# Inflationary solutions in asymptotically safe $f(R)$ theories

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# Outline:

## Asymptotic Safety

## Gravity

## Solutions

### Inflationary cosmology

### Reliability analysis

## Final remarks

# Asymptotic Safety



$$\text{Theory Space}: \ \Gamma(\phi) = \sum g_i \mathcal{O}_i(\phi)$$

$$\text{RG : vector field } \Gamma(\phi) \rightarrow \Gamma_k(\phi) = \sum g_i(k) \mathcal{O}_i(\phi)$$

present work based on RG *à la* Wilson

EFT at scale $k$ $\Rightarrow$ integration of modes $p > k$

$\mathcal{S} \rightarrow \mathcal{S} + \Delta\mathcal{S}_k$ mass term for $p < k$

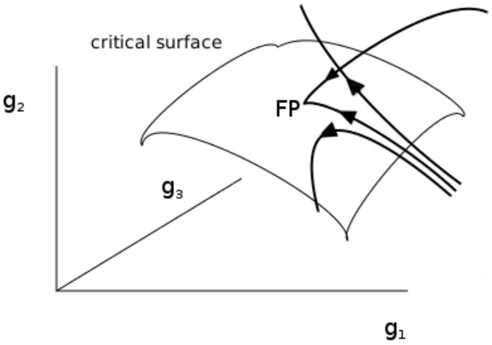
define generating functional $W_k(J) \ \rightarrow$ effective action $\Gamma_k(\phi)$

obeying ERGE $\quad k\frac{d\Gamma_k}{dk} = \frac{1}{2}\mathrm{Tr}\left(\Gamma_k^{(2)} + R_k\right)^{-1} k\frac{dR_k}{dk}$

where $k\frac{d\Gamma_k}{dk} = \sum \beta_i \mathcal{O}_i(\phi)$

## Asymptotic safety (define $\tilde{g}_i(k) = g_i(k)k^{-d_i}$)

- $\exists$ UV fixed point $\lim_{k\to\infty} \tilde{g}_i(k) = \tilde{g}_i^*$
- $M_{ij} = \left.\frac{\partial \beta_{\tilde{g}_i}}{\partial \tilde{g}_j}\right|_*$ has finite number of negative eigenvalues




$\Rightarrow$ **PREDICTIVE THEORY**

# Gravity

$$\Gamma = \int d^4x \sqrt{|g|} F(R) \quad \text{with} \quad F(R) = \sum_{i=0}^{n} g_i R^i$$

$\exists$ FP for $n = 2, \ldots, 8$ — 3 attractive directions

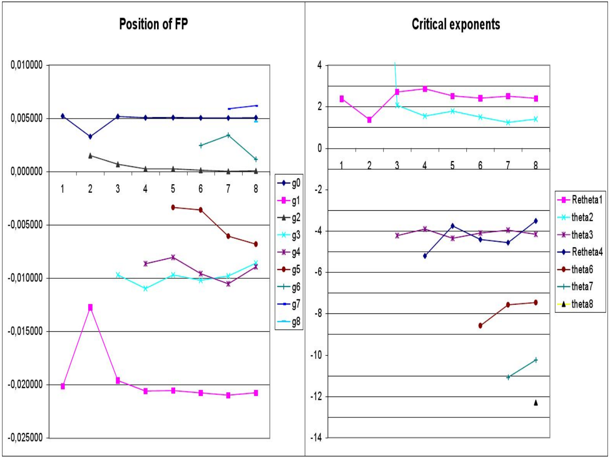
(we extended analysis to $n = 9, 10$)

finite truncation: self-consistant?

$\{\tilde{g}_i^*\}$ must be stable against the inclusion of new operators

as well as the critical exponents $\{\theta_i\}$

(Codello, Percacci, Rahmede '08)




stable FP + constant number of attractive directions

$\Rightarrow$ **RELIABLE TRUNCATION**

# What about cosmology?

Universe described by $\Gamma^{(n)} = \int d^4x \sqrt{|g|} \sum_{i=0}^{n} g_1^{(n)} R^i$

**A.** INFLATIONARY COSMOLOGY?

$\ddot{a} / a > 0$

**B.** HOW DIFFERENT FROM $\Gamma^{(n+1)}$?

reliable expansion of the "complete" action $\Gamma^{(\infty)}$

# **A.** Inflationary cosmology

Already studied by Weinberg

$$\text{Effective action } \Gamma_W = \int d^4x \sqrt{|g|} \sum_i \sum_j g_{i,j} \mathcal{O}^i_j(R, R_{\mu\nu}, ...)$$

$$\text{where } g_{i,j} = g_{i,j}(\Lambda) = \tilde{g}_{i,j}(\Lambda)\Lambda^{2(2-i)}$$

action $\Rightarrow$ FRW symmetry $\Rightarrow$ friedmann equations

$\Rightarrow$ dS SOLUTION ($\bar{H} = const.$)

$+$ minimization of radiative corrections by acting on $\bar{H}/\Lambda$

Last remark:

graceful exit obtained by perturbing dS $\rightarrow$ decaying sol.

(Weinberg '09)

A different procedure:

$$F(R) = \sum_{i=0}^{n} g_i(k) R^i \quad , \quad g_i(k) = \tilde{g}_i(k) k^{2(2-i)}$$

perform a *cutoff identification* $k \to k(t)$

in FRW $H(t)$ $\Rightarrow$ mass term in propagators / inverse of curvature radius

GOOD CANDIDATE $k = \xi H$

plug into equations to perform *RG improvement*

(restrict ourselves to FP action: $\tilde{g}_i(k) \simeq \tilde{g}_i^*$)

(Bonanno, AC, Percacci '10)

NO uniquely defined improvement procedure

Einstein-Hilbert case:

- take equations $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$
  and set $G = G(k(t))$ , $\Lambda = \Lambda(k(t))$

- take action $\Gamma = \int d^4x \sqrt{|g|} \mathcal{L}(R)$ , $\mathcal{L} = \frac{R-2\Lambda}{16\pi G}$ $\Rightarrow$
  $\mathcal{L}' R_{\mu\nu} - \frac{1}{2}\mathcal{L} - \nabla_\mu \nabla_\nu + g_{\mu\nu} \nabla^2 \mathcal{L}' = \frac{1}{2} T_{\mu\nu}$ $\Rightarrow$
  $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - G \left( \nabla_\mu \nabla_n - g_{\mu\nu} \nabla^2 \right) G^{-1} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$

General $F(R) = \frac{1}{16\pi G}\left(f(R) - 2\Lambda\right)$

FRW $\Rightarrow$ (modified) Friedmann equations

$$\begin{aligned} \mathcal{A}(H) &= 8\pi G\rho + \Lambda \\ \mathcal{B}(H) &= 8\pi G\rho(1-3w) + 4\Lambda \end{aligned}$$

that can be rearranged in the form

$$\begin{aligned} \mathcal{B}(H) &= (1-3w)\mathcal{A}(H) + 3(1+w)\Lambda \\ \rho &= \frac{1}{8\pi G}\left(\mathcal{A}(H) - \Lambda\right) \end{aligned}$$

de Sitter $a(t) \propto e^{\bar{H}t}$ , $\bar{H} = const.$

power law $a(t) \propto t^p$ , $H = p/t$

dS $\Rightarrow$ $w = -1$ so we reabsorb matter into $\Lambda$

find dS only for discrete values of $\xi$ (for $n = 2, 3, 6, \ldots, 10$)

PL $\Rightarrow$ $p = p(\xi; \tilde{g}_i^*, w)$

for $n = 2, 3, 6, \ldots, 10$ $\exists \xi : p \to \infty$

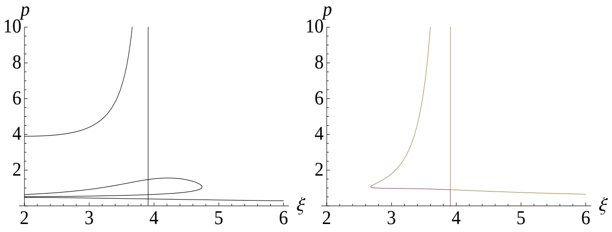



Figure: n=8

# B. Reliability analysis



define $f(R) = k^2 \tilde{f}(\tilde{R})$ , $\tilde{f}(\tilde{R}) = \sum_{i=1}^{n} \tilde{f}_i \tilde{R}^i$

$\tilde{f}^*(\tilde{R})\Big|_n$ reliable if it differs from $\tilde{f}^*(\tilde{R})\Big|_{n+1}$ by less than 5%

$$\tilde{R} \lesssim c$$

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $c$ | 0.40 | 0.24 | 0.45 | 1.07 | 1.21 | 0.90 | 0.79 | 0.92 | 1.09 | 1.09 |

$$\Rightarrow \quad c\xi^2 \gtrsim 6\left(2 + \dot{H}/H^2\right) = 6\,(2 - 1/p)$$

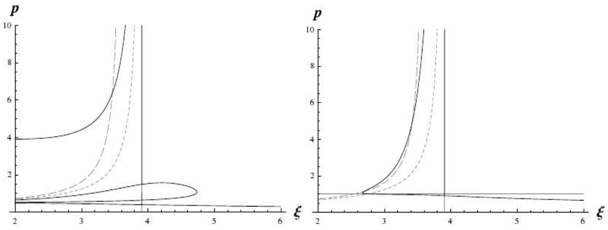



Figure: n=8

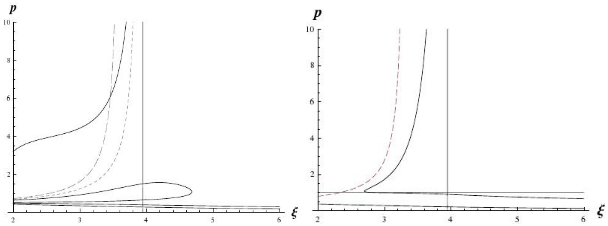



Figure: n=10

# Final Remarks

**1)** conservation law

define $\nabla_\mu = \tilde{\nabla}_\mu + \hat{\nabla}_\mu$ and $E_{\mu\nu} = -\frac{1}{\sqrt{|g|}} \frac{\delta \Gamma_k}{\delta g^{\mu\nu}}$

generalized Bianchi id.: $\tilde{\nabla}_\mu E^\mu{}_\nu = 0 \quad \Rightarrow$

$$\nabla_\mu E^\mu{}_\nu = -\frac{1}{2} \hat{\nabla}_\nu F(R) = -\frac{1}{2} \frac{\nabla_\nu k}{k} \sum \beta_{g_i} R^i$$

continuity equation: $\dot{\rho} + 3H(\rho + p) = \mathcal{P}$

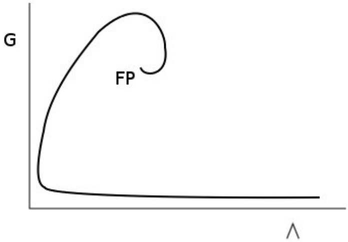
$$\mathcal{P} = -\frac{1}{8\pi G}\left[\dot{\Lambda} - \Lambda \frac{\dot{G}}{G} - \frac{1}{2}\left(\dot{f} - f'\dot{R}\right) + \frac{1}{2} f \frac{\dot{G}}{G}\right]$$

# 2)

graceful exit

RG trajectory is chosen in such a way that, at late limes, (dimensionful) couplings approach observed values

e.g.: Einstein-Hilbert truncation




when the trajectory exits the FP, inflation ends and the Universe evolves "classically"

(Bonanno, Reuter '07)

# CONCLUSIONS

- Increasing powers of $R$ seem to be a reliable, asymptotically safe truncation of the gravitational effective action
- A dynamical identification of the cutoff can account for solutions with varying energy scale (non-dS)
- Cosmological solutions describe accelerated expansion of the (power-law) scale factor $a(t)$
- Solutions become more and more stable against the widening of the truncation