Torsional shear oscillations of magnetized neutron stars

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Motivation

Quasi-periodic oscillations QPO in giant flares of SGR

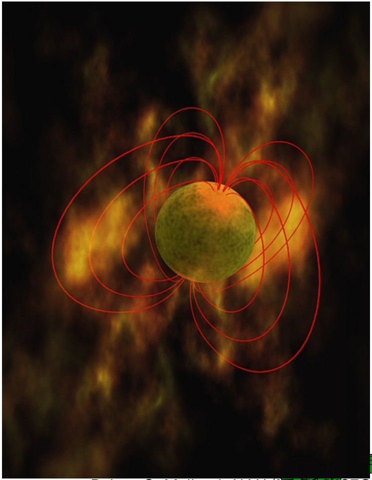
Soft Gamma Repeater

- repeated activity
- soft gamma ray spectrum
- slowly rotating ($P = 5 \ldots 10$s)
- rapid spin down of rotation

Magnetar and giant flares

(Duncan & Thompson 1992)

- highly magnetized ($\gtrsim 10^{15}$G)
- magnetic field produces stresses $\rightarrow$ crust brakes
- energy released $\rightarrow$ fireball



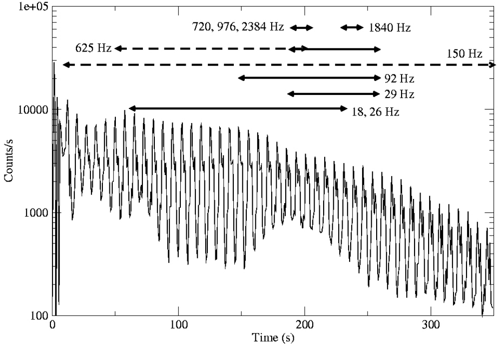
Robert S. Mallozzi, UAH/NASA MSFC

Observations

Giant flares

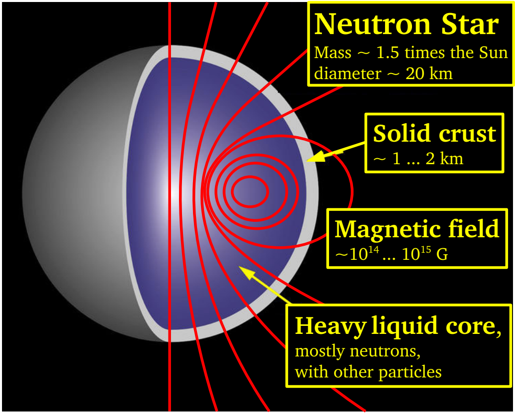
- SGR 0526-66 (1978), SGR 1900+14(1998), SGR 1806-20 (2004)
- Peak luminosity $10^{44} \ldots 10^{46}$ erg/s
- Low frequency modulation $\Rightarrow$ rotation period $(5 \ldots 10\text{s})$
- High frequency quasi periodic oscillations QPOs

(Israel et al. 2005, Strohmayer & Watts 2006)



Strohmayer & Watts 2006

Where do the QPOs come from?



- Torsional modes crust
- Alfvén modes core
- Oscillations of the magnetosphere

$\Rightarrow$ coupling of all three regions by magnetic field

Torsional Shear Oscillations of the Crust

Samuelsson & Andersson 2007

Observed frequency in Hz		torsional shear mode	
SGR 1806-20	SGR 1900+14	n	l
18		???	???
26		???	???
30	28	0	2
	53	0	4
92	84	0	6
150		0	10
	155	0	11
625		1	
1840		3	

(Schomaker & Thorne 1983, Piro 2005, Samuelsson & Andersson 2007)

- no magnetic field
- free slip / zero traction at crust core interface
- Newtonian estimates for ω:

$\Rightarrow$ $n = 0$:

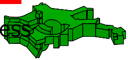
$$\boxed{\omega^2 \sim \frac{(l-1)(l+1)}{RR_c}}$$

R_c - radius of crust

$\Rightarrow$ $n > 1$:

$$\boxed{\omega \sim \frac{n}{\Delta}}$$

Δ - crust thickness

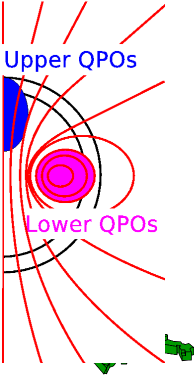


Alfvén oscillations of the core

Linear approximation (Sotani et al. 2008; Colaiuda et al. 2009)

2D MHD simulations in the anelastic approximation (Cerdá-Durán et al. 2009)

- Axisymmetric, poloidal magnetic field configuration
- Continuum of frequencies with two families of weakly damped long lived QPOs
- Lower QPOs near the closed field lines
- Upper QPOs near the pole
- Integer relation between different overtones may explain some of observed QPOs:
  - ▷ SGR 1900+14 : 28, 84, 155 Hz= 1, 3, (5)
  - ▷ SGR 1806-20 : 30, 92, 150 Hz= 1, 3, 5



Crust-core coupling

(the key ingredient)

Toy models (Glampedakis et al. '06; Levin '07; Van Hoven & Levin '10)

- Global torsional modes only with unphysically large dissipation
- Coupling introduces damping of crust modes
- Modes of Alfvén-continuum are preferably excited by crust modes of similar frequency
- QPOs appear at the edges or turning points of the continuum

Linear analysis (Sotani et al. '06)

- Avoided crossings between crustal and Alfvén modes with increasing field strength
- Magnetic field dominates for strength 10^{15} G
- Good agreement with observed QPOs of SGR

Equations

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$T^{\mu\nu} = (\rho h + b^2) u^\mu u^\nu + \left(P + \frac{1}{2} b^2\right) g^{\mu\nu} - b^\mu b^\nu - 2\mu_{Shear}\Sigma^{\mu\nu}$$

$$\boxed{\frac{1}{\sqrt{-g}}\left(\frac{\partial\sqrt{\gamma}\mathbf{U}}{\partial t} + \frac{\partial\sqrt{-g}\mathbf{F}^{\mathbf{i}}}{\partial x^i}\right) = 0}$$

In linear regime poloidal and toroidal perturbations decouple

$$\begin{aligned}
\mathbf{U} &= [S_\varphi, B^\varphi] \, , \\
\mathbf{F}^i &= \left[-\frac{b_\varphi B^i}{W} - 2\mu_S \Sigma^i{}_\varphi, -v^\varphi B^i\right] \\
\Sigma^{i\varphi} &= 1/2 g^{ii} \xi^\varphi{}_{,i}
\end{aligned}$$

$$\xi^i_{,t} = \alpha v^i \Rightarrow \boxed{\left(\xi^i_{,r}\right)_{,t} - \left(\alpha v^i\right)_{,r} = 0}$$

The numerical code - MCoCoA

- Ideal MHD code in dynamical space-time (Godunov type schemes + flux CT)
- Conformal Flatness Condition (CFC) approximation (Isenberg 1979/2008, Wilson 1989)
- Spherical polar coordinates in axisymmetry (2D)
- GRMHD anelastic approximation (Bonazzola et al. 2007)
  - ▷ Sound waves neglected
  - ▷ Valid close to the equilibrium
  - ▷ Courant condition for Alfvén time

Recovery of purely shear oscillations

The Fourier analysis of 2D simulations without magnetic field for different tabulated EOS provides this sample of frequencies:

Model	frequency in Hz for mode			
	n=0 (±1Hz)			n = 1
	l = 2	l = 3	l = 4	±20Hz
APR+DH 1.6	23.5 (23.4)	37.1 (37.0)	49.8 (49.6)	880 (860)
APR+DH 2.0	21.9 (21.3)	35.1 (33.6)	46.8 (45.1)	1070 (1083)
L+DH 1.6	20.5 (20.6)	32.5 (32.5)	43.8 (43.7)	590 (586)
L+DH 2.0	19.0 (18.9)	30.2 (29.9)	40.5 (40.2)	720 (713)
WFF3+DH 1.6	25.2 (25.2)	39.8 (39.9)	53.4 (53.5)	1130 (1101)

Very good agreement with linear analysis of Sotani et al. 2007

Coupled crust-core oscillations

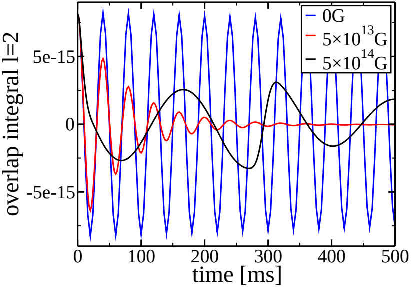
Representative model

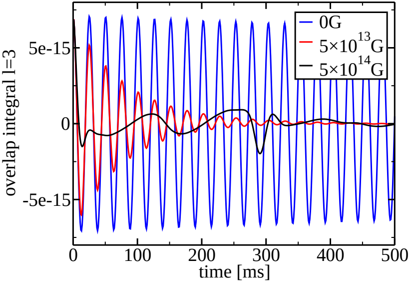
- EOS: APR + DH
- Mass: 1.4 solar masses
- Radius: 12.26km
- Magnetic field structure: dipolar
- Magnetic field strength (if not stated otherwise): 4×10^{14}G

Additional tool for analysis

- Adopted semi-analytical model of Cerdá-Durán et al. 2009: integration of a perturbation along the magnetic field lines to obtain the frequencies of the Alfvén continuum

Damping of crust modes



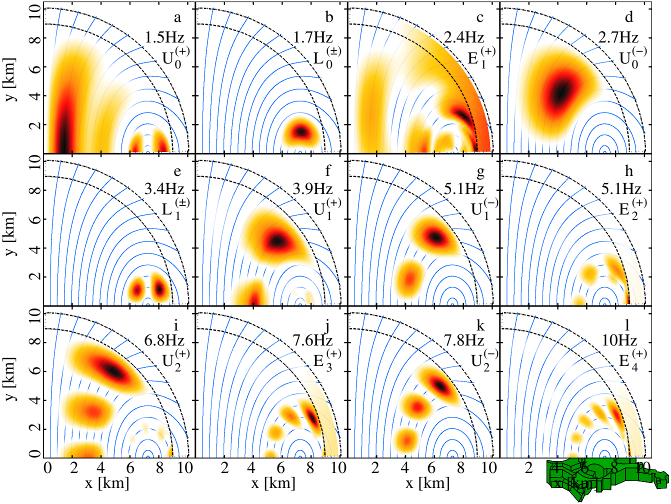


- Strong damping of $n = 0$ crust modes (timescale $1/10$s for 5×10^{13}G)
- After initial damping contributions to the overlap integrals are produced by coupled magneto-elastic oscillations

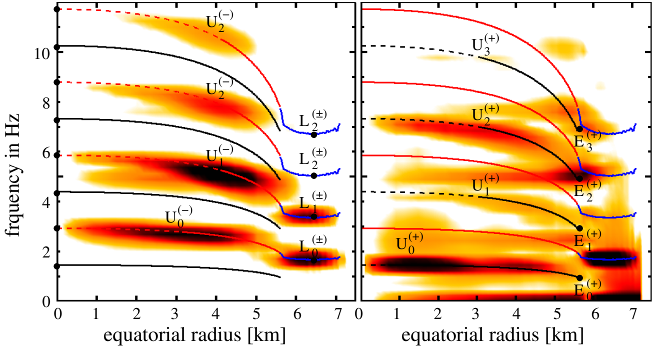
Fourier analysis of the simulation

Three different families of QPOs:

- Upper (U)
  a, d, f, g, i, k
- Edge (E)
  c, h, j, l
- Lower (L)
  b, e



Averaged amplitude of the FFT per field line



Comparison with semi-analytical approach (solid and dashed lines)

- L: at the turning points located at the closed field lines
- E: at the edges of the continuum which are not connected to the continuum of the closed field lines
- U: in the continuum of the simple semi-analytical model, but this only valid for perfect reflection

Conclusions

- The two extreme cases zero shear and zero magnetic field are recovered
- $n = 0$ crustal shear modes are damped rapidly for magnetic field strength well below that assumed for magnetars (timescale: $1/10\mathrm{s}$ for $5 \times 10^{13}\mathrm{G}$)
- $n = 1$ are damped less strongly, this is currently investigated
- At magnetar field strength we find Alfvén QPOs in the core:
  - Lower (turning point) QPOs are obtained as before, because the crust does not influence the closed field lines
  - Coupling introduced by the crust enables new edge QPOs
  - Influence of the crust changes the properties of the upper QPO