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Unlensing the CMB: a real space approach to extract the weak lensing potential

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Lensing of the CMB photons along the line of sight

The lensing potential ψ deflects the photons by $\boldsymbol{\alpha}(\boldsymbol{\theta}) = \nabla\psi$, which amounts to remaping the temperature anisotropy according to

$$\tilde{T}(\boldsymbol{\theta}) = T(\boldsymbol{\theta} + \nabla\psi) = T(\boldsymbol{\theta}) + \nabla\psi \cdot \nabla T(\boldsymbol{\theta}) + O[(\nabla\psi)^2].$$

For a line element

$$ds^2 = a^2(\eta)\left[-(1-2\Psi)d\eta^2 + (1+2\Psi)\left[dr^2 + f_k^2(r)\left(d\theta_x^2 + d\theta_y^2\right)\right]\right]$$

the lensing potential is given by

$$\psi(\boldsymbol{\theta}) = -2\int_0^{r_{LS}} dr\,\frac{f_k(r_{LS}-r)}{f_k(r_{LS})\,f_k(r)}\Psi(r\boldsymbol{\theta}, -r).$$

Relevance of lensing reconstruction for cosmology:

- probe the full-sky large scale structure distribution.
- allow to recover the primordial B-mode predicted by some inflationary models, of which lensing is major contaminant.

Estimator of the lensing potential (α)

Lensing effects are apparent in the power spectrum

$$\left\langle \tilde{T}(\boldsymbol{\ell}') \; \tilde{T}(\boldsymbol{\ell}' - \boldsymbol{\ell}) \right\rangle = \delta(\boldsymbol{\ell}) \; C^{TT}_{\ell'} + \boldsymbol{\ell} \cdot \left[\boldsymbol{\ell}' \; C^{TT}_{\ell'} + (\boldsymbol{\ell} - \boldsymbol{\ell}') \; C^{TT}_{|\boldsymbol{\ell} - \boldsymbol{\ell}'|} \right] \psi(\boldsymbol{\ell}).$$

An estimator for ψ is a weighted average of the $\boldsymbol{\ell} \neq 0$ term

$$\hat{\psi}(\boldsymbol{\ell}, \boldsymbol{\ell}') = \frac{\tilde{T}(\boldsymbol{\ell}') \; \tilde{T}(\boldsymbol{\ell} - \boldsymbol{\ell}')}{\left[\boldsymbol{\ell} \cdot \boldsymbol{\ell}' \; C_{\ell'} + \boldsymbol{\ell} \cdot (\boldsymbol{\ell} - \boldsymbol{\ell}') \; C_{|\boldsymbol{\ell} - \boldsymbol{\ell}'|} \right]}.$$

The optimal estimator is the convolution of $\tilde{T}(\boldsymbol{\ell})$ by $Q^{\psi}(\boldsymbol{\ell}, \boldsymbol{\ell}')$

$$\hat{\psi}(\boldsymbol{\ell}) = \int d^2\boldsymbol{\ell}' \; \tilde{T}(\boldsymbol{\ell}') \; \tilde{T}(\boldsymbol{\ell} - \boldsymbol{\ell}') \; Q^{\psi}(\boldsymbol{\ell}, \boldsymbol{\ell}')$$

where

$$Q^{\psi}(\boldsymbol{\ell}, \boldsymbol{\ell}') = \mathcal{N}_{\ell} \; \frac{1}{2} \frac{\boldsymbol{\ell} \cdot \boldsymbol{\ell}' C_{\ell'} + \boldsymbol{\ell} \cdot (\boldsymbol{\ell} - \boldsymbol{\ell}') C_{|\boldsymbol{\ell} - \boldsymbol{\ell}'|}}{\tilde{C}_{\ell'} \tilde{C}_{|\boldsymbol{\ell} - \boldsymbol{\ell}'|}}$$

$$\mathcal{N}_{\ell} = \left[\int \frac{d^2\boldsymbol{\ell}'}{(2\pi)^2} \; \frac{1}{2} \frac{[\boldsymbol{\ell} \cdot \boldsymbol{\ell}' \; C_{\ell'} + \boldsymbol{\ell} \cdot (\boldsymbol{\ell} - \boldsymbol{\ell}') \; C_{|\boldsymbol{\ell} - \boldsymbol{\ell}'|}]^2}{\tilde{C}_{\ell'} \; \tilde{C}_{|\boldsymbol{\ell} - \boldsymbol{\ell}'|}} \right]^{-1} \equiv C^{\hat{\psi}(\ell)}_{\ell}.$$

Estimator of the lensing potential (β)

Expression can be straightforwardly modified to include detector noise and finite beam width by replacing

$$\tilde{C}_{\ell'} \rightarrow \tilde{C}_{\ell'} + N_{\ell'} .$$

The noise power spectrum is the inverse-sum of the detector noise $n_i(\ell)$ of each channel i

$$N_\ell = \left(\sum_{i=0}^{\texttt{num\_chann}} \frac{1}{n_i(\ell)} \right)^{-1}$$

and $n_i(\ell)$ includes both the white noise amplitude and the beam profile attenuation factor

$$n_i(\ell) = \left(\theta_{fwhm_i} \sigma_{pix_i}\right)^2 \exp[+(\theta_{fwhm_i})^2 \ell(\ell+1)/(8 \ln 2)].$$

Do we need a new estimator?

Optimal reconstruction in harmonic space implicitly assumes full-sky coverage without:

- galactic cuts,
- bad pixels due to excision of point sources,
- nonuniform weighting for uneven sky coverage.

[Okamoto and Hu, Phys.Rev.D67, 083002 (2003)]

More sophisticated approaches based on a maximum likelihood estimator [Smith, Zahn and Doré, Phys.Rev.D76, 043510 (2007)] treat inhomogeneous sky-coverage in harmonic space [Hanson, Rocha and Górski, MNRAS 400, 2169 (2009)].

We consider a slightly less optimal estimator which:

- has compact support,
- acts in real space.

[Carvalho and Moodley, arXiv:1005.4288]

Measuring the lensing potential from the CMB

Three ways of describing the lensing distortion of the CMB:

- the lensing potential ψ,
- the deflection vector $\boldsymbol{\alpha} = \nabla\psi$,
- the convergence tensor $\boldsymbol{\kappa} = -\nabla\nabla\psi/2$ decomposed as

$$\boldsymbol{\kappa} = \begin{pmatrix} \kappa_0 + \kappa_+ & \kappa_\times \\ \kappa_\times & \kappa_0 - \kappa_+ \end{pmatrix}.$$

The descriptions of ψ and $\boldsymbol{\alpha}$ suffer from an ambiguity upon translation, since a patch of the sky and its translation have the same likelihood on account of isotropy. In contrast, the description of the convergence is locally well defined.

Reconstruct $\kappa_0(\boldsymbol{\theta}) = -\nabla^2\psi(\boldsymbol{\theta})/2$ with estimator $\hat{\kappa}_0(\boldsymbol{\ell}) = \ell^2\ \hat{\psi}(\boldsymbol{\ell})$

A new estimator for the lensing potential (α)

An estimator for the convergence in real space

$$\hat{\kappa}_0(\boldsymbol{\theta}) = \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2} \exp[i\boldsymbol{\ell}\cdot\boldsymbol{\theta}] \int d^2\boldsymbol{\ell}' \; \tilde{T}(\boldsymbol{\ell}') \; \tilde{T}(\boldsymbol{\ell}-\boldsymbol{\ell}') \; Q(\boldsymbol{\ell},\boldsymbol{\ell}')$$
$$\equiv \int d^2\boldsymbol{\theta}' \; \tilde{T}(\boldsymbol{\theta}') \int d^2\boldsymbol{\theta}'' \; \tilde{T}(\boldsymbol{\theta}'') \; Q(\boldsymbol{\theta},\boldsymbol{\theta}',\boldsymbol{\theta}''),$$

where we define the weight function in real space by

$$Q(\boldsymbol{\theta},\boldsymbol{\theta}',\boldsymbol{\theta}'') = \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2} \exp[i\boldsymbol{\ell}\cdot\boldsymbol{\theta}]$$
$$\times \int \frac{d^2\boldsymbol{\ell}'}{(2\pi)^2} \exp[-i\boldsymbol{\ell}'\cdot\boldsymbol{\theta}'] \exp[-i(\boldsymbol{\ell}-\boldsymbol{\ell}')\cdot\boldsymbol{\theta}''] \; Q(\boldsymbol{\ell},\boldsymbol{\ell}').$$

and $Q(\boldsymbol{\ell},\boldsymbol{\ell}') = \ell^2 Q^{\psi}(\boldsymbol{\ell},\boldsymbol{\ell}')$.

A new estimator for the lensing potential (β)

$Q(\boldsymbol{\ell}, \boldsymbol{\ell}')$ is a function of the lengths and the angle $\xi_{\boldsymbol{\ell}} = \phi_{\boldsymbol{\ell}} - \phi_{\boldsymbol{\ell}'}$ between ℓ and ℓ'. Hence we expand in terms of eigenfunctions

$$Q(\boldsymbol{\ell}, \boldsymbol{\ell}') = \sum_{m=-\infty}^{+\infty} \exp[im\xi_{\boldsymbol{\ell}}]\; Q_m(\ell, \ell')$$

$$Q_m(\ell, \ell') = \frac{1}{2\pi} \int d\xi_{\boldsymbol{\ell}}\; \exp[-im\xi_{\boldsymbol{\ell}}]\; Q(\boldsymbol{\ell}, \boldsymbol{\ell}').$$

We find that

$$Q(\theta', \theta'', \xi_{\boldsymbol{\theta}}) \equiv \sum_{m=-\infty}^{+\infty} \exp[im\xi_{\boldsymbol{\theta}}]\; Q_m(\theta', \theta'', \xi_{\boldsymbol{\theta}})$$

where $\xi_{\boldsymbol{\theta}} = \phi_{\boldsymbol{\theta}'} - \phi_{\boldsymbol{\theta}''}$ is the angle between $\boldsymbol{\theta}'$ and $\boldsymbol{\theta}''$, and

$$Q_m(\theta', \theta'', \xi_{\boldsymbol{\theta}}) = \frac{1}{(2\pi)^2} \int_0^{\infty} d\ell\; \ell\; J_m(\ell\theta'') \times \int_0^{\infty} d\ell'\; \ell'\; J_m(\ell'(\theta' - \theta'' \cos[\xi_{\boldsymbol{\theta}}]))\; Q_m(\ell, \ell').$$

Bias of the estimator

To the convergence map contribute:

- the convergence of the lensing potential
- the convergence of the unlensed CMB (Gaussian noise)

$$\hat{\kappa}_0 = \kappa_0|_\psi + \kappa_0|_{\psi=0}.$$

To remove bias of the estimated map:

- take pixel average $\langle \hat{\kappa}_0 \rangle$ from a single realization of the sky and subtract $\Rightarrow$ Gaussian noise averages out $\Rightarrow$ $\langle \hat{\kappa}_0 \rangle = \langle \kappa_0|_\psi \rangle$.

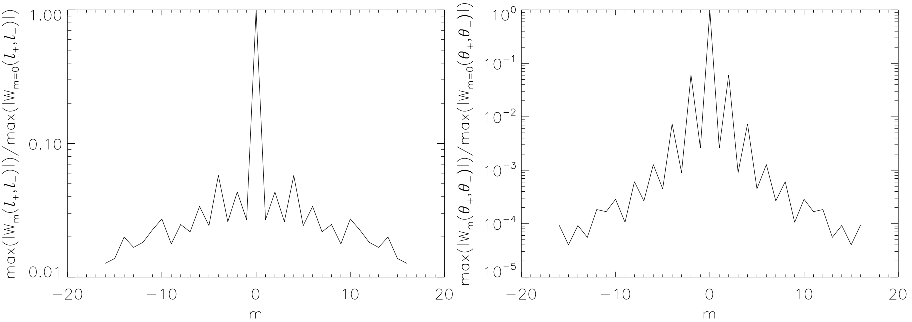
To remove bias of the estimated power spectrum:

- compute $\left\langle \kappa_0|_{\psi=0} \; \kappa_0|^*_{\psi=0} \right\rangle$ and subtract average variance of the unlensed Gaussan noise over several realizations of the CMB $\Rightarrow \left\langle \hat{\kappa}_0 \; \hat{\kappa}_0^* \right\rangle = \left\langle \kappa_0|_\psi \; \kappa_0|^*_\psi \right\rangle$.

Some technical details: number of m's

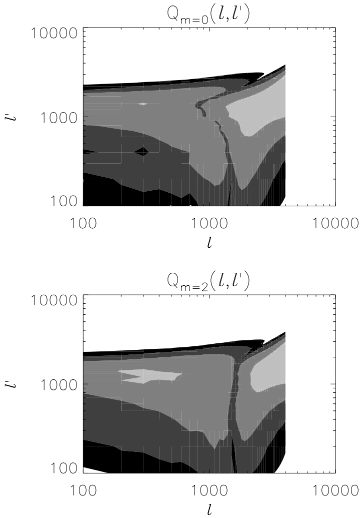
A TECHNICAL NOTE: We change to coordinates $(\boldsymbol{\ell}_+, \boldsymbol{\ell}_-)$ such that $\boldsymbol{\ell} = \boldsymbol{\ell}_+$ and $\boldsymbol{\ell}' = (\boldsymbol{\ell}_+ + \boldsymbol{\ell}_-)/2$.

$max(W_m(\ell_+, \ell_-))/max(W_{m=0}(\ell_+, \ell_-))$ and
$max(W_m(\theta_+, \theta_-))/max(W_{m=0}(\theta_+, \theta_-))$ versus m :

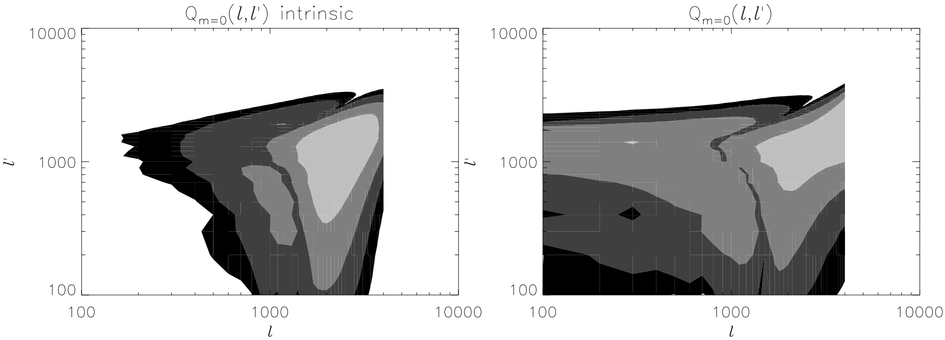


Some technical details: compactness of the kernel (α)

$$Q_m(\ell, \ell') = \frac{1}{2\pi} \int d\xi_\ell \; \exp[-im\xi_\ell] \; Q(\boldsymbol{\ell}, \boldsymbol{\ell}'), \quad \texttt{m\_max} = 16$$



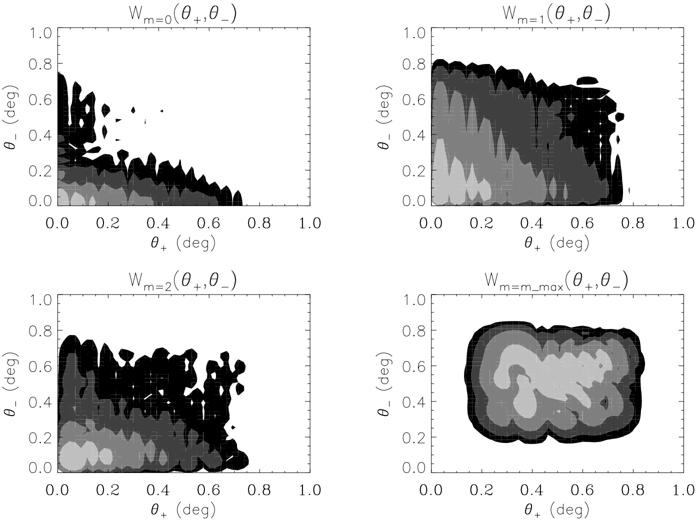
$$Q(\ell,\ell') = \ell^2 \mathcal{N}_\ell \, \frac{1}{2} \frac{\ell\cdot\ell' C_{\ell'} + \ell\cdot(\ell-\ell') C_{|\ell-\ell'|}}{\tilde{C}_{\ell'} \tilde{C}_{|\ell-\ell'|}} = \mathcal{N}_\ell \; Q^{intrinsic}(\ell,\ell')$$



The estimator variance $\mathcal{N}_\ell$ weights the different ℓ modes in such a way as to enhance the contribution of the highest ℓ modes accessible.

Some technical details: compactness of the kernel (γ)

$$W_m(\theta_+, \theta_-) = \frac{1}{2\pi} \int d\chi_{\boldsymbol{\theta}} \; \exp[-im\chi_{\boldsymbol{\theta}}] \; W(\boldsymbol{\theta}_+, \boldsymbol{\theta}_-), \qquad \texttt{num\_m} = 16$$

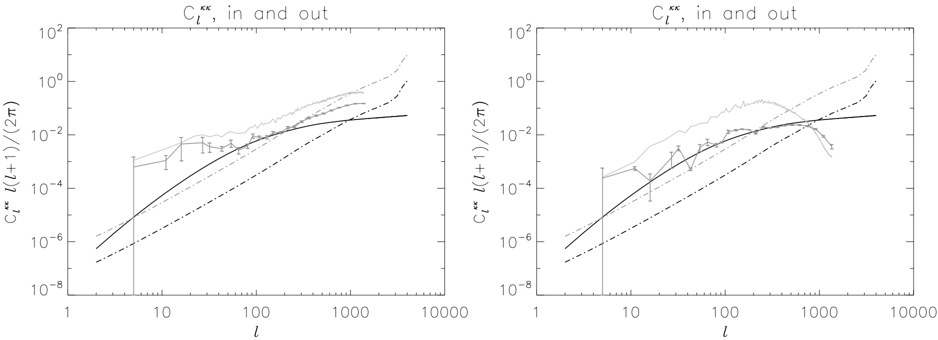


My little pipeline

1. Synthesise lensed map by pixel remapping, apodize, convolve with beam and add detector noise [$\theta_{fwhm} = 7.8'$, $FOV_{map} = 67^\circ$ and $\sigma_{pix} = 6.8\ f_{sky}^{1/2} \mu\mathrm{K}/\mathrm{rad}$ (PLANCK noise at $\nu = 143$ GHz)].
2. Deconvolve the lensed map with beam $\Leftrightarrow$ deconvolve the kernel in harmonic space.
3. Generate kernel in real space.
4. Convolve the lensed map with the kernel and subtract average value.
5. Compute the power spectrum $C_\ell^{\kappa\kappa}$ and subtract average power spectrum from unlensed realizations of the CMB.

Results for noiseless Planck-like experiment

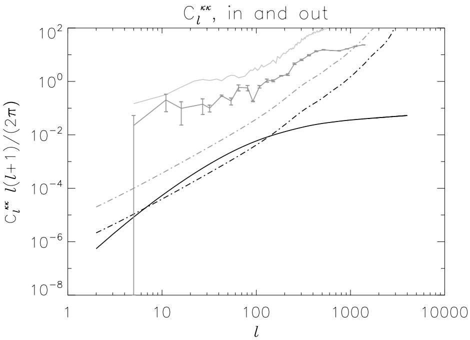
Harmonic space estimator — Real space estimator



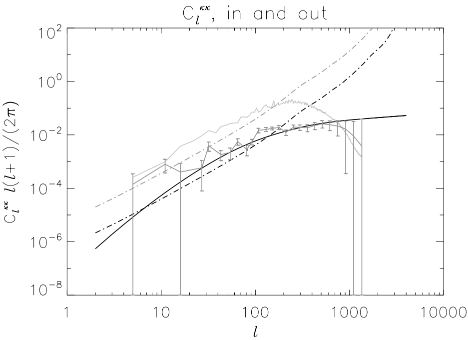
- Black solid: Input power spectrum
- Gray solid, light and dark: Output power spectrum (reconstructed) before and after removal of bias
- Black dashed-dotted: Total theoretical variance of the estimator

Results for noisy Planck-like experiment

Harmonic space estimator



Real space estimator

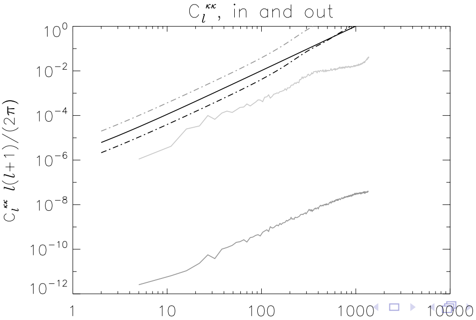


- Black solid: Input power spectrum
- Gray solid, light and dark: Output power spectrum (reconstructed) before and after removal of the bias
- Black dashed-dotted: Total theoretical variance of the estimator

Observations

Estimate of the convergence in each pixel is the sum of product of pairs of neighbouring pixels weighted by the kernel.

- Loss of power at small scales: in theory $\ell_{max} \sim 1/\theta_{fwhm}$ but in practice limited by $1/\theta_{kernel}$ which measures the smallest wavelengths probed by the kernel $\Rightarrow$ loss of recovered power on angular scales smaller than θ_{kernel}.
- No sensitivity to experimental noise: test with white noise only $\Rightarrow$ noise, independent in each pixel, is averaged out.



In progress...

Next step:

- apply to maps with excisions of points
- probe smaller angular scales ($\ell = 4000$) as expected for the Atacama Cosmology Telescope

Next studies:

- develop analogous estimator for the shear components of the convergence tensor
- complement estimation with the polarization field to optimize the reconstruction from Planck