Observational consequences of Loop Quantum Cosmology

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- LQG = non-perturbative and background-independent quantization of GR which agree with GR at large scale.
- GR with Ashtekar variables : SU(2) valued connections and conjugate densitized triads
  $\rightarrow$ the theory is similar to a gauge theory
  $\rightarrow$ methods of Quantum field theory with an Hamiltonian formulation
  no metric dependance
  $\rightarrow$ constraints (WDW), no external time
- Quantization through holonomies and fluxes of densitized triads
- adding symetries (FLRW metric) at the universe leads to LQC

$$ds^2 = dt^2 - a^2(t) \cdot d\vec{x}.d\vec{x}$$

Some advantage of LQC

- using holonomies : modified Friedmann equation

$$H^2 = \frac{\kappa}{3} \cdot \rho \cdot \left(1 - \frac{\rho}{\rho_c}\right) \tag{1}$$

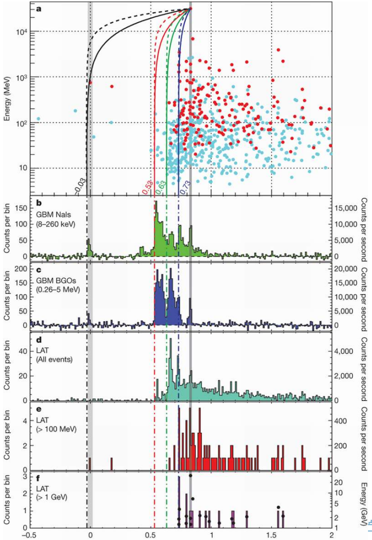
  $\rightarrow$ solve the initial singularity problem by a bounce (finite volume)

- LQC can lead somehow naturally to an inflation phase
  $\rightarrow$ solve the problem of horizon and flatness

$\rightarrow$ bring questions : What in the other side ? Is it deterministic ? ....

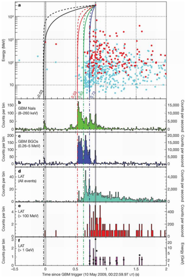
How to test such a theory ?

- break of the Lorentz invariance
  (Amelino-Camelia *et al*, Nature **462** (2008))



How to test such a theory ?

- Lorentz invariance measure



amelino-camelia *et al* (2008)

- Quantum perturbations during the bounce on
  - scalar perturbations (see : Bojowald,...)
  - tensorial perturbations

$\rightarrow$ fingerprints on the CMB (PLANCK, BPOL, BICEP, ...)

$$\frac{d^2}{d\eta^2}h_a^i + 2aH\frac{d}{d\eta}h_a^i - \nabla^2 h_a^i + m_Q^2 h_a^i = 0, \tag{2}$$

where h_a^i are gravitational perturbations, η is the conformal time and the factor due to the holonomy corrections is given by

$$m_Q^2 := 16\pi G a^2 \frac{\rho}{\rho_c}\left(\frac{2}{3}\rho - V\right). \tag{3}$$

Considering the Tensor Power Spectrum given by

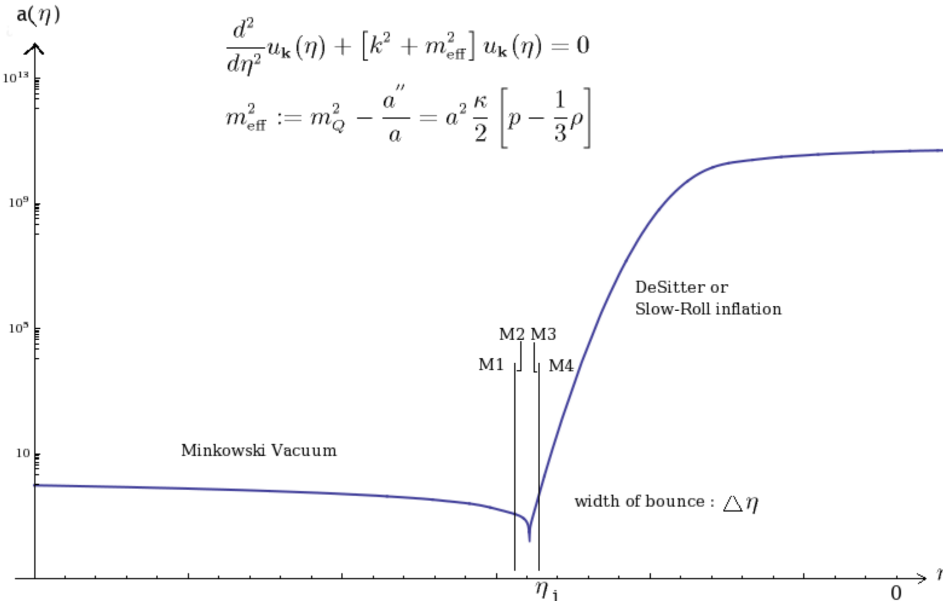
$$\langle 0|\hat{h}_b^a(\mathbf{x},\eta)\hat{h}_a^b(\mathbf{y},\eta)|0\rangle = \int_0^\infty \frac{dk}{k}\mathcal{P}_{\mathrm{T}}(k,\eta)\frac{\sin kr}{kr}, \tag{4}$$

$$\mathcal{P}_{\mathrm{T}}(k,\eta) \approx \frac{64\pi G}{a^2(\eta)}\frac{k^3}{2\pi^2}|h_k(\eta)|^2. \tag{5}$$

Analytical model

$$\frac{d^2}{d\eta^2}u_{\mathbf{k}}(\eta) + \left[k^2 + m_{\rm eff}^2\right] u_{\mathbf{k}}(\eta) = 0$$

$$m_{\rm eff}^2 := m_Q^2 - \frac{a''}{a} = a^2\,\frac{\kappa}{2}\left[p - \frac{1}{3}\rho\right]$$



$$\mathbf{M_1} = \begin{bmatrix} \frac{e^{-ik(\eta_i-\Delta\eta)}}{\sqrt{2k}} & \frac{e^{ik(\eta_i-\Delta\eta)}}{\sqrt{2k}} \\ -i\sqrt{\frac{k}{2}}e^{-ik(\eta_i-\Delta\eta)} & i\sqrt{\frac{k}{2}}e^{ik(\eta_i-\Delta\eta)} \end{bmatrix}, \quad (6)$$

$$\mathbf{M_2} = \begin{bmatrix} \frac{e^{-i\Omega(\eta_i-\Delta\eta)}}{\sqrt{2\Omega}} & \frac{e^{i\Omega(\eta_i-\Delta\eta)}}{\sqrt{2\Omega}} \\ -i\sqrt{\frac{\Omega}{2}}e^{-i\Omega(\eta_i-\Delta\eta)} & i\sqrt{\frac{\Omega}{2}}e^{i\Omega(\eta_i-\Delta\eta)} \end{bmatrix}, \quad (7)$$

$$\mathbf{M_3} = \begin{bmatrix} \frac{e^{-i\Omega\eta_i}}{\sqrt{2\Omega}} & \frac{e^{i\Omega\eta_i}}{\sqrt{2\Omega}} \\ -i\sqrt{\frac{\Omega}{2}}e^{-i\Omega\eta_i} & i\sqrt{\frac{\Omega}{2}}e^{i\Omega\eta_i} \end{bmatrix} \quad (8)$$

$$\mathbf{M_4} = \text{function of} \quad \sqrt{-\eta}\sqrt{\frac{\pi}{4}}e^{i\pi(2\nu+1)/4}H_\nu^{(1)}(-k\eta) \quad (9)$$

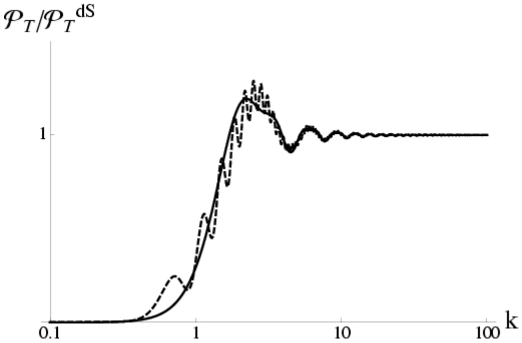
where

$$\Omega = \sqrt{k^2 + k_0^2}. \quad (10)$$

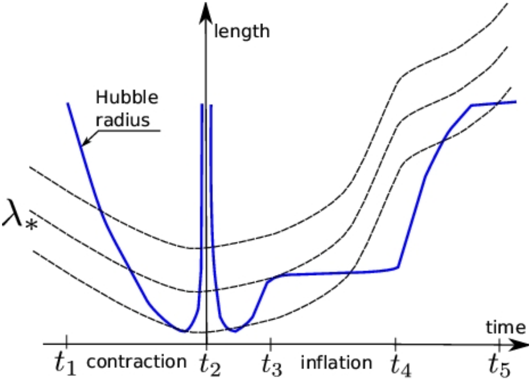
$$k_0^2 \approx m_{eff}^2(\eta_{bounce}) \quad (11)$$

analytical tensorial power spectrum at the end of Inflation

- suppressed power in IR : feature of the bounce
- UV behavior as standard GR picture
- oscillations between these two areas
- huge oscillation = bump
- $k_0 \rightarrow$ amplitude of oscillations
  $\Delta\eta \rightarrow$ frequency of oscillations



tricky horizon history

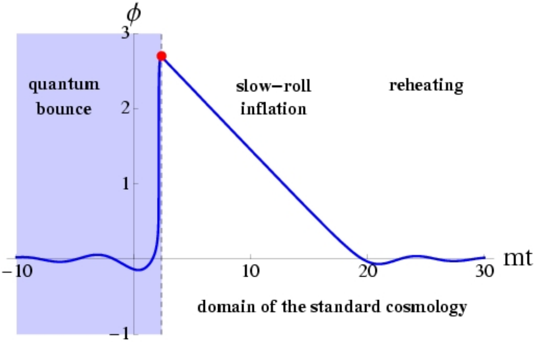


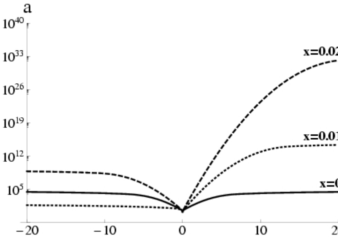
- $R_H = \frac{1}{H}$
- $\lambda_* = a(t)\frac{2\pi}{k_*}$

$$\ddot{\Phi} + 3 \cdot H \cdot \dot{\Phi} + m^2 \cdot \Phi = 0$$

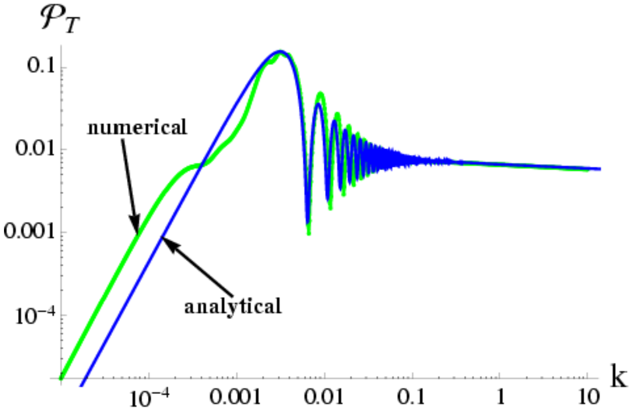
$$H^2 = \frac{\kappa}{3} \cdot \rho \cdot \left(1 - \frac{\rho}{\rho_c}\right)$$

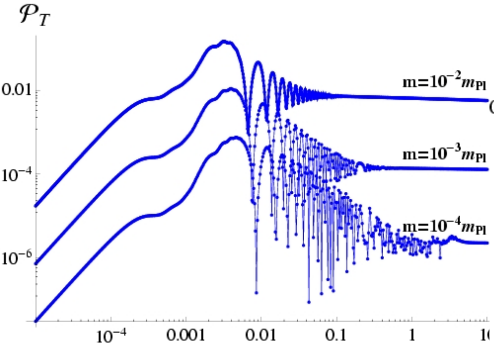
Shark fin scenario

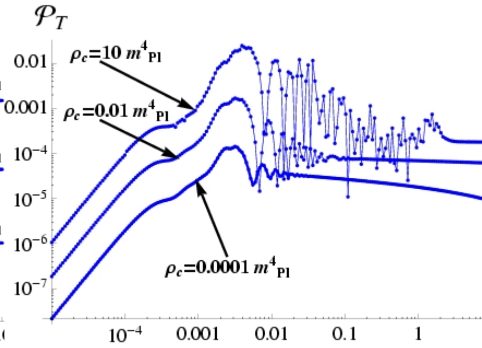


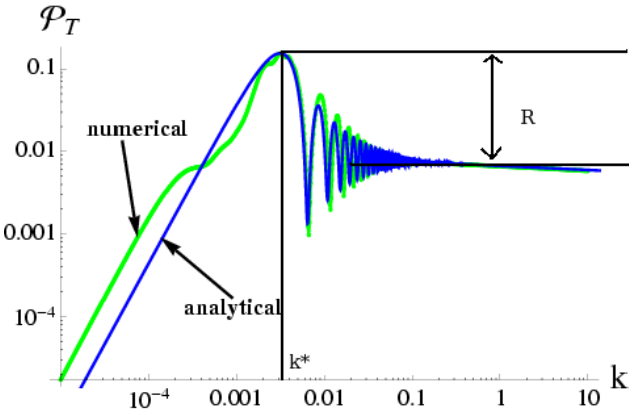


Numerical simulations




$\mathcal{P}_T$
0.01
10^{-4}
10^{-6}
$m=10^{-2}m_{Pl}$
$m=10^{-3}m_{Pl}$
$m=10^{-4}m_{Pl}$
10^{-4}
0.001
0.01
0.1
1

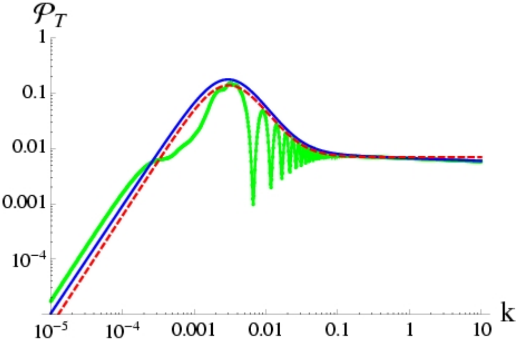

$\mathcal{P}_T$
0.01
0.001
10^{-4}
10^{-5}
10^{-6}
10^{-7}
$\rho_c=10\ m^4_{Pl}$
$\rho_c=0.01\ m^4_{Pl}$
$\rho_c=0.0001\ m^4_{Pl}$
10^{-4}
0.001
0.01
0.1
1



$\mathcal{P}_T$
0.1
0.01
0.001
10^{-4}
numerical
analytical
R
k*
10^{-4}
0.001
0.01
0.1
1
10
k

Approximation of the spectrum

$$\mathcal{P}_\mathrm{T} = \frac{16}{\pi}\left(\frac{H}{m_\mathrm{Pl}}\right)^2 \frac{\left(\frac{k}{aH}\right)^{-2\epsilon}}{1+(k_*/k)^2}\left[1+\frac{4R-2}{1+(k/k_*)^2}\right], \qquad (12)$$



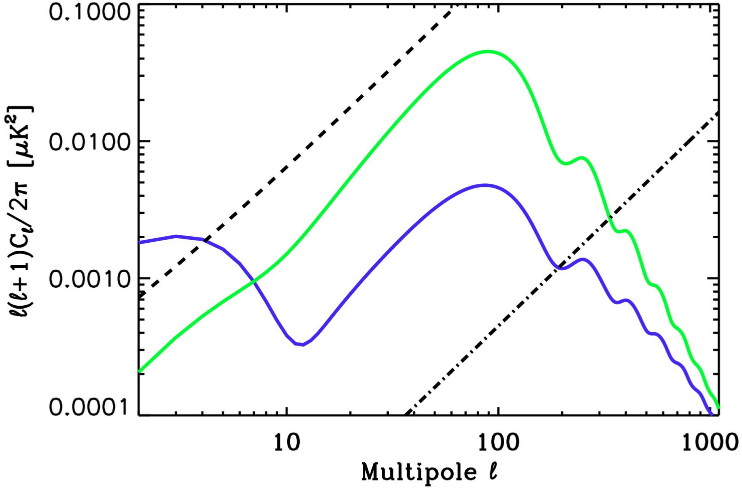


FIGURE: Julien Grain (IAS Orsay) via CAMB

conclusion

- take into account the back reaction
- compute the scalars perturbations in LQC
- compare with the datas (in progress)

$\rightarrow$ validate or not the theory of Loop Quantum Gravity

$\rightarrow$ thus give informations on fondamental parameters $(m_\phi, \phi_{max}, \rho_c, ...)$

But we have to calculate them (scalar)
or observe (tensor) which implies a huge experimental challenge.

Thank You !