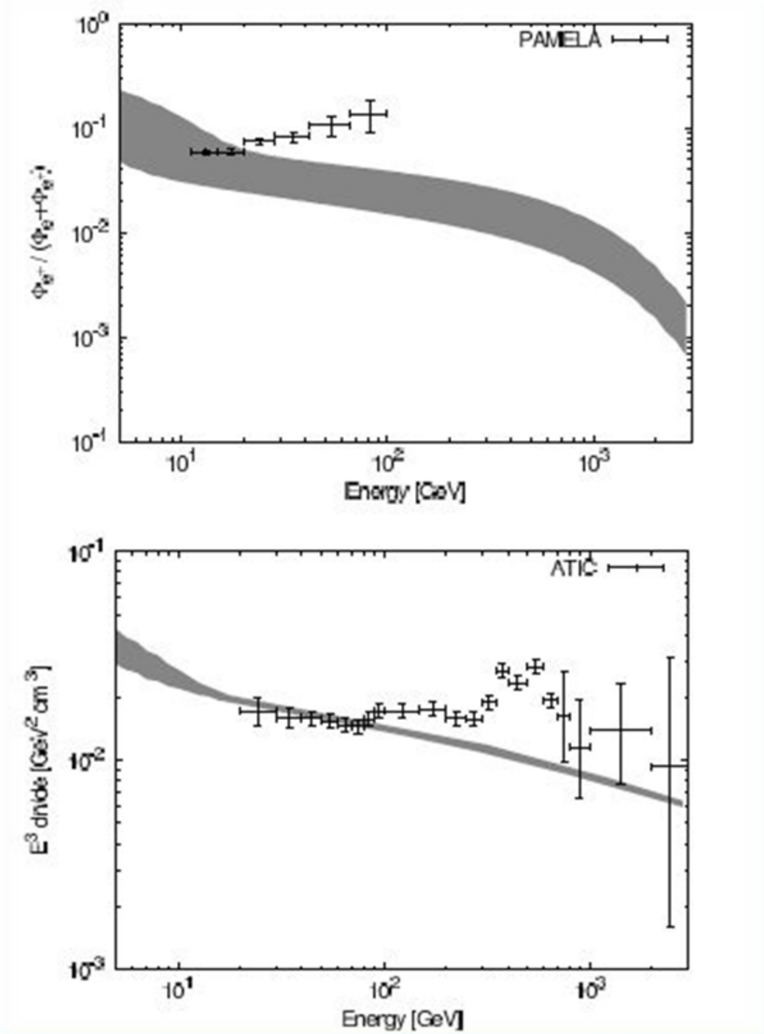
Candidates for the dark energy:

- ✓ Cosmological constant $p = - \rho c^2$
- ✓ Quintessence $- \rho c^2 < p < 0$
- ✓ Other (more exotic) scalar fields $p < - \rho c^2$

Are there any **conventional candidates** ?

A convenient one would be the **Interacting Dark Matter** (Spergel & Steinhardt 2000)

Indeed, recent results from high–energy–particle's tracers revealed an unusually–high electron – positron production in the Universe.



Among the best candidates for such high–energy events are the **annihilations of WIMPs**, i.e.:

The DM constituents can be slightly collisional.

(see, e.g., Arkani–Hamed et al. 2009, Cirelli et al. 2009, Cohen & Zurek 2010)

Our model

Motivated by there results, we have considered **a more conventional approach to the dark energy concept.**

If the DM constituents interact with each other frequently enough, so that their (kinetic) energy is re-distributed, i.e., **the DM itself possesses, also, some sort of thermodynamical properties**, a conventional extra energy component **does exist** in the Universe:

- It is the energy of the **internal motions** of the collisional–DM fluid!

Based on such an assumption:

We study the **evolution** and the **dynamical characteristics** of a **spatially–flat** cosmological model

$$ds^2 = S^2(\eta)\left[c^2 d\eta^2 - \left(dx^2 + dy^2 + dz^2\right)\right]$$

$$\eta = \int \frac{dt}{S(t)} \quad \text{is the conformal time}$$

in which (in principle) there is no DE at all !

- The matter–energy content consists of : (i) DM (**dominant**)
(ii) Baryonic matter (**subdominant**)

- These two constituents form a gravitating perfect fluid of **positive** pressure

$p = w\,\rho\,c^2$, where $0 \leq w = (c_s/c)^2 \leq 1$

the volume elements of which perform adiabatic flows.

- Together with all the other physical characteristics, the energy of this fluid' s internal motions is (also) taken into account, as a source of the universal gravitational field.

The **total** energy – density is given by

$$\varepsilon = \underbrace{\rho c^2}_{rest-mass} + \underbrace{\rho \Pi}_{\substack{internal \\ motions}}$$

Π : internal energy per unit mass (energy within a specific volume)

Cosmological Evolution

- The **first law of thermodynamics** for adiabatic flows yields:

$$d\Pi + p\,d\left(\frac{1}{\rho}\right) = 0 \Rightarrow \Pi = \Pi_0 + wc^2 \ln\left(\frac{\rho}{\rho_0}\right) \qquad \rho_0, \Pi_0 : \textit{present - time} \text{ values}$$

- The **continuity equation** results in:

$$T^{0\nu}{}_{;\nu} = 0 \Rightarrow \dot{\varepsilon} + 3\frac{\dot{S}}{S}(\varepsilon + p) = 0 \Rightarrow \rho = \rho_0 \left(\frac{S_0}{S}\right)^3$$

- The **Friedmann equation (for k = 0) with Λ = 0** :

$$\left(\frac{H}{H_0}\right)^2 = \Omega_M \left(\frac{S_0}{S}\right)^3 \left\{1 + \frac{\Pi_0}{c^2} + 3w\ln\left(\frac{S_0}{S}\right)\right\}$$

At the **present epoch**, where **$S = S_0$ & $H = H_0$** :

$$\Pi_0 = \left(\frac{1}{\Omega_M} - 1\right) c^2$$

$$\Omega_0 = \frac{\varepsilon_0}{\varepsilon_c} = \frac{\rho_0 c^2}{\rho_c c^2} + \frac{\rho_0 \Pi_0}{\rho_c c^2} = \Omega_M + \Omega_M \frac{\Pi_0}{c^2} = 1$$

Therefore:

✓ **In principle, the extra (dark) energy, needed to flatten the Universe, can be compensated by the energy of the internal motions of a collisional–DM fluid!**

In this case:

$$\left(\frac{H}{H_0}\right)^2 = \left(\frac{S_0}{S}\right)^3 \left[1 + 3w\Omega_M \ln\left(\frac{S_0}{S}\right)\right]$$

$$0 \le w\Omega_M \le 0.3$$

can be solved in terms of the ***error function .***

- For $w\Omega_M << 1:$

$$S = S_0 \left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w\Omega_M}}$$

  a natural generalization of the **E-dS** model ($S \sim \eta^2$).

- The **Hubble parameter** (in terms of z):

$$H \simeq H_0 (1+z)^{\frac{3}{2}(1+w\Omega_M)}$$

  it is **functionally similar** to the corresponding result regarding a **DE fluid**.

However, in our model **w ≥ 0**.

✓ On the approach to z = 0, H(z) decreases !

- **Deceleration parameter**:

$$q = \frac{1}{2}\left(1 + 3w\Omega_M\right) > 0$$

A cosmological model filled with collisional–DM
necessarily decelerates its expansion!

This model is inadequate for confronting the apparent accelerated expansion!

In fact, it does not have to!

Mistreating DM as collisionless

We distinguish two kinds of observers, **based on their perception** about the Cosmos :

- Those who **treat the DM as collisional**

$$ds^2 = S^2(\eta)\left[c^2 d\eta^2 - \left(dx^2 + dy^2 + dz^2\right)\right]$$

  Accordingly, the various motions in this model are (in principle) **hydrodynamic flows of the volume elements** of the collisional-DM fluid.

- Those who insist in adopting the **collisionless–DM approach**

$$d\tilde{s}^2 = R^2(\eta)\left[c^2 d\eta^2 - (dx^2 + dy^2 + dz^2)\right]$$

As far as these observers are concerned, the various motions in the Universe (necessarily) take place along **geodesic trajectories of <u>test</u> particles** receding from each other.

These two models (describing the same Universe) can be related by a **conformal transformation** (Kleidis & Spyrou 2000):

$$d\tilde{s} = f(x^\kappa)ds$$

Upon consideration of **isentropic flows**, the conformal factor is given by

$$f\left(x^{\kappa}\right)=\frac{\varepsilon+p}{\rho c^{2}}$$

In terms of z, takes on the functional from

$$f(z)=\frac{1+w\Omega_{M}\left[1+3\ln(1+z)\right]}{1+w\Omega_{M}}$$

We can express several cosmologically–significant parameters of the **collisional–DM** model (**un-tilde variables**) in terms of the traditional, **collisionless–DM** approach (**tilde variables**).

- The **cosmological red shift**:

$$1+\tilde{z}=\frac{1+w\Omega_{M}}{1+w\Omega_{M}\left[1+3\ln(1+z)\right]}(1+z)$$

At relatively–low values of z **(z < 5)** :

$$1+\tilde{z}\approx(1+z)^{1-3w\Omega_{M}}$$

✓ For every fixed value of **z** the corresponding collisionless–DM quantity is **a little bit smaller** $(\tilde{z}<z)$.

- The **luminosity distance**:

$$d_L = \tilde{d}_L (1 + z)^{3w\Omega_M}$$

As long as w ≠ 0 , $d_L > \tilde{d}_L$, for every z > 0.

- The corresponding **distance modulus** :

$$\mu = \tilde{\mu} + 15 w \Omega_M \log_{10}(1 + z)$$

From the point of view of an observer who adopts the collisionless–DM scenario any light–emitting source of the collisional–DM Universe (w ≠ 0) **appears to be dimmer** than expected, i.e.,

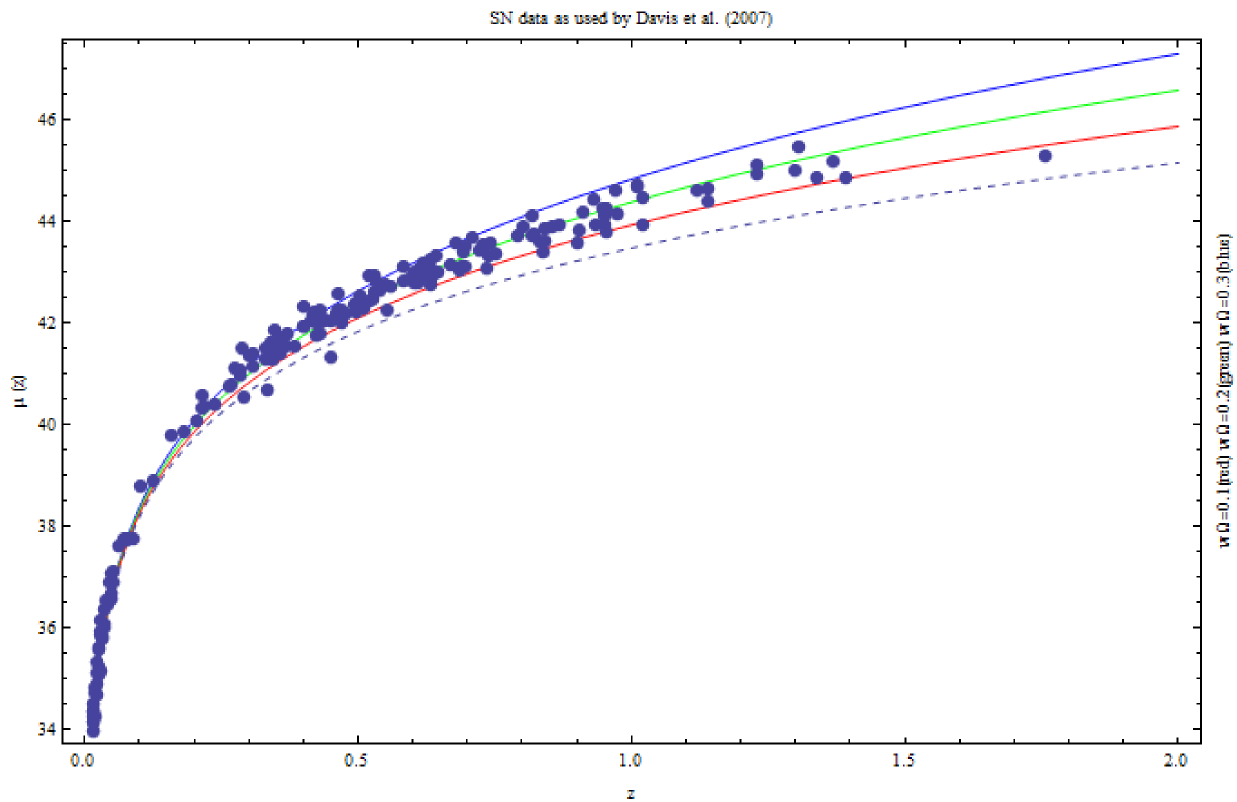
$$\tilde{\mu} < \mu$$

Comparison with the SN Ia data:

An extended sample of 192 SN Ia events used by Davis et al. (2007), consisting of

45	nearby events
57	events from SNLS (Astier et al. 2006)
60	events from ESSENCE (Wood – Vasey et al. 2007)
30	events from *Gold-07* (Riess et al.2007)

http://www.ctio.noao.edu/essence
http://braeburn.pha.jhu.edu/~ariess/R06



So … what's the catch in it ?

- w Ω_M = 0.1 => **w = 1/3** The DM consists of **relativistic particles**!
- w Ω_M = 0.2 => **w =** 2/3 The **best fit** is achieved for **w ≈ 2/3 (?)**
- w Ω_M = 0.3 => **w = 1** The DM consists of **stiff matter** !

- **The Hubble parameter**:

$$\tilde{H} = H\,\frac{d}{dz}\left(\frac{1+z}{f(z)}\right)$$

- **The deceleration parameter:**

$$\tilde{q}(z) = \frac{1}{2}\left\{\frac{1-4w\Omega_M+6w\Omega_M\ln(1+z)+O(w\Omega_M)^2}{1-10w\Omega_M+6w\Omega_M\ln(1+z)+O(w\Omega_M)^2}\right\}$$

The condition for **accelerated expansion** is:

$$\tilde{q}(z) < 0 \Leftrightarrow \quad 1-14w\Omega_M+12w\Omega_M\ln(1+z) < 0$$

$$z < z_t = e^{\frac{14w\Omega_M-1}{12w\Omega_M}} - 1$$

If the Universe matter–content consists of a collisional–DM fluid with

$$w > w_c = 0.238$$

so that:

$$\left(w\Omega_M\right) > \left(w\Omega_M\right)_c = \frac{1}{14} ; \; 0.0714$$

then, **from the point of view of someone who treats the DM as collisionless**, there exists **a transition value** (z_t) of the cosmological red shift, **below which** the **Universe is accelerating!**

Adopting the observational result

$$\mathbf{z_t = 0.46 \pm 0.13} \qquad \text{(Riess et al. 2004)}$$

we arrive at

$$\mathbf{w\Omega_M = 0.106 \pm 0.012}.$$

Once again

$$\mathbf{w \approx 1/3}$$

i.e., **the DM – fluid consists of relativistic particles** (**Hot Dark Matter ?)**

SUMMARIZING

We have considered **a cosmological (toy) model**, i.e., not necessarily reflecting our own Universe, in which:

- **The DM constitutes a fluid of relativistic particles**

- Interacting with each other frequently enough, **thus attributing to this fluid some sort of thermodynamical properties.**

This model:

(i)Could provide a **conventional explanation for the extra (dark) energy** needed to flatten the Universe:

It **can be compensated by the energy of the internal motions** of the collisional–DM fluid.

(ii) Could **account for the observed dimming** of the distant light–emitting sources:

It can be due to the **misinterpretation of several cosmologically–relevant parameters by an observer who (although living in a collisional–DM model) insists in adopting the (traditional) collisionless–DM approach**.

(iii) Could **explain the apparent accelerated expansion** of the Universe without suffering from the **coincidence problem.**

Although speculative,

the idea that the DE could be attributed to the internal physical characteristics of a collisional–DM fluid

is (at least) intriguing

and should be further explored and scrutinized in the search for

conventional alternatives to the DE concept !

THANK YOU
FOR
YOUR ATTENTION!